RESEARCH ARTICLE

On the Volatility and Market Inefficiency of Bitcoin During the COVID-19 Pandemic

Cesar C. Rufino
De La Salle University, Manila, Philippines
cesar.rufino@dlsu.edu.ph

The COVID-19 pandemic has been causing unprecedented economic downturn worldwide. As it wreaks havoc on every aspect of global economic activities, stakeholders are wondering how its impact can be quantified to craft viable responses. In the exotic field of cryptocurrencies, prior to the pandemic, everyone was excited about Bitcoin and its multitude of potentials. However, a day after COVID-19 was officially announced by the World Health Organization as a pandemic, the rate of return to Bitcoin dropped by an unheard-of one-day decline of -46.5%, and people started to rethink the prospects of Bitcoin. A day after this steep decline, Bitcoin recovered and started a sustained bull run which lasted for almost a year and even posted an all-time high daily uptick of 59.6%. By the end of July 2021, the price reached its all-time high but lost more than half of it at the end of the sample period. This study aims to empirically analyze the risk-return profile and the market efficiency of Bitcoin utilizing a 1,306-day data set conveniently subdivided into pre-pandemic and pandemic periods. The general conclusion of the study is: During the pandemic, Bitcoin is extremely volatile and does not subscribe to the efficient market hypothesis.

Keywords: Bitcoin, COVID-19 Pandemic, GARCH Variants, Efficient Market Hypothesis, Martingale Difference Sequence

JEL Classification: C58, G11, G32

The consecutive outbreaks of devastating financial disasters and the crushing economic harm triggered by the COVID-19 pandemic have prompted investors to explore exotic, non-traditional, profitable, but safe investment opportunities. The introduction of Bitcoin at the height of the 2009 financial crisis presented a much-needed alternative that harnessed the seemingly endless potentials of the high-technology era. It ushered in the age of the so-called "cryptocurrencies." Being the pioneer, Bitcoin is the "King" of virtual currencies and has become a standard means of payment over the internet. Investors are attracted to Bitcoin as an investment vehicle due to its perceived desirable features: simplicity, transparency, exceptionally high average return, extreme volatility, accessibility even during weekends, and low correlation with traditional assets—features that offer significant diversification benefits (Briere et al., 2015)

This paper aims to empirically validate investors' perception of the desirability of Bitcoin as an investment alternative even during the devastation wrought by the COVID-19 pandemic. Employing the state-of-the-art variants of the generalized autoregressive conditional heteroscedasticity (GARCH) model and stylized facts statistical analyses and testing, the study attempts to provide stakeholders with empirically sound bases in examining Bitcoin as an attractive investment alternative by virtue of its extreme volatility. The analysis is also done to ascertain whether the Bitcoin market is informationally efficient, in other words, whether it subscribes to the principles of the efficient market hypothesis (EMH) before and during the pandemic.

Theoretical and Operational Framework

One important characteristic of financial assets that a lot of market players are attracted to is the immutable trade-off between the return from the asset and the associated risk in holding it. Mainstream financial and economic theories predict a positive but nonlinear relationship. In formulating sound investment strategies for Bitcoin, this trade-off must be taken into consideration. The following techniques are employed in the study to analyze the risk-return dynamics of Bitcoin:

Stylized Facts Analysis

Daily closing prices (P) and rate of returns (*rr*) of Bitcoin within the sample horizon are subjected to a battery of graphical and descriptive analyses of their first four moments (central tendency, variability, symmetry, and tail density). In quantifying the returns series, the following formula is used in this study:

$$rr_t = 100*1\text{n}\,(P_t/P_{t-1}) \qquad (1)$$

To confirm the susceptibility of the return series to econometric modeling and to verify one requirement of market efficiency, a battery of unit root tests is implemented. These tests determine the order of integration of price series, and if its natural logarithm is shown to be *I(1)*, the first difference (identical to *rr* per formula (1)), is deemed to be *I(0).* The following unit root tests are used: augmented Dickey-Fuller (ADF), Philips-Perron (PP), and the ERS (Elliot, Rothenberg, and Stock) point optimal tests.

The ARCH/GARCH Models

The autoregressive conditional heteroscedasticity (ARCH) effect (Engle, 1982) is an almost unique phenomenon associated with modeling returns to financial assets. In classical regression analysis, the presence of ARCH is a complete anathema to all the classical model stands for. Hence, instead of just modeling the mean return equation (or the population regression function (PRF) of the return series), the conditional variance equation is likewise specified owing to the expected presence of the time-varying second moment. The basic ARCH(q) model is specified as follows:

Mean equation:

$$E_{t-1}(rr_t) = c + \psi\, rr_{t-1} + u_t \qquad (2)$$

Conditional variance equation:

$$h_t = \omega + \sum^{q} \alpha_i u^2_{t-i} \qquad (3)$$

Bollerslev (1986) saw the need to generalize the ARCH effect to augment the current conditional variance with its past values up to lag p. The conditional variance equation for the classic GARCH(q,p) is now:

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i u^2_{t-i} + \sum_{j=1}^{p} \beta_j h_{t-j} \qquad (4)$$

Contrary to the standard ARMA model, the above structure allows the noise to be a function of its past values and the past values of the square of stochastic disturbance. Hence, the variance of the noise element is not homoscedastic but time-varying and needs to be modeled as well as Equation (4).

The GARCH Variants

To check for the presence of certain special volatility effects (e.g., leverage effect, asymmetric effects, etc.), two different families of GARCH models are introduced in the literature: the APARCH (asymmetric power ARCH) and the EGARCH (exponential GARCH) models. The former is specified in a straightforward manner, with the left-hand side of the equation as either the conditional variance or the conditional standard deviation, but the LHS of the latter is expressed as natural logarithms of the conditional variance (Francq & Zakoian, 2010).

The APARCH Family (Ding et al., 1993)

The APARCH family of the GARCH model can accommodate various asymmetric effects and power transformations of the conditional variance. The general specification of the conditional volatility equation of the APARCH family is as follows (Lucchetti & Balietti, 2022):

$$\sigma_t^\delta = \omega' z_t + \sum_{i=1}^{q} \alpha_i (|u_{t-i}| - \gamma_i u_{t-i})^\delta + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta \tag{5}$$

where $\sigma_t^2 = h_t$

The parameter δ is either 1 or 2 and performs a Box-Cox transformation and γ captures the asymmetric effects. Specific values for δ's and γ's give rise to different variants of the APARCH models:

- ARCH (Engle, 1982) - β's, γ's = 0, δ=2
- GARCH (Bollerslev, 1986) γ's = 0, δ=2
- GARCH (Taylor, 1986 and Schwert, 1990) γ's = 0, δ=1
- GJR (Glosten et al., 1993) δ=2
- TARCH (Zakoian, 1994) δ=1
- NARCH (Higgins and Bera, 1992) β's, γ's = 0

The EGARCH (Nelson, 1991)

The EGARCH introduced by Nelson in 1991, with the variance equation expressed in terms of log volatility, captures the asymmetric effect as a function of standardized innovations. Thus, the conditional variance equation is specified as:

$$\ln h_t = \omega' z_t + \sum_{i=1}^{q} \left[\alpha_i (|\varepsilon_{t-1}| + \gamma_i \varepsilon_{t-i}) + \sum_{j=1}^{p} \beta_j \ln h_{t-j} \right] \tag{6}$$

with $\varepsilon_t = u_t / \sqrt{h_t} \sim N(0,1)$ or a suitable distribution. (7)

Table 1. *Alternative Conditional Distributions of the Normalized Error*

Name	Density	Parameters
Normal	$\frac{1}{\sqrt{2\pi h_t}} \exp\left\{-\frac{\varepsilon_t^2}{2}\right\}$	none
Student's t	$\frac{K(\nu)}{\sqrt{h_t}} \left[1 + \frac{\varepsilon_t^2}{\nu-2}\right]^{-(\nu+1)/2}$	$\nu > 2$
Generalised Error Distribution (GED)	$C(\nu) \exp\left\{-\left|\frac{\varepsilon_t}{\kappa_\nu}\right|^\nu\right\}$	$\nu > 0$
Skewed t	$\frac{bK(\nu)}{\sqrt{h_t}} \left[1 + \frac{\zeta_t^2}{\nu-2}\right]^{-(\nu+1)/2}$	$\nu > 2, \xi \in \Re$
Skewed GED	$\begin{cases} D(\nu)\exp\{-\beta_1 \lvert\varepsilon_t - m\rvert^\nu\} & \varepsilon_t < m \\ D(\nu)\exp\{-\beta_2 \lvert\varepsilon_t - m\rvert^\nu\} & \varepsilon_t \geq m \end{cases}$	$\nu > 0, \xi \in \Re$

$$\begin{aligned}
K(\nu) &= \frac{\Gamma[(\nu+1)/2]}{\sqrt{\pi(\nu-2)}\Gamma(\nu/2)} \\
C(\nu) &= \frac{\nu}{2}\sqrt{\frac{\Gamma(3/\nu)}{\Gamma(1/\nu)^3}} \\
\kappa_\nu &= \sqrt{2^{-\frac{2}{\nu}}\frac{\Gamma(1/\nu)}{\Gamma(3/\nu)}} \\
a &= K(\nu)\cdot 4\lambda\left(\frac{\nu-2}{\nu-1}\right) \\
b &= \sqrt{1+3\lambda^2-a^2} \\
\lambda &= \tanh(\xi) \\
\zeta_t &= \begin{cases} \frac{b\varepsilon_t+a}{1-\lambda} & \text{for } \varepsilon_t < -a/b \\ \frac{b\varepsilon_t+a}{1+\lambda} & \text{for } \varepsilon_t > -a/b \end{cases} \\
D(\nu) &= \frac{1}{\Gamma(1/\nu)}\sqrt{\frac{\Gamma(3/\nu)}{2\cdot\Gamma(1/\nu)}\left(\frac{1+\lambda}{1-\lambda}\right)^3 - (2\lambda\Gamma(2/\nu))^2}
\end{aligned}$$

Taken from Lucchetti & Balietti (2022)

The Autoregressive Mean Equation

Because what is being modeled in this study is the return series, which is basically mean reverting, the population regression function is assumed to follow a "first-order auto-regressive scheme." Hence, the mean equation is modelled as an AR(1) equation, in contrast to many studies (e.g., Alberg et al., 2008; Zivot, 2008; Bolllerslev et al., 1992) where the mean equation is a constant. For all the variants of the GARCH model considered in this study, the first-order autoregressive mean equation is used.

The Conditional Error Distribution

All the GARCH variants are invariably estimated using the maximum likelihood (or pseudo maximum likelihood) procedure, bringing to the fore the need to choose the most appropriate distribution of normalized error ε_t , see equation (7). In this study, five alternative error distributions are considered depending on the shape of the empirical distribution of the standardized residuals. These distributions are presented in Table 1 (taken from Lucchetti & Balietti, 2022).

Thus, in analyzing an empirical model for Bitcoin daily return, three specifications should be formulated:

1. The mean equation (first-order autoregressive)
2. The conditional variance equation (the GARCH variants), and
3. The error distribution (chosen empirically from the distributions in Table 1)

Data

Daily historical data on the closing price of Bitcoin in US$ per coin over the uninterrupted period of July 26, 2018, to February 20, 2022, involving 1,306 daily observations, constitutes the database of the study. This data set is conveniently subdivided into sub-periods labeled pre-pandemic and pandemic eras. The day March 12, 2020, when the World Health Organization (WHO) officially declared the onset of the pandemic, serves as the breakpoint. Source of data is www.CoinMarketCap.com

To Assess Whether Bitcoin (BTC) is Informationally Efficient or Not

The empirical strategy employed in the study to demonstrate BTC's informational efficiency is to show that it exhibits the martingale difference sequence (MDS) properties during the periods under review (pre-pandemic and pandemic). This can be accomplished by showing that Bitcoin has: (a) a unit root component and there is (b) the presence of uncorrelated increments of returns for each period.

MDS is a special form of the random walk under the stylized fact of volatility clustering of returns observed in most financial markets (Escanciano & Lobato,

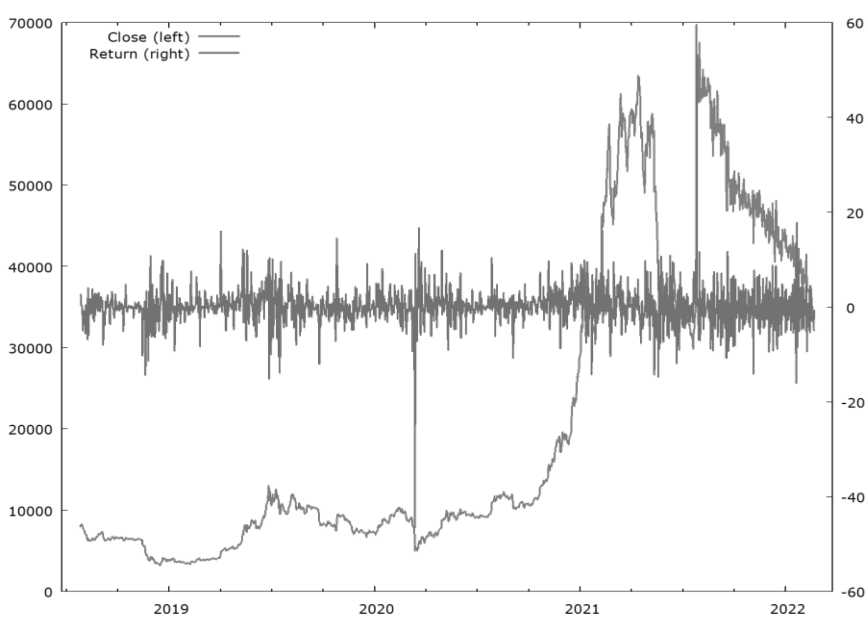

Figure 1. *Bitcoin Daily Closing Price and Continuously Compounded Rate of Return July 26, 2018 to February 20, 2022*

2009), and random walk is a hallmark of the so-called efficient market hypothesis (EMH; (Samuelson, 1965; Fama, 1970). Thus, market prices meeting the above two MDS requirements are deemed to be informationally efficient of the weak kind where supposedly, all available information is already factored into the market prices, and no one can take advantage of publicly available information to amass abnormal profit.

Variance Ratio Tests to Test for Uncorrelated Returns

Lo and MacKinlay (1988) provided the seminal basis for the variance ratio (VR) test, which has been used extensively in testing the market efficiency of the weak form. This empirical procedure explores the validity of the random walk hypothesis (RWH) by testing the property that the variance of random walk increments is linear in all sampling intervals (that is, the variance of q-period return is q times the variance of one-period return; Charles & Darne, 2009; Smith & Hyun-Jung Ryoo, 2003. Hence, the VR at lag q, which is defined as the ratio between (1/q) of the q-period return to the variance of the one-period return, should equal to 1 for all q.

If rr_t is assumed to be a realization of a stochastic process R_t that follows an MDS, which is known to be uncorrelated and may or may not be conditionally heteroscedastic, Lo and MacKinlay (1988) formulated two test statistics to undertake the VR test for the RWH. The first statistic works under the strong assumption of *i.i.d.* (identically and independently distributed) return with constant variance, whereas the other statistic downgraded the *i.i.d.* assumption to permit general types of time-varying volatility, which are often seen in financial time series (aka ARCH effect or volatility clustering). The associated null hypothesis under the heteroscedastic assumption is presented below.

From the variance ratio (VR) statistic (where r_t is rr_t in our study)

$$VR(r_t;\mathrm{q}) = \frac{\sum_{t=q}^{T}[\mathrm{r}_t + \ldots + r_{t-q+1} - q\bar{r}]^2}{q\sum_{t=1}^{T}[\mathrm{r}_t - \bar{r}]^2} \quad \text{with} \quad \bar{r} = \sum_{t=1}^{T} r_t / T \tag{8}$$

the VR test statistic $M(r_t,\mathrm{q})$ (shown to be asymptotic standard normal z) under the assumption of conditional heteroscedasticity (MDS null) proposed by Lo and Mackinlay (1988) is given by

$$z = M(r_t,\mathrm{q}) = \frac{VR(r_t,\mathrm{q}) - 1}{\sqrt{\psi(q)}} \quad \text{with } \psi(\mathrm{q}) = \sum_{k=1}^{q-1}\left[\frac{2(\mathrm{q}-k)}{q}\right]^2 \xi(k)$$

$$\xi(k) = \frac{\sum_{t=k+1}^{T}[r_t - \bar{r}]^2[r_{t-k} - \bar{r}]^2}{\sum_{t=1}^{T}[r_t - \bar{r}]^2} \tag{9}$$

As the variance ratio restriction holds for every q difference (or logarithmic difference) of the underlying series, for the holding period q ≥ 1, it is customary to evaluate this test statistics at several selected values of q (in this study q = 2, 4, 8, and 16). Chow and Denning (1993) proposed a test statistic used to examine the absolute values of a statistic set of multiple variance ratio statistics (for the different set values of q). The main purpose of this is to control the size (type I or the incorrect rejection of null hypothesis error probability) of a joint variance ratio test to be implemented. The null hypothesis for the Chow-Denning multiple VR test is set as the joint statement:

$$VR(q_i) = 1 \text{ for i} = 1, 2, \ldots, \text{m} \tag{10}$$

Against the alternative hypothesis that $VR(q_i) \neq 1$ for some holding period q. The Chow-Denning test statistic can be written as:

$$CD = \max\left|M(\mathrm{r}_t; q_i)\right| = \max(|\, z\,|) \tag{11}$$

For $VR(q_i) \neq 1$ where

$$MV(\mathrm{r}_t; q_i) = (VR(q_i) - 1)\left\{\sum_{j=1}^{q-1}\left[\frac{2(q_i - j)}{q_i}\right]^2 \xi_{q_i}\right\}^{-0.5}$$

$$\text{and} \quad \xi_{q_i} = \frac{\sum_{t=q_i+1}^{T}[r_t - \bar{r}]^2[r_{t-k} - \bar{r}]^2}{\sum_{t=1}^{T}[r_t - \bar{r}]^2} \tag{12}$$

To implement the Chow-Denning test, the maximum absolute value of the individual VR tests in Equation (9), which follows a studentized maximum modulus (SMM) distribution with m and T degrees of freedom (Chow & Denning, 1993), whose critical values are tabulated in Stoline and Uri (1979). The p-value for the SMM distribution is bounded above, with T approaching infinity. To apply the individual and joint MDS tests, the wild bootstrapping procedure (Mammen, 1993) is used as an alternative to the normal approximation because of its superior small sample properties (see Charles et al., 2009).

Results and Discussion

Applying the empirical strategy outlined in the Methodology section, I began with the descriptive stylized facts analysis of both the closing price and rate of return to Bitcoin over the periods in review. I then proceeded with the analytical tests and other inference procedures as applied to the database. Looking at Figure 1, daily returns for the whole sample horizon somewhat cluster around a constant value and, to some extent, exhibit autoregressive behavior. Taking a hint at this observed stationary behavior, the mean equation of the return series may be specified as a first-order autoregressive linear equation plus a time-varying noise element. The time graph of the return series also reveals a phenomenon of volatility clustering, as evidenced by episodes of wild swings and tranquil periods. As seen here, wild swings exceed calm episodes. It may be noted that the pre-pandemic and pandemic eras are separated conveniently by an extreme negative return, which occurred a day after the pandemic announcement by WHO (March 12, 2020). That day also appears to start a steady bull run in Bitcoin price, which culminated in all-time highs towards the earlier parts of 2021 but lost steam during the middle of that year, only to register record highs in the second half of August 2021 after which, steadily declined until the end of the sample period.

Based on the results of a battery of individual unit root tests on the price and the return series of Bitcoin, exhibited in Table 2 over the periods in review, the price series during the pre-pandemic and the pandemic eras is non-stationary with a single unit root. Meanwhile, all the unit root tests on the return series during the two periods produced highly significant stationarity. Without loss of generality, it may be concluded that the daily closing price of Bitcoin in both periods is *I(1)* series, while the daily return is *I(0)*.

Table 3 presents the important descriptive statistics and empirical tests central in verifying certain important aspects of analysis relevant to the study. These tests include the normality test of the return series via the Jarque-Bera (JB) test (the result of which indicates the absence of normality of return during both eras), the Ramsey RESET procedure, which empirically validated the adequacy of the mean equation for the two periods; and the highly important ARCH-LM test which resulted in the significant presence of the ARCH effects during the two eras.

Table 2. *Individual Unit Root Tests on the Daily Price and Return on Bitcoin During Pre-Pandemic and Pandemic Periods (p-values are in brackets)*

Time Period	ADF	Unit Root Tests Philips-Perron	ERS
Pre-Pandemic (7/26/18 – 3/12/20)			
BTC Close	-2.081774	-2.023250	28.91920
	[0.5537]ns	[0.5862]ns	[>0.10]ns
BTC Daily Return	-19.36095	-19.36095	0.520623
	[0.0000]***	[0.0000]***	[<0.01]**
Pandemic (3/13/20 – 2/20/22)			
BTC Close	-0.883783	-0.919497	51.99991
	[0.9554]ns	[0.5862]ns	[>0.10]ns
BTC Daily Return	-21.18889	-21.17266	0.82396
	[0.0000]***	[0.0000]***	[<0.01]**

Null Hypothesis: Closing Price/Return is I(1)
ns – not significant ***significant at 0.01 level* ****significant at 0.001 level*

Table 3. *Stylized Facts and Relevant Statistical Tests*

Time Period	Mean	StDev	Min	Max	JB-Stat (p-value)	ARCH-LM(7) RESET (p-value)	Ramsey (p-value)
Pre-pandemic (T =596)							
Daily Price (US$)	7165.19	2377.25	3236.76	13016.23	23.02456***	1.519739	1.539546^{n}
Daily Return (%)	-0.0008	0.0382	-0.4647	0.1601	29619.15***	(0.1577)	(0.1511)
Pandemic (T =710)							
Daily Price (US$)	31963.45	18749.09	5014.48	67566.83	63.91396***	2.89983	1.265348^{ns}
Daily Return (%)	0.002628	0.0475	-0.1608	0.0475	36056.13***	(0.0054)	(0.26535)

***p<0.01 ***p<0.001 ns-not significant (p>0.10)*

Table 4. *Estimates of the Alternative GARCH(1,1) Models for the Daily Returns for Bitcoin Using Generalized Error Distribution (GED) During the Pre-Pandemic Period*

Coefficients/ Models	GARCH (Bollerslev)	GARCH (Taylor/ Schwert)	NARCH (Higgins and Bera)	GJR (Glosten, et. al.)	TARCH (Zakoian)	EGARCH (Nelson)
Mean Equation						
Constant	0.0672287 0.0000 ***	0.0733939 0.0000 ***	0.0743691 0.0000 ***	0.0649216 0.0000 ***	0.0737051 0.0000 ***	0.0674831 0.0000 ***
AR(1) (ψ)	-0.0636674 0.0000 ***	-0.0427571 0.0000 ***	-0.0425670 0.0000 ***	-0.0793203 0.0000 ***	-0.0426965 0.0000 ***	-0.0634438 0.0000 ***
Conditional Variance Equation						
Omega (ω)	0.436821 0.4603^{ns}	0.558412 0.1271^{ns}	0.728761 0.0930*	0.466989 0.4108^{ns}	0.557635 0.1275^{ns}	-0.0725984 0.2725^{ns}
Alpha (α)	0.150413 0.2187^{ns}	0.166344 0.0002***	0.132783 0.0654*	0.155838 0.1778^{ns}	0.166887 0.0002***	0.268234 0.0002***
Beta (β)	0.850684 3.97e-013***	0.857522 3.85e-092***	0.857600 3.32e-145***	0.843981 2.03e-014***	0.857034 1.81e-091***	0.960710 8.92e-209 ***
Gamma (γ)				0.0604310 0.5837^{ns}	0.0444318 0.7620^{ns}	-0.0201452 0.5774^{ns}
Information Criteria						
Log Likelihood	-1466.56476	-1463.46225	***-1462.78405***	-1466.36137	-1463.41721	-1464.21291
AIC	2945.12953	***2938.92450***	2939.56810	2946.72274	2940.83442	2942.42582
BIC	2971.45080	***2965.24578***	2970.27625	2977.43089	2971.54258	2973.13398
HQC	2955.38048	***2949.17545***	2951.52754	2958.68218	2952.79386	2954.38526

Modeling the Conditional Variance Through GARCH Variants

Using the stylized facts uncovered in the descriptive analysis, together with the results of the different statistical tests, modeling the conditional variance in tandem with the mean equation model has become imperative. The seven different GARCH variants discussed earlier as the alternative conditional variance formulations for the autoregressive mean equation are implemented for the pre-pandemic and pandemic periods. In addition, the three different specifications on the error distribution give rise to a total of 21 alternative models for the daily returns of the Bitcoin in each subperiod. To remain parsimonious, these models consider only p = 1 and q = 1 for good reasons. For one, GARCH (1,1) has been considered as the "gold standard" in the literature because adding more ARCH and GARCH terms (i.e., p, q > 1) rarely adds more information and more significant coefficients to infer the data-generating process (DGP) of the series. Furthermore, this parsimonious representation has been known to be robust in modeling countless applied

Table 5. *Estimates of the Alternative GARCH (1,1) Models for the Daily Returns for Bitcoin Using Generalized Error Distribution (GED) During the Pandemic Period (3/13/19 -2/20/22)*

Coefficients/Models	GARCH (Bollerslev)	GARCH (Taylor/Schwert)	NARCH (Higgins and Bera)	GJR (Glosten, et. al.)	TARCH (Zakoian)	EGARCH (Nelson)
Mean Equation						
Constant	0.304556 0.0000***	0.304555 0.0000***	Did not converge	0.300544 0.0000***	0.305395 0.0000***	Did not converge
AR(1) (ψ)	-0.161615 0.0000***	-0.161615 0.0000***	Did not converge	-0.157913 0.0000***	-0.161685 0.0000***	Did not converge
Conditional Variance Equation						
Omega (ω)	0.110108 0.3826	0.184567 0.3524^{ns}	Did not converge	0.138224 0.2806	0.171456 0.3786	Did not converge
Alpha (α)	0.0401731 0.0546*	0.0641230 0.0156**	Did not converge	0.0345290 0.0556*	0.0644821 0.0155	Did not converge
Beta (β)	0.956487 0.0000***	0.946734 5.54e-279***	Did not converge	0.959023 0.0000***	0.947140 1.85e-289***	Did not converge
Gamma (γ)				0.239770 0.2649	-0.0964862 0.5264	Did not converge
Information Criteria						
Log Likelihood	-1974.98667	-1966.38692	Did not converge	-1973.82792	***-1966.23630***	Did not converge
AIC	3961.97334	***3944.77383***	Did not converge	3961.65584	3946.47260	Did not converge
BIC	3989.36493	***3972.16542***	Did not converge	3993.61270	3978.42945	Did not converge
HQC	3972.55485	***3955.35535***	Did not converge	3974.00094	3958.81770	Did notconverge

phenomena (Engle, 2001; Bollerslev et al., 1992; Zivot, 2008). Tables 4 and 5 show the results of implementing the different GARCH variants using the GED, which is found to be the most appropriate error distribution for both eras (model selection results are not shown here for lack of space). The skewed versions of the Student's t and GED are not considered because of the observed symmetry of the returns.

The outcome of modeling the return series during the pre-pandemic and the pandemic periods led to the use of the GED for the normalized error during both eras. Table 4 shows the modeling results of the variants of GARCH (1,1) during pre-pandemic, whereas Table 5 gives the results after the pandemic was announced. It can be seen in both tables that the mean equation appears to be statistically adequate for all variants as the parameters are all significant at the highest level, supplementing the results of the RAMSEY Reset in Table 3; although during the Pandemic era, both the NARCH and the EGARCH models failed to converge. For empirical comparison, four information criteria were used to adjudge the best empirical model. These are the log likelihood (max), the Akaike information criterion (AIC) (min), the Bayesian information criterion (BIC) (min), and the Hannan-Quinn criterion (HQC) (min). Applying the above criteria to choose the best GARCH (1,1) variant, Tables 4 and 5 turned in the verdict—the GARCH (1,1) model of Taylor and Schwert won the empirical comparison almost unanimously during both eras in all criteria, except log likelihood. Hence, the most inference may be based on the estimated form of this model.

Based on the GARCH results, it may be inferred that there existed no asymmetric effects before and during the pandemic. This is also evident by the insignificant gamma (γ) parameter of all asymmetric models (GJR, TARCH, and EGARCH) that converged in both eras. Further examining Tables 4 and 5, it may become clearer that volatility clustering is valid for Bitcoin returns by virtue of the significant estimates of the α and β parameters in all variants regardless of whether or not there is a pandemic. Volatility persistence is evident as captured by the estimate of $\alpha + \beta$ in GARCH (1,1) of Taylor (1986) and Schwert (1990) (TS), which is not significantly greater than 1 across eras. In fact, the estimate of TS gives $\alpha + \beta$ approximately equal to

Table 6. *Variance Ratio Tests for Random Walk of Daily Closing Price of Bitcoin (BTC) Using Wild Bootstrap*

Ho: BTC_PrePandemic is a Martingale		Lo-MacKinlay Individual Variance Ratio Tests (7/26/18 – 3/12/20)				
Chow-Denning Joint Variance Ratio Test		Holding Period (q)				
		Period	2	4	8	16
		Var. Ratio	0.876096	0.852688	0.864128	0.866783
Max |z|	1.640314	**Std. Error**	0.075537	0.137695	0.208214	0.286624
Degrees of Freedom	595	**z-Statistic**	-1.640314	-1.069841	-0.652562	-0.464782
p-value	0.4440	**p-value**	0.3500	0.4960	0.6520	0.7320

1 (0.0402 plus 0.9565) during the pandemic, implying that the unconditional variance tends to infinity; hence volatility is persistent. Mathematically, if the ratio of ω / (1- (α + β)) is infinite, as in TS during the pandemic, the future unconditional variance is not constant (or persistent). This may lead to extremely high or extremely low returns, which offers a lot of earning opportunities to long-term investors (a.k.a. the "HODLers").

Informational Efficiency of Bitcoin Market

Potential investors of Bitcoin during the pandemic are expectant of unlimited earning possibilities of this cryptocurrency. Some even speculate that BTC may even replace gold as a refuge asset during uncertain times, like what the world is now facing (e.g., see Chemkha et al., 2021). One way of providing the empirical content to this expectation is to show that BTC is informationally efficient. In this study, I endeavor to ascertain that there has been a transition of BTC from informationally efficient before the pandemic into one that is informationally inefficient during the pandemic. Using the empirical strategy adopted for this purpose, I venture to prove that BTC was a "Martingale" before the pandemic, but has transitioned into a non-Martingale stochastic process during the pandemic. Table 6 provides the empirical results.

Conclusion

The emergence of the COVID-19 pandemic has been leaving in its wake countless broken lives, devastated institutions, and compromised processes—widespread suffering among various stakeholders of the global economy. Prior to the pandemic, Bitcoin was hailed as the future of money with its numerous desirable attributes. Many are wondering about the future of Bitcoin in light of the pandemic, which for more than two years now has been devastating the world. This study, through stylized facts analysis, statistical testing, and cutting-edge econometric modeling, hopes to contribute to the understanding of this most dominant cryptocurrency. The general conclusion of the study: the pandemic further strengthen the volatility structure of Bitcoin with no leverage effects, and the market for this prime cryptocurrency has become informationally inefficient, which may give rise to tremendous profit opportunities for long-term investors.

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