AN INTEGER PROGRAMMING APPLICATION TO THE CAPITAL BUDGETING PROBLEM IN A LARGE MULTINATIONAL FIRM IN METRO MANILA*

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THIS STUDY concerns endeavors to formulate an integer programming model for a certain capital budgeting problem. The company considered is contemplating replacement of one or more of its production lines by new lines which would be able to produce the required amounts with lower costs. For this purpose, the present values of existing lines and new possible lines have been estimated, and a comparison has been carried out in the 0-1 integer programming model. The results obtained, with the help of Branch and Bound technique, specify clearly which lines are to be replaced and which may be closed entirely, and the exact monetary benefits to be accrued quantitatively.

Capital budgeting is the allocation of financial resources to various possible investments. It requires the selection from different profitable investments, each having a cost/benefit pattern in future time periods, of a combination of investments that will maximize profit while remaining within cost budgets established for each of the future time periods. The decision to the problem may be based on the payback period (length of time required for the initial capital investments to be recovered from the incremental cash flow of the proposal), accounting rates of return (evaluates the profitability of the proposal), net present value (based on the concept of cash flow analysis and time value of money), and internal rate of return (equates the present value of the net cash inflows to the original investments).

In a real-world situation, especially in a complicated business world, these techniques are sometimes not enough in justifying an investment. Some companies would have to consider constraints which may alter the decision but cannot be observed by the above methods. One possible way of treating both theoretical and practical considerations is the use of linear programming. Linear programming, specifically integer programming (IP), can determine the optimal set of investment proposals subject to certain constraints.

In a typical, practical capital budgeting situation, the objective may be to maximize the net present value or the internal rate of return, subject to the constraints of cash available for investment, limitations for particular proposals, and other constraints.

There are different techniques in solving a capital budgeting problem. However, in a replacement problem, where a "yes-or-no" decision is required, integer programming is needed.

Mathematical Formulation

Consider a company planning its capital spending for the replacement of its production lines. There are N machines being considered for replacement. The existing production lines are represented by the subscripts 1, 2, ..., N. On the other hand, the new lines are represented by the subscripts

$$N + 1, N + 2, ..., 2N.$$

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Let $x_j = <$ 1 if the production line j is in operation; 0 otherwise

where j = 1, 1, ..., 2N

Let P_j be the net present value (NPV) of the future (net) cash flows (inflows) from production line 1 to production line 2N. The objective is to maximize total net present value subject to the budget limitations and market demand.

Formulation 1

$$\text{Maximize NPV} = \sum_{j=1}^{2N} P_j x_j$$

subject to

$$\left.\begin{array}{l} x_1 + x_{N+1} = 1 \\ x_2 + x_{N+2} = 1 \\ \quad \vdots \\ x_N + x_{2N} = 1 \end{array}\right\} \text{limit on project acceptance}$$

$$\sum_{j=N+1}^{2N} C_j x_j \le M \quad \text{Budget Availability}$$

$$\sum_{j=1}^{2N} Q_j x_j \ge D \quad \text{Supply/Demand Constraint}$$

Formulation 2

$$\text{Maximize NPV} \sum_{j=1}^{2N} = P_j x_j$$

subject to

$$\left.\begin{array}{l} x_1 + x_{N+1} \le 1 \\ x_2 + x_{N+2} \le 1 \\ \quad \vdots \\ x^N + x^{2N} \le 1 \end{array}\right\} \text{limit on project acceptance}$$

$$\sum_{j=n+1}^{2N} C_j x_j \le M \quad \text{Budget Availability}$$

$$\sum_{j \le 1}^{2N} Q_j x_j \le D \quad \text{Supply/Demand Constraint}$$

where: P_j = net present value of machine over its useful life

x_j = machine/production line j that is in operation

C_j = cash outlay of machine j

Q_j = installed capacity for each machine

M = budget available for replacement

D = market demand

It should be noted that the IP formulation of capital budgeting/problem may have additional constraints aside from those previously indicated above, depending on the resources of the company.

APPLICATION TO COLORS MANUFACTURING COMPANY

The COLORS manufacturing company, a leading personal and home care product manufacturer, produces detergent bars. The company has five production lines/machines. These machines can produce a total of 625 cases of detergent bars per hour. Each case contains 48 480-gram bars; each bar can be sold at P8.50. The average economic life of these machines is 10 years with no resale value if sold at the end of its lifetime. The frequent use of machines increases its variable cost; however, the fixed cost remains constant for all the machines.

Since the company has been experiencing cost overruns in the past years, the management is thinking of replacing some of the machines. A replacement of one machine would mean a 40% reduction in the variable cost but not in the fixed cost, although the same quantity of detergent bars could be produced. Each new machine costs P19,800,000, but the company has set a budget of P41,440,000; hence, it cannot possibly engage in the replacement of all five machines. Now, the management wants to determine which of these machines is/are to be replaced that would satisfy the market demand of P28,500 ton/yr given its variable resources. Also the company is considering the possibility of scrapping machines (that is, using fewer machines for production). Table 1 shows the ages of the machines and the corresponding cost of acquisition, and Table 2 shows the yearly sales and yearly contribution.

TABLE 1
Age and Acquisition Cost of Each Machine

Machine	Age (in years)	Cost of Acquisition (in 000 Pesos)
1	10	9,000
2	8	9,500
3	4	15,000
4	4	15,000
5	7	10,000

TABLE 2
Actual Annual Sales and Contribution Margin

Year	Sales (in P)	Contribution Margin (in P)
1979	173,700	45,800
1980	260,100	62,900
1981	294,100	56,100
1982	323,800	77,900
1983	341,000	610,000
1984	692,000	225,900
1985	628,600	163,300

The problem is solved through integer programming using the *Branch-and-Bound (BAB) Method*. The objective is to select the proper machine to be replaced which will maximize the total net present value of the future cash flows of all the machines in operation constrained by limited capital and market demand.

Certain calculations are required to come up with the integer formulation of the replacement problem. Below are the steps in the formulation of x_j's in the objective function. For the old machines:

1. Project sales and variable cost until 1996 using linear regression:

variable cost = sales revenue - contribution margin

2. Compute cash flow (see Table 3)

Cash Flow = Projected Sales - Projected Variable Cost - Fixed Cost

3. Compute the share of each machine to the total cash flow by multiplying the cash flow of machine j to certain weight W_j which is

$$W_j = \frac{1}{(\text{age of machine } j)} \Big/ \left(\frac{1}{10} + \frac{1}{8} + \frac{1}{4} + \frac{1}{4} + \frac{1}{7}\right)$$

4. Get the present value of individual annual cash flow using 20% as the discount rate

$$\text{Present Value} = \frac{1}{(1 + i)_n}$$

where i = interest rate
n = number of year.

5. Get the total of the present value for each machine. The net present value of each machine now becomes the coefficient of x^J S (j = 1, 2, ..., 5) in the objective function. Table 5 gives the present value of cash flow generated by each machine.

TABLE 3
Projected Sales, Projected Variable Cost and Cash Flows

Year	Projected Sales (in P)	Projected Variable Cost in P	Cash Flows (in P)
1986	633,700	502,400	131,000
1987	709,800	587,700	122,100
1988	849,600	703,500	146,100
1989	1,180,500	977,500	203,000
1990	1,074,100	995,300	78,800
1991	1,160,900	1,079,700	81,200
1992	1,247,600	1,164,100	83,500
1993	1,334,400	1,248,500	85,900
1994	1,421,200	1,322,800	88,400
1995	1,508,000	1,417,200	90,800
1996	1,594,800	1,501,600	93,200

TABLE 4
Share of Each Machine to Cash Flows (in P) Machine

Year	1	2	3	4	5
1986	15,095	18,868	37,737	37,737	21,564
1987	14,070	17,586	35,173	35,173	20,099
1988	16,875	21,043	42,087	42,087	24,049
1989	23,403	29,253	58,507	58,507	33,432
1990	9,080	11,350	22,700	22,700	12,971
1991	9,357	11,695	23,390	23,390	13,366
1992	9,492	12,026	24,054	24,054	13,765
1993	9,898	12,372	24,745	24,745	14,140
1994	10,221	12,232	25,465	25,465	14,552
1995	10,463	13,078	26,157	26,157	14.947
1996	10,739	13,427	26,848	26,848	15,342

TABLE 5
Present Value of Cash Flow Generated by Each Machine (in pesos)

Machine Year	1	2	3	4	5
1986	15,095	18,868	37,737	37,737	21,564
1987	11,725	14,625	29,311	29,311	16,749
1988	11,691	14,613	29,227	29,227	16,700
1989	13,543	16,929	33,858	33,858	19,347
1990	4,379	5,474	10,948	10,948	6,255
1991	3,760	4,700	9.400	9,400	5,371
1992	3,222	4,027	8,057	8,057	4,603
1993	2,762	3,453	6,906	6,906	3,946
1994	2,377	2,961	5,922	5,922	3,384
1995	2,028	2,535	5.069	5,069	2,897
1996	1,734	2,170	4,336	4,336	2,478
TOTAL	73,316	90,385	186,693	186,693	103,294

For the new machines:

1. Compute cash flow. Due to the 40% reduction in the variable cost, only 60% of the variable cost of the old machine is used for the computation.

$$\text{Cash Flow} = \text{Sales Revenue} - \left(\frac{60}{100}\right) \text{variable cost}$$

2. Compute the share of each new machine to the total cash flow.

$$\text{Share of each new machine} = \frac{\text{Annual cash flow}}{5}$$

3. Get the present value of cash flow (d.r. 20%). (See Table 6 for summary.)

4. Get the total of the present values for each machine.

5. Add the NPV of the cash flow for the new machine to the resale value of the corresponding machine to be replaced.

$$\text{Resale Value} = \frac{\text{Cost of Acquisition}}{\text{economic life}} \left(\text{economic life} - \text{age of machine}\right)$$

The result becomes the coefficient of the x^js (j = 6, 7, ..., 10) of the objective function. Table 7 gives us the resale value for each machine.

The formulation of the constraints does not entail too many calculations as in the formulation of the objective function. However, all units must first be converted to kilogram for consistency and simplicity in computation.

The total installed capacity of the machines in operation must at least be equal to the total market demand.

Annual installed capacity (kg/yr)	=	Individual capacities (case/hr)	x	No. of working days (hr/day)	x	No. of working days (day/yr)
			x	No. of bars per case (bar/case)	x	No. of kg. per bar (kg/bar)

The total market demand per year which the company is wanting to satisfy is computed as follows:

total market demand (kg/yr)	=	market demand (ton/yr)	x	equivalent in kilogram (kg/ton)

The capital outlay of each machine for replacement must at most be equal to the total money or capital available to the company. Also, one machine from machines 1 and 6 must be in operation. Only one must be selected from the two machines.

TABLE 6
Present Value of Cash Flow from New Machine (in pesos)

Year	Present Value
1986	66,452
1987	59,530
1988	59,375
1989	68,760
1990	45,999
1991	41,239
1992	36,785
1993	32,669
1994	28,909
1995	23,554
1996	20,797
TOTAL	484,069

TABLE 7
Resale Value for Each Machine (in pesos)

Machine	Resale Value
1	0
2	1,900,000
3	9,000,000
4	9,000,000
5	3,000,000

Since machines 3 and 4 have only been in operation for four years, the company is not willing to replace them. Hence the formulation of the replacement problem is as follows:

Maximize Z

$$72{,}316x_1 - 90{,}385X_2 + 186{,}693x_3 + 186{,}693x_4 + 103{,}294x_5 + 484{,}069x_6 + 2{,}384{,}069x_7 + 9{,}484{,}069x_8 + 9{,}484{,}069x_9 + 3{,}484{,}069x_{10}$$

subject to

$$25{,}228{,}800\,(x_1 + x_2 + \ldots + x_{10}) > 116573300$$

$$19{,}800{,}000\,(x_6 + x_7 + \ldots + x_{10}) > 41440000$$

$x_1 + x_6 = 1$ $\quad$ $x_4 + x_9 = 1$

$x_2 + x_7 = 1$ $\quad$ $x_5 + x_{10} = 1$

$x_3 + x_8 = 1$ $\quad$ $x_8 = 0; x_9 = 0$

This problem was solved using *simplex model.* However, it yielded an unbounded solution. Different interim models were constructed to come up with one that is a true representation of the capital budgeting/ replacement problem of COLORS Manufacturing Company. Below is the summary of interim models constructed.

Interim Model	Description	Solution
1	$x_1 = 0$ us appened to the original model	unbounded
2	Mutually exclusive alternative is changed to a less restrictive constraint	unbounded
3	Projected Sales for the year 1986 was used as the RHS of the Supply/ demand constraint	integer solution
4	Projected Sales for the year 1996 was used as the RHS of the Supply/ demand constraint	integer solution

From interim models 3 and 4, a final model was formulated. Although the company maintained that the installed capacity is the same for both the old and the new machines, the researchers thought it logical to assume that the installed capacities would be different. Table 8 gives the assumed installed capacity for each machine.

TABLE 8
Assumed installed capacities for each machine

Machine	Individual Capacities case/hr	kg/yr
1	96	19375718
2	104	20990362
3	120	24219648
4	120	24219648
5	108	21797683

The final model for the year 1986 is formulated by changing the left hand side of the supply/demand constraint of interim model 3 by the assumed installed capacity of the machine. Also, the final model for the year 1996 was obtained by replacing the left hand side of the supply/demand constraint of interim model 4 with the assumed installed capacities of the machines. Both the two final models gave an integer solution similar to that of the interim models from which they are derived. Following is the summary and interpretation of the final solution.

Decision Variable	Interpretation
$x_1 = 0; x_6 = 0$	the whole line is totally scrapped
$x_2 = 0; x_7 = 1$	the second machine is replaced by a new one
$x_3 = 1; x_8 = 0$	the third machine is retained
$x_4 = 1; x_9 = 0$	the fourth machine is retained
$x_5 = 0; x_{10} = 1$	the fifth machine is replaced by a new one

Conclusion

Zero-one programming, through its "yes-or-no" decision, enabled the researchers to solve the replacement issue of capital budgeting problem. Through the BAB method, the optimal replacement policy in terms of profitability was determined by looking at all the feasible combinations of machines that will yield a maximum net present value. The researchers determined not only the integer solution to the company's replacement problem but also an alternative production policy. Through the use of interim models formulated one after another, the possibility of maintaining four machines instead of the usual five machines for production came out. This means that one production line is removed without replacement.