Structural Health Fuzzy Classification of Laguna Bridge Based on Hybrid Inspections Results

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Abstract

Bridge structural health monitoring system (BSHMS) is an aid for systematized decision-making and planning for bridge infrastructure assessment and recondition. One of the critical efforts is to have some criteria to show the current health condition of the bridge based on the inspection results. The conventional way of classification is morphologically and linguistically rated which shows impreciseness and uncertainties in evaluations. The paper proposed a new fuzzy system based on the hybrid (subjective and objective) inspection data results. The optimum value of parameters based on reconstructed data is selected as ambiguous inputs with membership functions using the concept of the statistical distributions and cognitive limitations. The fuzziness of health classification rating is calculated by the fuzzy arithmetic rules inherent in the fuzzy expert system. The proposed Health Classification System, based on hybrid data, yielded a 90 % accuracy in comparison with the conventional inspection method. Thus, the proposed study proved that it can be used for structural health monitoring.

***Index Terms* — Structural Health Monitoring (SHM), Bridge fuzzy logic, membership functions, the optimal parameter**

I. Introduction

The Department of Public Works and Highways (DPWH) in the Philippines developed a manuscript called DPWH Atlas containing tables and graphs of recent statistics and conditions of roads and bridges nationwide. It provides data for Bridge Management System (BMS), which is a conventional tool for standardized decision making and planning/scheduling for bridge infrastructure inspection, maintenance, and repair or retrofit [1]the design of system development of Sihui BMS(SH-BMS.

Combining the statistics of all regions in the country, 32.67% are Excellent, 43.93% is good, 15.48% is fair, 7.42% are low and 1.51% need further assessment.

Visual methods are the fundamental approach to classify and evaluate the condition of a structural bridge. Considering as subjective analysis, it has a great estimation that may have to be a significant influence on the health classification of bridges proven through the eventualities and developments [4]. The main advantage of this method is it can be done in non destructive manner [5]. However, the trade-off is the uncertainties and vagueness of the data. Dissimilarity and disparity of reports between the authorized inspector can cause great adversity.

To overcome this ambiguity, the objective approach is defined using appropriate nondestructive testing (NDT) methods. Despite the possibility that these methods are more accurate comparing to the subjective process, there are still drawbacks to them [6]. Clarification and interpretation of the NDT results need compliance in material and bridge elements. In essence, the uncertainty of the products and data might cause faulty preference and conclusion.

Table 1 illustrates a representative case of health classification data from Laguna 1st Bridges [7]. The extent between the minimum and maximum substantially shows the illogical approach of the rating.

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Table 1
Case Sample of Statistical Bridge Classification Information per District Engineering Office (Laguna 1st)

ID	Name	Average	Standard Deviation	Mode	Rating
B02464LZ	San Antonio	5.8	0.81	6	5
B02465LZ	Pila	4.9	0.94	5	4
B02466LZ	Labuin	5.2	0.92	6	4
B02467LZ	Pagsawitan	4.8	0.94	5	5
B02468LZ	Salasad	4.5	0.74	5	4

Given those sets of circumstances, the fuzzy set theory can be an inspiration to overcome the shortcomings and uncertainties ([8] - [10]. This paper proposed an expert structural health fuzzy classification of bridges that will classify them into excellent, good, fair, bad, and failed. It is achievable by the hybrid application of subjective and objective measurements.

II. Bridge structural Health Monitoring

A. Conventional parameters and their relations

1. Spalling and Delamination

Visual inspection of the bridge utilized the spalling and delamination method [12]. The Field-cast of a structured bridge showed some evidence of deterioration through spalling and delamination. Spalling is a visual failure of a concrete structure showing fractures formed in running surfaces as a result of surface or sub-surface fatigue [13]. Delamination in correlation to spalling is also a visible or perceptible fracture seen in layers rather than cracks. It is a more defined break of the entire structure composition compared to spalling [14]. Although both are visual in prospect, modern days use nondestructive techniques to classify these failure properties by allowing a frequent and extensive inspection of the slabs without damaging structures [15].

2. Structure Temperature and Ultrasonic Velocity

Ambient parameters are significantly considered in classifying the health of a structure. Ultrasonic velocity also referred to as structure vibration, which is technically termed as seismic noise, represents continuous relative vibration on the surface of the Earth. The tiny figures of this vibration determine the modal properties of structures. Moreover, this can be used to evaluate the linear behavior of the degree of damage incurred in the structure. Transient and forced load conditions contribute to vibration characteristics [12].

Structure temperature is also a modal parameter. Several papers proved that both vibration and temperature rea crucial in classifying the condition of a structure [13-16]. Distinctively speaking, these two parameters drew an analogy in between.

In civil infrastructures, the temperature usually affects bridge decks in gradient. Temperature loading affects bridges causing expansion and contraction throughout the deck [17]. In worst cases, the temperature gradient over the surface causes depth to distort. It adds strain to the structure. This strain variation changes the vibration pattern of the structure. As a basis for understanding the relationship of thermal stress and temperature gradient, here is a mathematical representation:

$$\sigma = E\alpha\Delta T \tag{1}$$

The thermal stress (σ) is the product of modulus of elasticity (E), coefficient of thermal expansion (α), and temperature gradient (ΔT) or temperature change. Considerably, the thermal strain is directly proportional to the temperature gradient. The thermal strain (f) is the product of the coefficient of thermal expansion (α), temperature gradient (ΔT), and span of the beam (L) or material being tested as shown in the equation below. Both thermal stress and strain have a linear effect with temperature gradient considering any material being used.

$$f = \alpha\Delta TL \tag{2}$$

Young's modulus, otherwise known as elastic modulus, is a mechanical property of solid materials that are linear elastic, such as steel. It is technically defined as the ratio of stress, force per unit area, and strain, the ratio of deformation per initial length, along an axis. It is a measure of how stiff the solid material is.

Thus, there is an infinite Young's modulus in a perfectly rigid material because it requires endless force to deform it. With increasing temperature, Young's modulus of concrete decreases, giving the material vulnerability to damage [18]. In conclusion, an increase in structural temperature results in the reduction of ultrasonic velocity. The change of modulus of the material is the cause of varying natural frequency [19]. Thus, they suggested that structural bridge and ultrasonic velocity measurement is necessary and must be well understood to provide correct subjective structural condition identification [20].

B. Fuzzy Inference System

Fuzzy logic is an easy-to-use method for practical inference problems in engineering because it relates significance and precision very well [21]. To outline the mathematical background of the proposed method in this paper, the following general definitions and theory of fuzzy sets are used: Let X be the universe of discourse, and its elements are denoted as x. In the fuzzy theory, fuzzy set A of universe X is defined by function mAðxÞ called the membership function of set [22].

m$_A$ðxÞ : *X* 2 ½0; 1] where **1**$_A$ ðxÞ ¼ 1 if *x* is totally in *A*;
1$_A$ ðxÞ ¼ 0 if *x* is not in *A*;
0 < **1**$_A$ ðxÞ < 1 if x is partly in A:

This mathematical representation of sets allows a sequence of possible sets. For any element x of universal X, membership function mA x equals the degree to which x is an element of set A. From that derived degree, represents the degree of membership and also called membership value, of element x in set A.

It has a strict value between 0 and 1. Any universal set of discourse consists of some sets describing some attributes to the output.

The gist and essence of the utilized fuzzy set theory depicted and defined from the mathematical representation stated above are dealing with the linguistic or subjective parameters acquired from the data results of roads and bridges conventional inspections of in the Philippines. To make the case in point, the statement "a is b" implies that the linguistic parameter takes the linguistic value of b and it is utilized as the fuzzy rules.

The range of possible values of linguistic parameters represents the universal set of that parameter. In the process, the emergence of the fuzzy rule can be defined as a conditional statement in the form:

"IF ðx is aÞ THEN ðy is bÞ where x and y are linguistic variables; and a and b are linguistic values determined by fuzzy sets on the universal set of X and Y, respectively."

The essential characteristic of fuzzy systems is that fuzzy rules relate fuzzy sets to each other. Fuzzy sets provide the basis for the output estimation model. The model is based on relationships among some fuzzy input parameters [22].

In this paper, fuzzy logic-based inference systems are utilized to decode and translate the subjective linguistic level of bridge data from the National Roads and Bridges Inventory 2019 [11].

To enhance the capabilities of the health classification system, objective inspection data results are correlated. These data results are derived out of advanced nondestructive testing performed by DPWH Bridge Health Assessment Team.

III. Methodology

This paper used the Mamdani method for describing the bridge health classification in a more intuitive, more humanlike manner by establishing necessary hybrid conditions of subjective and objective inspections. The Mamdani-style fuzzy inference process is performed in four steps shown in Fig. 1 [24].

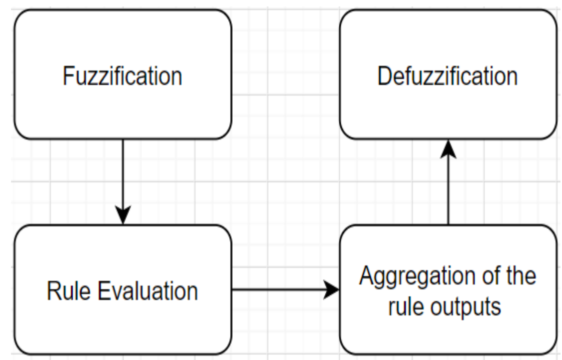

Fig. 1. Mamdani-style Inference Process

A. Fuzzification

The initial step was to acquire data inputs, x1 and y1, and determine the degree to which these inputs belong to each of the appropriate fuzzy sets (subjective or objective). National Roads and Bridges Inventory 2019 on DPWH Atlas [11] provided the subjective data result of inspections. A segment of data statistics is shown in Table 2 and Table 3.

Table 2
Case Sample of Subjective Inspection Data of Bridges in Quezon 1st District
(Spalling, Delamination, and Temperature)

ID	Name	Spalling	Delamination	Temp
B01193LZ	Pakil	4.3	4.0	4.3
B01580LZ	Lumban	4.2	3.8	4.4
B01589LZ	Abuyon	4.8	4.3	5.1
B01774LZ	Pacabit	4.3	4.1	3.9
B01775LZ	Tagbacan	5.5	4.9	5.8

Table 3
CASE SAMPLE OF SUBJECTIVE INSPECTION DATA OF BRIDGES IN QUEZON 1ST DISTRICT (VELOCITY AND CORROSION RATING)

ID	Name	Velocity	Corrosion Rating
B01193LZ	Pakil	4.8	4.4
B01580LZ	Lumban	4.0	4.1
B01589LZ	Abuyon	4.6	4.7
B01774LZ	Pacabit	4.2	4.3
B01775LZ	Tagbacan	5.3	5.4

While objective data results were derived out of advanced nondestructive testing (NDT) from DPWH Bridge Health Assessment Team, Raw Data, and Corrosion Rate of Nondestructive inspection is shown in Table 4.

Table 4
RAW DATA AND CORROSION RATE OF NDT INSPECTION RESULTS

USPV	COV>MET	½ CELL	HVT	Corrosion Rate
-0.00838	-0.01027	0.883178	27.50846	3
-0.0084	-0.00977	0.882935	27.50846	3
-0.00822	-0.01003	0.882883	27.50846	2
-0.00857	-0.00983	0.882827	27.50846	3
-0.00852	-0.00966	0.882872	27.50846	2
0.014481	-0.01194	0.87572	27.39735	0

It was believed that hybrid encoding of observed symptoms into bridge health classification rating through inaccurate subjective data with an objective evaluation of the NDT results was an excellent tool to guess the condition rating practically. Figure 2 shows the segmentation of the data inputs into subjective and objective.

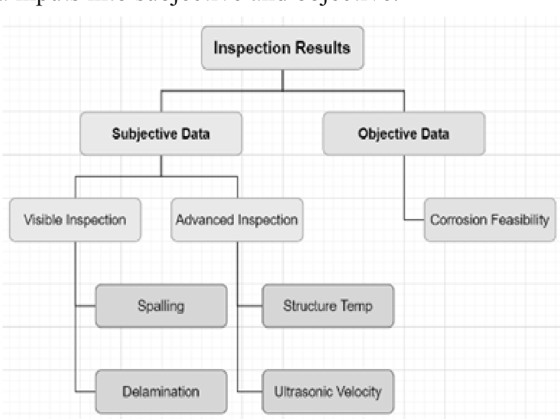

Fig. 2. Segmentation of Data

B. Rule Evaluation

Fuzzified inputs were acquired and applied to the quantifications of the fuzzy rules.

The representation of parameters used was presented below as distinguished inputs of discourse X, which consists of evidence from inspection techniques:

X f *Spalling*; *Delamination*; *Ultrasonic Velocity or Vibration*; *Structural Temperature Corrosion Feasibility*

The next step was to identify the fuzzy ranges of each. Fuzzy sets of universe *X* are:

A_1 f*No*; *Yes*
A_2 f*No*; *Maybe*; *Yes*
A_3 f*Low*; *Moderate*; *High*
A_4 f*Low*; *Moderate*; *High*
A_5 f*Low*; *Moderate*; *High*

The whole fuzzy system is depicted in Figure 3. While Figures 4-9 illustrate the membership functions for inputs and output fuzzy bridge health classification. In the description of the membership functions, it is basically in Gaussian membership function shown below:

$$f(x;\sigma,c) = e \wedge (-(x-c)^2 / 2\, \sigma^2) \tag{3}$$

Where f is the membership value, x is the input value, c is the mean, and σ is the standard deviation.

The only advantage is that it was made with the assistance of statistical distributions together with accomplished and experienced evidence/data for regulations. Similar to the fuzzy proceedings of some technical papers ([26] - [28]). Therefore, the membership functions are stretched to show a more average range in the Gaussian membership functions.

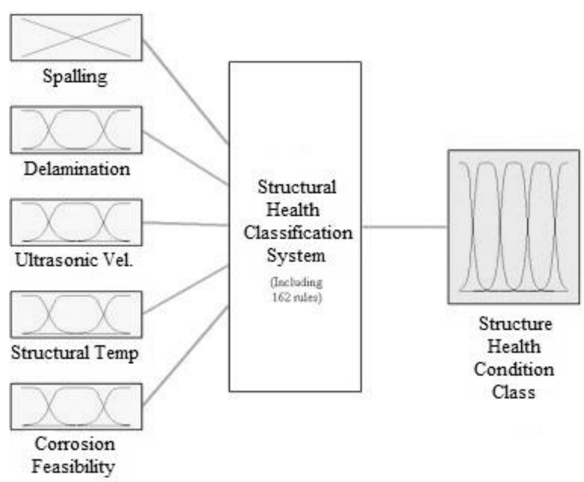

Fig. 3. Fuzzy System for Structural Health Classification of Bridges

In this paper, 162 different rules were used in the knowledge-based from the subjective and objective approach of inspections.

A typical rule is:

IF [(Spalling is No) AND
(Delamination is No) AND
(Ultrasonic Velocity is low) AND
(Structural temp is High) AND
(Corrosion Feasibility is Low)] THEN
[(Concrete Bridge Deck Condition Rating is Excellent)].

All of the 162 rules were eviewed and checked to correlate input parameters to the outputs.

For classicality, all weights are considered the same and equal to one. The main reason for utilizing the knowledge rule is that it is flexible at any time. Changes and modifications can be improved by new findings, experience, and data.

C. Aggregation of the rule outputs

In this procedure, all of the outputs of the amended fuzzy rules are unified. We take the membership functions of all rule consequents previously found as the input and combine them into a single fuzzy set for each output variable. The technical procedure of the fuzzy system and its concept were explained and detailed above. The final result is a definite characterization of numbers, which shows the structural health conditions of a concrete bridge.

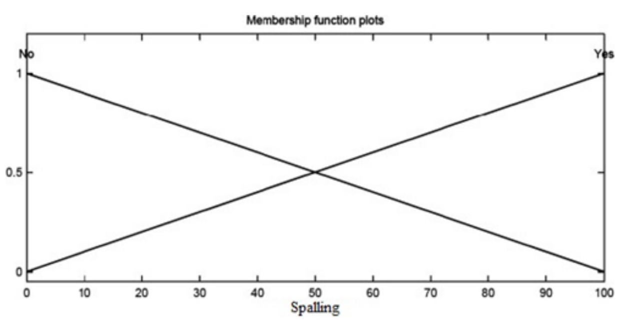

Fig. 4. Spalling Membership Function

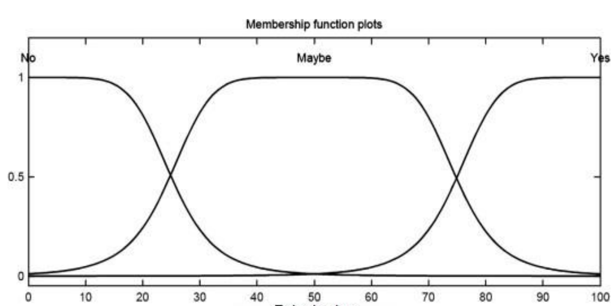

Fig. 5. Delamination Membership Function

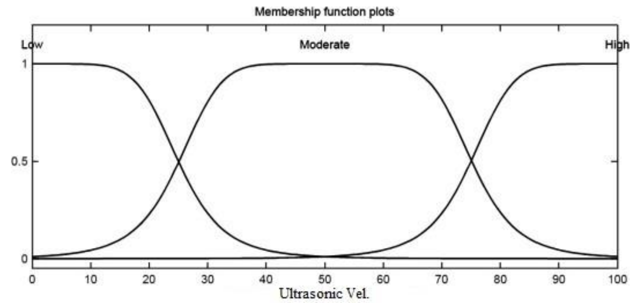

Fig. 6. Ultrasonic Membership Function

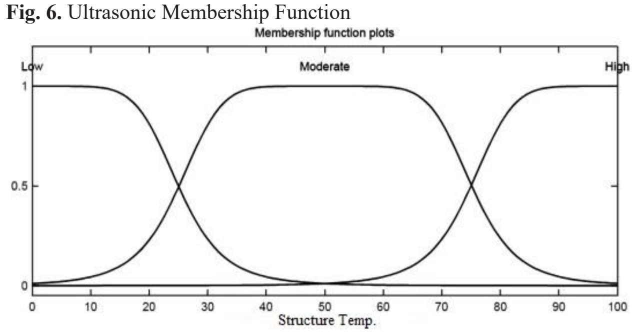

Fig. 7. Structure Temperature Membership Function

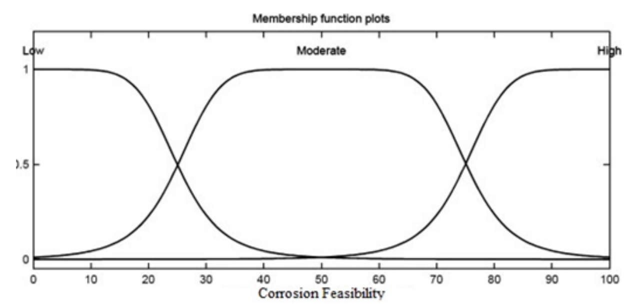

Fig. 8. Corrosion Feasibility Membership Function

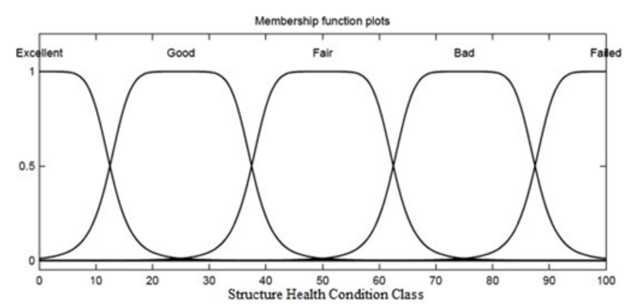

Fig. 9. Structure Health Condition Class Membership Function

D. Defuzzification

The fourth and final step is defuzzification, which deals with converting fuzzy sets based on the fuzzy inference engine into numbers, remarkably crisp values. It helps us to evaluate the rules. The aggregate output fuzzy set is the input for the defuzzification process, and the output is a single crisp number/value. The defuzzification employed by the system uses the centroid method that is mathematically

defined by Eq. 4, wherein X is the activation function. The centroid method is otherwise known as the center of the area or center of gravity method.

$$x* = x.\mu C(x)dx / (\mu C(x)dx) \quad (4)$$

IV. Results and Discussions

The advancement and privileges of the fuzzy expert system can be thought-out as an innovative approach for Bridge Structural Health Monitoring System (BSHMS). Figures 10 to 14 illustrate some of the relationships formed from the different parameters and fuzzy structural health classification system. These figures exhibit that the rules show proximate real situations in practical issues of bridge subjective and objective inspections. Experts in this field can verify these relationships in this fuzzy model by his apprehension and experience. Significant points depict a great nonlinear relationship between parameters. Smoothness in the surfaces also indicates the noise tolerance of the system. It also shows contempt on the uncertainty, faulty, and unprecise input data. Lastly, just like what is mentioned in the above concept, the classification output can be considered valuable for decision-makers.

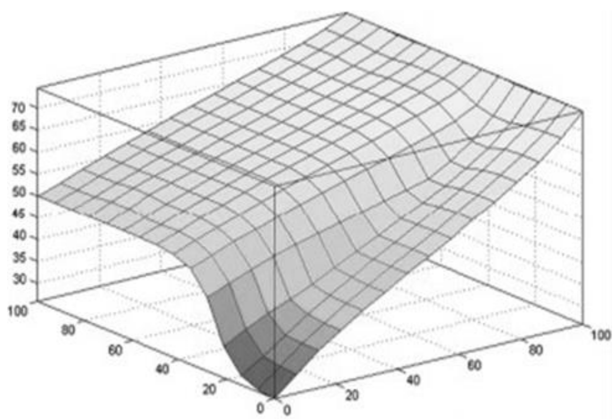

Fig. 10. Structure temperature and corrosion feasibility scores related to structure health condition class

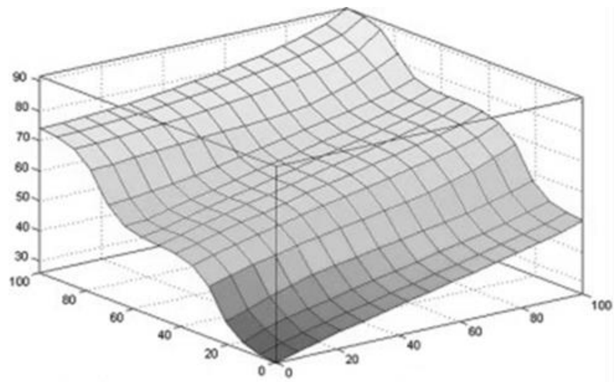

Fig. 11. Ultrasonic velocity and corrosion feasibility scores related to structure health condition class

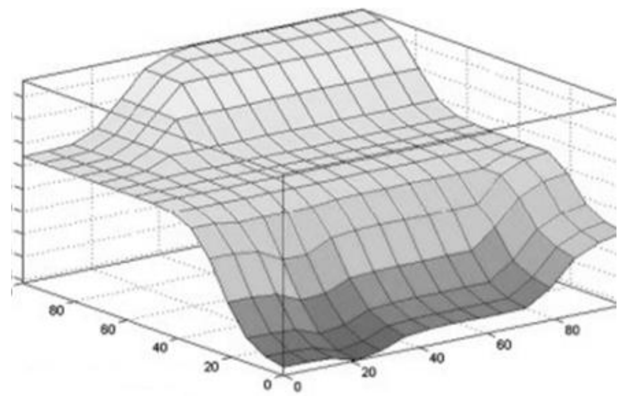

Fig. 12. Spalling and corrosion feasibility scores related to structure health condition class

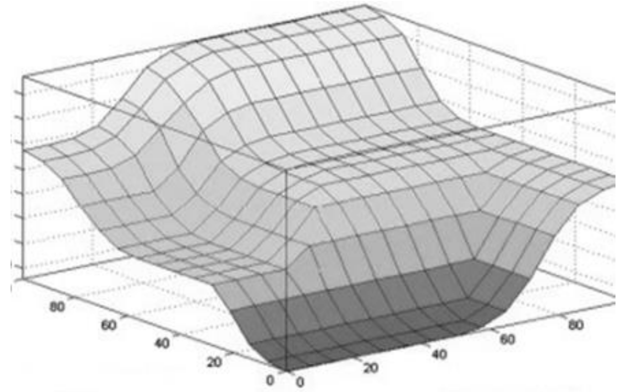

Fig. 13. Delamination and hammer tapping scores related to bridge deck condition rating

Table 5

Rating Comparison of Conventional and Proposed Structural Health Classification

Bridge ID No.	Proposed Structural Health Fuzzy Classification of Bridge		Bridge Health Classification Rating, DPWH Atlas 2019 (record ref.)	
	Numerical value	Linguistical Index Rating	Numerical value	Linguistical Index Rating
B02465LZ	72.3	Bad	63.5	Poor Condition
B02466LZ	76.1	Bad	78.4	Poor Condition
B02467LZ	58.7	Fair	49.3	Good Condition
B02468LZ	80.2	Bad	82.1	Poor Condition
B02469LZ	53.3	Fair	56.4	Good Condition

Table 5 shows a partial comparison of the proposed fuzzy method to the conventional classification rating method of DPWH. It shows that it can estimate the bridge health classification reasonably. The main asset of the proposed fuzzy method is that a complicated method and calculation are not necessary to classify a bridge. It can be done thru the inspector's judgment.

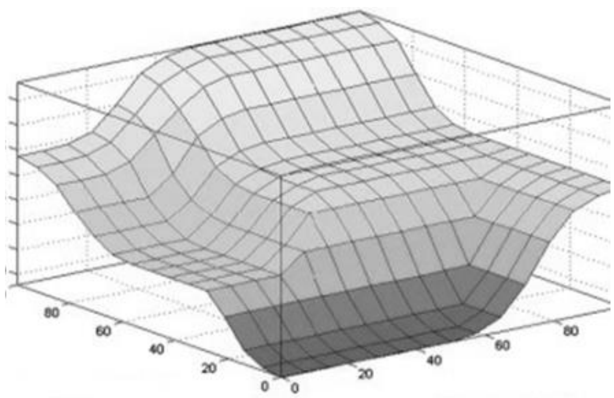

Fig. 14. Comparison of Proposed Fuzzy System to Fuzzified Atlas Output

In some of the cases judgment is incorrect, but still close to real condition. Figure 18 shows some misclassifications of the system. It presents the discrepancy between the proposed fuzzy classification system and the conventional and standard Atlas. Trials 13, 16, and 17 spikes their crisp output to 80 levels that must be, on the contrary, near or equal to 50. These are considered misclassifications of the Proposed fuzzy system. For the other trial points, the consistency is considerably noticeable especially for trials 20 to 30.

Based on the experimental results, there are 27 correct classifications out of 30 trials, which is equivalent to 90.00% accuracy. Nevertheless, the proposed method can classify the health of a typical bridge close to the real condition and without a major difference from a functional point of view.

V. Conclusion

The fuzzy logic inference method is utilized in this paper to calculate the bridge's structural health condition. It optimistically assists the management system of bridges. It can be suitable for prioritization of bridge repair and budgeting tasks in which adequate and feasible reasoning is required for inspectors and decision-makers.

The main capability to give a precise and systematic measurement of the health classification of bridges shows reasonable and substantial use in the future. Engaging in this system provides privilege to the inspector as well as the non-expert to classify the health condition of bridges without complicated processes and advanced machinery. Measuring the dimensions of the defect area and further calculation is not needed. The only necessity is the inspector's judgment.

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