Manila Journal of Science ***11*** (2018)*, pp. 58–67*

Bound-State Solutions of the Schrödinger Equation with Woods–Saxon Plus Attractive Inversely Quadratic Potential via Parametric Nikiforov–Uvarov Method

Benedict Iserom Ita[1], Nelson Nzeata-Ibe[1], Thomas Odey Magu[1], and Louis Hitler[2],*

[1]Physical/Theoretical Chemistry Research Group, Department of Pure and Applied Chemistry, University of Calabar, Calabar, Cross River State, Nigeria

[2]CAS Key Laboratory for Nanosystem and Hierarchical Fabrication, CAS Centre for Excellence in Nanoscience, National Centre for Nanoscience and Technology, Beijing, China

*Corresponding author:louis@nanoctr.cn

ABSTRACT

We study the bound-state solutions of the Schrödinger equation with Woods–Saxon plus attractive inversely quadratic potential using the parametric Nikiforov–Uvarov method. We obtained the bound-state energy eigenvalues and the corresponding normalized eigenfunctions expressed in terms of hypergeometric functions. Two special cases of this potential are discussed. Numerical values of the energy eigenvalues are also computed for some values of n at $l = 0$ with $\alpha = 0.01$, 0.03, 0.1, 2, and 5 using python 3.6 programming.

Keywords: Bound-state solutions, Schrödinger equation, Nikiforov–Uvarov method, Woods–Saxon potential, attractive inversely quadratic potential

INTRODUCTION

Over the years, theoretical physics and chemistry have been successful in explaining the behavior of different particles in different potentials. This has been made possible through obtaining exact or approximate solutions of the nonrelativistic and relativistic wave equations for different physical systems of interest (Louis et al., 2016). In

nonrelativistic quantum mechanics, one of the interesting problems is to obtain exact solutions of the Schrödinger equation. In order to do this, a real potential is normally chosen to derive the energy eigenvalues and eigenfunctions of the Schrödinger equation (Magu et al., 2017). The Schrödinger equation thereby reveals that the eigenfunctions of the system can furnish us with information regarding the behavior of such a system. Hence, if the system is exactly solvable for a given potential, the wave function can describe such a system completely. Recently, the study of exponential-like-type potentials has attracted much attention from different authors. However, the bound-state solutions of the Schrödinger equation of some of these potentials are possible for few cases such as the Coulomb potential, Woods–Saxon potential, Hulthen potential, Yukawa potential, Hellmann potential, Manning–Rosen potential, Eckart potential, Mie-type, and Morse potential (Ita et al., 2016). Furthermore, our group has attempted to study the bound-state solutions of the Klein-Gordon, Dirac, and Schrödinger equations using combined or mixed potentials. Some of which include the Woods–Saxon plus attractive inversely quadratic (WSAIQ) potential (Ita et al., 2017), Manning–Rosen plus a class of Yukawa potential (Ita et al., 2017), generalized Woods–Saxon plus Mie-type potential (Magu et al., 2017), and Kratzer plus reduced pseudoharmonic oscillator potential (Louis et al., 2016).

The Woods–Saxon potential, either in its spherical or in its deformed form, has been used in nuclear calculations, and this potential has been studied and many works had been published by various scientists (Louis et al., 2016). The inversely quadratic Yukawa potential (IQYP) was first studied in 2012 by Hamzavi et al., when they obtained approximate spin and pseudospin solutions to the Dirac equation with the potential including a tensor interaction. The IQYP expressed as $V_{IQYP} = -\frac{V_0' e^{2\alpha r}}{r^2}$ is related to the inversely quadratic potential (IQP) when $\alpha \to 0$, i.e., $V_{IQP} = -\frac{V_0'}{r^2}$. The IQP has been used by Oyewumi and Bangudu (2002) in combination with a isotropic harmonic oscillator in N-dimension spaces. Since then, several papers on the potential have appeared in the literature (Ita & Ikeuba, 2015)

The main aim of this paper is to use the proposed approximation in Ita and Ikeuba (2015) and the Nikiforov–Uvarov (NU) method to obtain the bound-state solutions of the Schrödinger equation with WSAIQ potential defined as

$$V(r) = \frac{-V_0}{1+ e^{2\alpha r}} - \frac{V_0'}{r^2} \quad (1)$$

where V_0, V_0' are the strength of the potential for the Woods–Saxon and inversely quadratic potentials, respectively; α = screening parameter; and r = radius.

The rest of the paper is organized as follows. In section 2, the parametric NU method is presented. The factorization method is presented in section 3. In section 4, solutions of the radial part of Schrödinger equation with WSAIQ potential is presented. We discuss the results of our work in section 5. Finally, we present a brief conclusion in section 6.

The scientific significance of this research paper includes giving an insight into possible eigensolutions of atoms and molecules moving under the influence of the WSAIQ potential. Secondly, the resulting eigenenergy equations can be used to study the spectroscopy of some selected diatomic atoms and molecules.

REVIEW OF PARAMETRIC NU METHOD

The NU method is based on the solutions of a generalized second-order linear differential equation with special orthogonal functions. The hypergeometric NU method has shown its power in calculating the exact energy levels of all bound states for some solvable quantum systems.

$$\Psi_n''(s) + \frac{\tilde{\tau}(s)}{\sigma(s)}\Psi_n'(s) + \frac{\bar{\sigma}(s)}{\sigma^2(s)}\Psi_n(s) = 0 \tag{2}$$

where σ(s) and $\bar{\sigma}$(s) are polynomials of at most second degree and $\tilde{\tau}$(s) is a first-degree polynomial. For a given potential, the Schrödinger or Schrödinger-like equation is given as

$$\psi''(x) + \big(E - V(x)\big)\psi(x) = 0 \tag{3}$$

where E = total energy and $V(x)$ = potential depth.

The parametric generalization of the NU method is given by the generalized hypergeometric-type equation

$$\Psi''(s) + \frac{c_1 - c_2 s}{s(1 - c_3 s)}\Psi'(s) + \frac{1}{s^2(1 - c_3 s)^2}[-\epsilon_1 s^2 + \epsilon_2 s - \epsilon_3]\Psi(s) = 0 \tag{4}$$

Thus, equation (2) can be solved by comparing it with equation (4), and the following polynomials are obtained:

$$\tilde{\tau}(s) = (c_1 - c_2 s),\ \sigma(s) = s(1 - c_3 s),\ \bar{\sigma}(s) = -\epsilon_1 s^2 + \epsilon_2 s - \epsilon_3 \tag{5}$$

The parameters obtainable from equation (4) serve as important tools to finding the energy eigenvalue and eigenfunctions. They satisfy the following sets of equation respectively:

$$c_2 n - (2n+1)c_5 + (2n+1)\left(\sqrt{c_9} + c_3\sqrt{c_8}\right) + n(n-1)c_3 + c_7 + 2c_3 c_8 + 2\sqrt{c_8 c_9} = 0 \tag{6}$$

$$(c_2 - c_3)n + c_3 n^2 - (2n+1)c_5 + (2n+1)\left(\sqrt{c_9} + c_3\sqrt{c_8}\right) + c_7 + 2c_3 c_8 + 2\sqrt{c_8 c_9} = 0 \tag{7}$$

while the wave function is given as

$$\Psi_n(s) = N_{n,l} s^{c_{12}}(1 - c_3 s)^{-c_{12} - \frac{c_{13}}{c_3}} P_n^{\left(c_{10}-1, \frac{c_{11}}{c_3} - c_{10} - 1\right)}(1 - 2c_3 s) \tag{8}$$

where

$$\begin{aligned} &c_4 = \tfrac{1}{2}(1 - c_1), c_5 = \tfrac{1}{2}(c_2 - 2c_3),\ c_6 = {c_5}^2 + \epsilon_1,\ c_7 = 2c_4 c_5 - \epsilon_2,\ c_8 = {c_4}^2 + \epsilon_3, \\ &c_9 = c_3 c_7 + {c_3}^2 c_8 + c_6,\ c_{10} = c_1 + 2c_4 + 2\sqrt{c_8},\ c_{11} = c_2 - 2c_5 + 2\left(\sqrt{c_9} + c3\sqrt{c_8}\right) \\ &c_{12} = c_4 + \sqrt{c_8},\ c_{13} = c_5 - \left(\sqrt{c_9} + c3\sqrt{c_8}\right) \end{aligned} \tag{9}$$

and P_n is the orthogonal polynomial.

Given that

$$P_n^{(\alpha,\beta)} = \sum_{r=0}^{n} \frac{\Gamma(n+\alpha+1)\Gamma(n+\beta+1)}{\Gamma(\alpha+r+1)\Gamma(n+\beta-r+1)(n-r)!r!} \left(\frac{x-1}{2}\right)^r \left(\frac{x+1}{2}\right)^{n-r} \tag{10}$$

this can also be expressed in terms of Rodriguez's formula:

$$P_n^{(\alpha,\beta)}(x) = \frac{1}{2^n n!}(x-1)^{-\alpha}(x+1)^{-\beta}\left(\frac{d}{dx}\right)^n\left((x-1)^{n+\alpha}(x+1)^{n+\beta}\right) \tag{11}$$

FACTORIZATION METHOD

In a spherical coordinate, the Schrödinger equation with the potential $V(r)$ is given as

$$-\frac{\hbar^2}{2\mu}\left(\frac{1}{r^2}\frac{\partial}{\partial \mathrm{r}}\left(r^2\frac{\partial}{\partial \mathrm{r}}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\square^2}\right)\psi(r,\theta,\phi)+V(r)\psi(r,\theta,\phi) = E\psi(r,\theta,\phi) \tag{12}$$

Using the common ansatz for the wave function as

$$\psi(r,\theta,\phi) = \frac{R(r)}{r}Y_{lm}(\theta,\phi) \tag{13}$$

and substituting equation (13) into equation (12), we obtain the following sets of equations:

$$R_{nl}{}''(r) + \frac{2\mu}{\hbar^2}\left[E - V(r) - \frac{\lambda\hbar^2}{2\mu r^2}\right]R_{nl}(r) = 0, \tag{14}$$

$$\frac{d^2\Theta(\theta)}{d(\theta)^2} + cot\theta\frac{d\Theta(\theta)}{d(\theta)}\left(\lambda - \frac{m^2}{sin^2\theta}\right)\Theta(\theta)=0, \tag{15}$$

$$\frac{d^2\Phi(\phi)}{d(\phi)^2} + m_l{}^2\Phi(\phi) = 0, \tag{16}$$

where $\lambda = l(l+1)$ and $m_l{}^2$ are the separation constants and $Y_{lm}(\theta,\phi) = \Theta(\theta)\Phi(\phi)$ is the solution of equation (15) and equation (16).

SOLUTIONS OF THE RADIAL PART OF SCHRÖDINGER EQUATION WITH WSAIQ POTENTIAL

Substituting the potential of equation (1) into the radial Schrödinger equation of equation (14), we obtain

$$R_{nl}{}''(r) + \frac{2\mu}{\hbar^2}\left[E + \frac{V_0}{1+e^{2\alpha r}} + \frac{V_0{}'}{r^2} - \frac{l(l+1)\hbar^2}{2\mu r^2}\right]R_{nl}(r) \tag{17}$$

It is well known that the Schrödinger equation of equation (17) cannot be solved exactly for $l \neq 0$ by any known method. The way out is to use approximation for the centrifugal term. On this note, we invoke a good approximation for the centrifugal barrier

$$\frac{1}{r^2} = \frac{4\alpha^2}{(1+ e^{2\alpha r})^2}, \tag{18}$$

similar to other related work.[2, 5, 6, 14]

By substituting equation (18) into equation (17) and using the transformations = $s(r) = -e^{2\alpha r}$, we have

$$R_{nl}{}''(s) + \frac{(1-s)}{(1-s)s} R_{nl}{}'(s) + \frac{1}{s^2(1-s)^2}\left[\frac{\mu E}{2\alpha^2\hbar^2}s^2 + \left(-\frac{2\mu E}{2\alpha^2\hbar^2} - \frac{\mu V_0}{2\alpha^2\hbar^2}\right)s + \frac{\mu E}{2\alpha^2\hbar^2} + \frac{\mu V_0}{2\alpha^2\hbar^2} + \frac{2\mu V_0{}'}{\hbar^2} - \lambda\right] R_{nl}(s) = 0 \tag{19}$$

$$let\ \beta^2 = \frac{-\mu E}{2\alpha^2\hbar^2};\ B = \frac{\mu V_0}{2\alpha^2\hbar^2};\ C = \frac{2\mu V_0{}'}{\hbar^2} - \lambda \tag{20}$$

$$R_{nl}{}''(s) + \frac{(1-s)}{(1-s)s} R_{nl}{}'(s) + \frac{1}{s^2(1-s)^2}[-\beta^2 s^2 + (2\beta^2 - B)s + (-\beta^2 + B + C)]R_{nl}(s)=0 \tag{21}$$

$$R_{nl}{}''(s) + \frac{(1-s)}{(1-s)s} R_{nl}{}'(s) + \frac{1}{s^2(1-s)^2}[-\beta^2 s^2 + (2\beta^2 - B)s - (\beta^2 - B - C)]R_{nl}(s)=0 \tag{22}$$

$$\epsilon_1 = -\beta^2$$

$$\text{where}\ \ \epsilon_2 = 2\beta^2 - B \tag{23}$$

$$\epsilon_3 = \beta^2 - B - C$$

Comparing equation (4) with equation (23), we obtain the following parameters:

$$\begin{aligned} &c_1 = c_2 = c_3 = 1,\ c_4 = 0,\ c_5 = -\frac{1}{2},\ c_6 = \frac{1}{4} + \beta^2,\ c_7 = (-2\beta^2 + B),\ c_8 = \beta^2 - B - C \\ &c_9 = \frac{1}{4} - C,\ c_{10} = 1 + 2\sqrt{\beta^2 - B - C},\ c_{11} = 2 + 2\left(\sqrt{\frac{1}{4} - C} + \sqrt{\beta^2 - B - C}\right), \\ &c_{12} = \sqrt{\beta^2 - B - C},\ c_{13} = -\frac{1}{2} - \left(\sqrt{\frac{1}{4} - C} + \sqrt{\beta^2 - B - C}\right) \end{aligned} \tag{24}$$

Substituting equation (24) into the energy eigenvalues equation of equation (7), we obtain the energy equation for this system as

$$\begin{aligned} &n^2 - (2n+1)\left(-\frac{1}{2}\right) + (2n+1)\left(\sqrt{\frac{1}{4} - C} + \sqrt{\beta^2 - B - C}\right) + B - 2\beta^2 + 2\beta^2 - 2B - 2C + \\ &2\sqrt{(\beta^2 - B - C)\left(\frac{1}{4} - C\right)} = 0 \end{aligned} \tag{25}$$

Solving equation (25) explicitly, we obtain the energy eigenvalues of the system as

$$E_{n,l} = -\frac{2\alpha^2\hbar^2}{\mu}\left[\left(\frac{n^2+n+\frac{1}{2}-\frac{\mu V_0}{2\alpha^2\hbar^2}-\frac{4\mu V_0{}'}{\hbar^2}+2l(l+1)+(2n+1)\left(\sqrt{\frac{1}{4}-\frac{2\mu V_0{}'}{\hbar^2}+l(l+1)}\right)}{(2n+1)+2\sqrt{\frac{1}{4}-\frac{2\mu V_0{}'}{\hbar^2}+l(l+1)}}\right)^2 + \frac{\mu V_0}{2\alpha^2\hbar^2} + \frac{2\mu V_0{}'}{\hbar^2} - l(l+1)\right] \tag{26}$$

Using equations (8) and (24), the wave function of this system is obtained as

$$\Psi_{n,l}(s) = N_{n,l} S^{\epsilon}(1-S)^{\frac{1}{2}+\upsilon} P_n^{(2\epsilon,2\upsilon)}(1-2s) \tag{27}$$

where

$$\epsilon = \sqrt{\beta^2 - B - C}, \text{ and } \upsilon = \sqrt{\frac{1}{4} - C}. \tag{28}$$

Furthermore, the relation between the hypergeometric function and the Jacobi polynomials are

$$P_n^{(a,b)}(z) = \frac{\Gamma(n+a+1)}{n!\Gamma(a+1)} \times {}_2F_1\left(-n, n+a+b+1; 1+a; \frac{1-z}{2}\right) \tag{29}$$

with $a = 2\epsilon > -1, b = 2\nu > -1$ under the transformation $z = (1-2s)$.

The normalization constant $N_{n,l}$ can be found from normalization condition as[13]

$$\int_0^{\infty} |R(r)|^2 \, dr = \alpha^{-1} \int_0^1 \frac{1}{s} \left|R_{n,l}(r)\right|^2 ds = 1 \tag{30}$$

By using the following integral formula

$$\int_0^1 (1-z)^{2(\delta+1)} z^{2\lambda-1} \{F_1 - n, n+2(\delta+\lambda+1); 1+2\lambda; z\}^2 dz = \frac{(n+\delta+1)n!\Gamma(n+2\delta+2)\Gamma(2\lambda)\Gamma(2\lambda+1)}{(n+\delta+\lambda+1)\Gamma(n+2\lambda+1)\Gamma(2(\delta+\lambda+1)+n)} \tag{31}$$

(if $z = 1-2s$)

with the help of equation (31) and after some calculations, the normalization constant $N_{n,l}$ is obtained as

$$N_{n,l} = \sqrt{\frac{n!2\epsilon\left(n+\nu+\frac{1}{2}+\epsilon\right)\Gamma\left(2\left(\nu+\frac{1}{2}+\epsilon\right)+n\right)}{\alpha\left(n+\nu+\frac{1}{2}\right)\Gamma(n+2\epsilon+1)\Gamma(n+2\nu+1)}} \tag{32}$$

Finally, the total normalized wave function, $\psi(r,\theta,\phi)$ of the Woods–Saxon potential plus attractive inversely quadratic potential is obtained as

$$\psi(r,\theta,\phi) = \frac{R(r)}{r} Y_{lm}(\theta,\phi)$$

$$\psi(r,\theta,\phi) = \sqrt{\frac{n!2\epsilon\left(n+\nu+\frac{1}{2}+\epsilon\right)\Gamma\left(2\left(\nu+\frac{1}{2}+\epsilon\right)+n\right)}{\alpha\left(n+\nu+\frac{1}{2}\right)\Gamma(n+2\epsilon+1)\Gamma(n+2\nu+1)}} \times \frac{1}{r}(-e^{2\alpha r})^{\epsilon}(1+e^{2\alpha r})^{\frac{1}{2}+\nu} \frac{\Gamma(n+2\square+1)}{n!\Gamma(2\square+1)} \times {}_2F_1\left(-n, n+2\epsilon+2\nu+1; 1+2\epsilon; \frac{1-z}{2}\right) Y(\theta,\phi)^{[1]} \tag{33}$$

DISCUSSION

Considering the proposed potential in equation (1), the radial Schrödinger equation has been solved, and the energy eigenvalues are obtained in equation (26) as

$$E_{n,l} = -\frac{2\alpha^2\hbar^2}{\mu}\left[\left(\frac{n^2+n+\frac{1}{2}-\frac{\mu V_0}{2\alpha^2\hbar^2}-\frac{4\mu V_0{}'}{\hbar^2}+2l(l+1)+(2n+1)\left(\sqrt{\frac{1}{4}-\frac{2\mu V_0{}'}{\hbar^2}+l(l+1)}\right)}{(2n+1)+2\sqrt{\frac{1}{4}-\frac{2\mu V_0{}'}{\hbar^2}+l(l+1)}}\right)^2+\frac{\mu V_0}{2\alpha^2\hbar^2}+\frac{2\mu V_0{}'}{\hbar^2}-l(l+1)\right]$$

With different screening parameters (α), the energy spectrum of the WSAIQ potential is reported numerically for various states in Table 1.

If $V_0{}' = 0$ in equation (17), the potential turns back into the Woods–Saxon potential expressed as

$$E_{n,l}^{WSP} = -\frac{2\alpha^2\hbar^2}{\mu}\left[\left(\frac{n^2+n+\frac{1}{2}-\frac{\mu V_0}{2\alpha^2\hbar^2}+2l(l+1)+(2n+1)\left(\sqrt{\frac{1}{4}+l(l+1)}\right)}{(2n+1)+2\sqrt{\frac{1}{4}+l(l+1)}}\right)^2+\frac{\mu V_0}{2\alpha^2\hbar^2}-l(l+1)\right] \text{at} l \neq 0, \text{and}$$

$$E_{n,0}^{WSP} = -\frac{2\alpha^2\hbar^2}{\mu}\left\{\left[\frac{\mu V_0}{4\alpha^2\hbar^2(n+1)}\right]^2+\left[\frac{n+1}{2}\right]^2+\frac{\mu V_0}{4\alpha^2\hbar^2}\right\} \text{at } l = 0.$$

Here, the index is non-negative integers with $\infty > n \geq 0$, and the above equation indicates that we deal with a family of the standard Woods–Saxon potential. By imposing appropriate changes in the parameters α and V_0, the index describes the quantization of the bound states and the energy spectrum.

If $V_0 = 0$, the energy eigenvalues for the attractive inversely quadratic potential becomes

$$E_{n,l}^{AIQP} = -\frac{2\alpha^2\hbar^2}{\mu}\left[\left(\frac{n^2+n+\frac{1}{2}-\frac{4\mu V_0{}'}{\hbar^2}+2l(l+1)+(2n+1)\left(\sqrt{\frac{1}{4}-\frac{2\mu V_0{}'}{\hbar^2}+l(l+1)}\right)}{(2n+1)+2\sqrt{\frac{1}{4}-\frac{2\mu V_0{}'}{\hbar^2}+l(l+1)}}\right)^2+\frac{2\mu V_0{}'}{\hbar^2}-l(l+1)\right] \text{at } l \neq 0, \text{and}$$

$$E_{n,0}^{AIQP} = -\frac{2\alpha^2\hbar^2}{\mu}\left[\left(\frac{n^2+n+\frac{1}{2}-\frac{4\mu V_0{}'}{\hbar^2}+(2n+1)\left(\sqrt{\frac{1}{4}-\frac{2\mu V_0{}'}{\hbar^2}}\right)}{(2n+1)+2\sqrt{\frac{1}{4}-\frac{2\mu V_0{}'}{\hbar^2}}}\right)^2+\frac{2\mu V_0{}'}{\hbar^2}\right] \text{at } l = 0.$$

The numerical values of the energy are obtained at different principal and orbital quantum numbers as shown in Table 1**.** The behaviors of the superposed potential at various values of the screening parameter are presented in Figures 1 to 3. The energy eigenvalues increase with n and l values and drop as l reduces for very low α values (α = 0.01, α = 0.03, and α = 0.1) and decreases abruptly at large α values (α = 2 and α = 5). Thus, the particles with a lower angular momentum can be easily separated from the WSAIQ potential. These results could be used in the study of the nuclear structure within the shell model of an atom in a nonrelativistic framework.

Table 1: Energy Eigenvalues $E(eV)$ of the WSAIQ pPotential for ħ=μ=1, $V_0 = 0.2$, $V'_0 = 0.1$ with Different α Values.

$\|n,l\rangle$	$E_{n,l}^{WSAIQP}$				
	$\alpha = 0.01$	$\alpha = 0.03$	$\alpha = 0.1$	$\alpha = 2$	$\alpha = 5$
$\|0,0\rangle$	-95.62974941	-10.74881358	-1.098111629	-2.140583888	-2.140583888
$\|0,1\rangle$	-13.45063995	-1.544529482	-0.230749915	-18.05210063	-112.551819266
$\|0,2\rangle$	-5.742611968	-0.675656881	-0.175867240	-34.033930195	-212.533810328
$\|0,3\rangle$	-3.196744092	-0.38868765	-0.181883736	-50.025259859	-312.525193282
$\|0,4\rangle$	-2.039681218	-0.25915525	-0.205268925	-66.020139956	-412.520097580
$\|1,0\rangle$	-16.93726919	-1.978295946	-0.291957248	-6.875722281	-42.4103580626
$\|1,1\rangle$	-5.896828321	-0.731420553	-0.204090920	-26.823792825	-167.232762720
$\|1,2\rangle$	-3.253093071	-0.427339394	-0.206020471	-44.512219613	-277.8700955764
$\|1,3\rangle$	-2.076941316	-0.290884728	-0.226225165	-61.548307887	-384.4022845879
$\|1,4\rangle$	-1.445804729	-0.217659671	-0.254104334	-78.253598986	-488.8506868257
$\|2,0\rangle$	-6.843417703	-0.855142273	-0.209216567	-15.749717146	-97.89555125
$\|2,1\rangle$	-3.323740234	-0.456762674	-0.221042527	-40.225433857	-250.944599082
$\|2,2\rangle$	-2.110594191	-0.315936401	-0.248577472	-60.808359923	-379.6507287233
$\|2,3\rangle$	-1.471171294	-0.240696632	-0.280737109	-79.3405561345	-499.5867532509
$\|2,4\rangle$	-1.088724595	-0.195550472	-0.315305741	-98.3405561345	-614.3168191017
$\|3,0\rangle$	-3.708294136	-0.508545145	-0.208844500	-28.6377975985	-178.453113065
$\|3,1\rangle$	-2.149754089	-0.333996925	-0.257434980	-57.804020576	-360.7886685771
$\|3,2\rangle$	-1.493461251	-0.258475741	-0.302071884	-81.813417724	-510.89437829
$\|3,3\rangle$	-1.107136973	-0.213372442	-0.345329223	-103.804237549	-648.378799343
$\|3,4\rangle$	-0.857618963	-0.184172282	-0.387765379	-124.47679346	-777.618169816

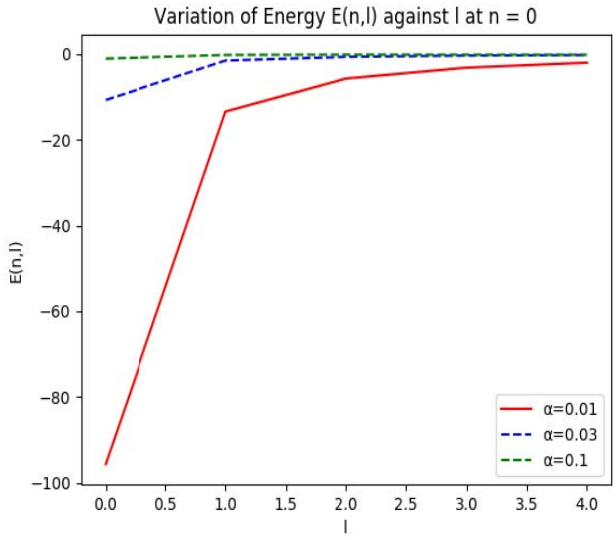

Figure 1: Variation of $E_{n,l}$ against l at $n = 0$.

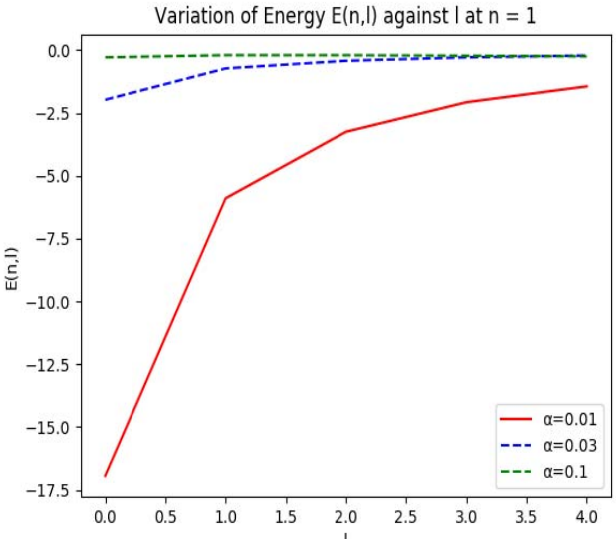

Figure 2: Variation of $E_{n,l}$ against l at $n = 1$.

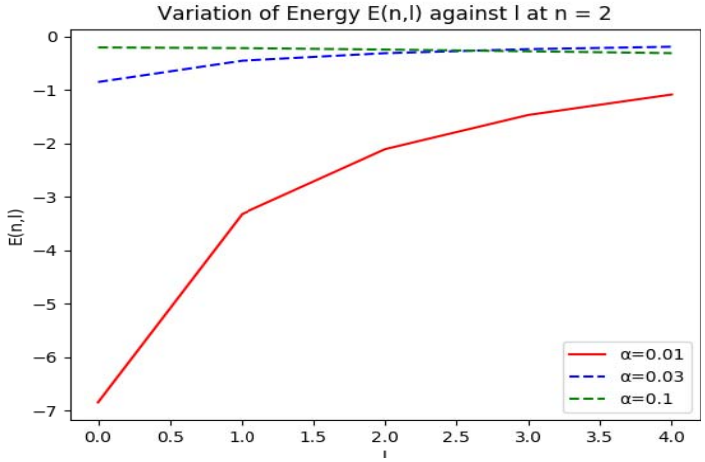

Figure 3: Variation of $E_{n,l}$ against l at $n = 2$.

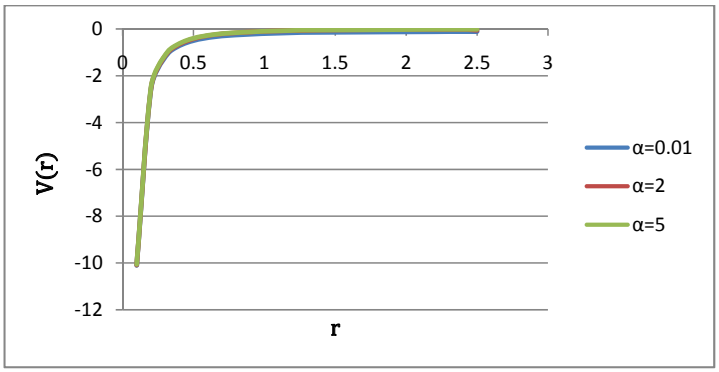

Figure 4: Variation of the WSAIQ potential $V(r)$ against r at different α.

CONCLUSION

In this paper, we have obtained the bound-state solution of the Schrödinger equation with WSAIQ potential via the parametric NU method. The energy eigenvalues and the corresponding total normalized wave functions expressed in terms of the hypergeometric functions for the system are also obtained. The energy eigenvalues were evaluated as a function of the screening parameter in Table 1. The behaviors of our potentials are discussed in Figure 3. Interestingly, the Schrödinger and Dirac equations with the arbitrary angular momentum values for this potential can be solved by this method. The resulting eigenenergy equations can be used to study the spectroscopy of some selected diatomic atoms and molecules.

REFERENCES

Antia, A.D., Essien, E., Umoren, B, & Eze, C. C. (2015). Approximate solution of the non-relativistic Schrödinger equation with inversely quadratic Yukawa plus mobius square potential via parametric Nikiforov–Uvarov method. *Advances in Physics Theories and Application*, *44*.

Ita, B.I., Ikeuba, A.I., & Obinna, O. (2015). Solution of Schrödinger equation with inversely quadratic Yukawa potential plus Woods–Saxon potential using parametric Nikiforov–Uvarov method. *Journals of Advance in Physics*, *18*(2), 2094–2098.

Ita, B.I., & Ikeuba, A. I. (2015). Solutions to the Klein–Gordon equation with inversely quadratic Yukawa plus inversely quadratic potential using Nikiforov–Uvarov method. *Journals of Theoretical Physics and Cryptography*, *8*.

Ita, B.I., Nyong, B. E., Alobi, N.O., Louis, H., & Magu, T.O. (2016). Bound state solution of the Klein–Gordon equation for modified Echart plus inverse square molecular potential with improved new approximation scheme to centrifugal team. *Equatorial Journal of Computational and Theoretical Sciences*, *1*(1), 55–64.

Ita B.I., Louis, H., Magu, T.O., & Nzeata-Ibe, N.A. (2017). Bound state solutions of Klein–Gordon equation with Woods–Saxon plus attractive inversely quadratic potential via parametric Nikifarov–Uvarov method. *WSN, 74*, 280–287.

Ita, B.I., Louis, H., Magu, T.O., & Nzeata-Ibe, N.A. (2017). Bound state solutions of the Schrödinger equation with Manning–Rosen plus a class of Yukawa potential using Pekeris-like approximation of the Coulombic term and parametric Nikiforov–Uvarov method. *World Scientific News (WSN), 70*(2), 312–319.

Louis, H., Ita, B.I., Nyong, B.E., Magu, T.O., Alobi, N.O., & Nzeata-Ibe, N.A. (2016). Approximate solution of the *N*-dimensional radial Schrödinger equation for Kratzer plus reduced pseudoharmonic oscillator potential within the frame work of N-U method. *Journal of NAMP, 36*(2), 199–204.

Louis, H., Ita, B.I., Nyong, B.E., Magu, T.O., Nzeata-Ibe, N.A., & Barka, S. (2016). Radial solution of the s-wave *D*-dimensional non-relativistic Schrödinger equation for generalized Manning–Rosen plus Mie-type Nuclei potential within the framework of Nikiforov–Uvarov method. *Journal of NAMP, 36*(2), 193–198.

Magu, T.O., Ita, B.I, Nyong, B.E, Louis, H. (2017). Radial solution of the s-wave Klein–Gordon equation for generalized Woods–Saxon plus Mie-type potential using Nikiforov–Uvarov method. *Journal of Chemical Society of Nigeria, 41*(2), 21–26.