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Stock Diversification and Integer Programming

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This study shows how investors can maximize returns by preparing and monitoring their own stock portfolio by using an integer programming model with an algorithm that can be computed in spreadsheet and linear programming software. Further, the study emphasizes the importance of diversifying stockholdings to reap optimal returns and minimum volatility/risk. It also suggests employing behavioral portfolio theory where goals/aspirations of investors are combined with their reward-to-volatility profile.

Keywords: Portfolio theory, mixed-integer programming, zero-one integer programming, reward-to-volatility ratio

Modern Portfolio Theory (MPT) is based on a simple assumption that risk is defined by volatility. According to the theory, investors are risk averse: they are willing to accept more risk (volatility) for higher payoffs and will accept lower returns for a less volatile investment. The theory is simple and elegant, and can lead further into ingenious mathematical proofs and equations, which probably has a lot to do with why it has become so widely accepted.

When Harry M. Markowitz and William F. Sharpe – pioneers of Modern Portfolio Theory – needed a definition of risk, they chose to define risk as volatility: the greater the volatility of the portfolio, measured either in terms of standard deviation or beta, the greater the risk. They concluded that volatility was a good measure of risk when an observation was made that the share market, which is generally thought to be more risky than cash investments, had the highest volatility. The principle was adopted generally without further evidence that volatility was a good way to measure risk.

Reward-to-Volatility Ratio

A ratio developed by Jack L. Treynor measures returns earned in excess of that which could have been earned on a riskless investment per unit of market risk. The Treynor ratio (*Treynor ratio*, 2008) is a risk-adjusted measure of return based on systematic risk. It is used to calculate that extra amount of return that is in excess to what would have been earned through safe investments or without taking any kind of risk. It is also termed as reward-to-volatility ratio as it measures the earnings made while facing the risks. Whenever the Treynor ratio is on the higher side, it denotes that the investor is provided with high yields by each unit of market risk.

Mean-Variance Portfolio Theory

In mean-variance portfolio theory, the optimal level of diversification is determined by marginal analysis; that is, diversification should be increased as long as the marginal benefits exceed its marginal costs. The benefits of diversification in mean-variance portfolio theory are in the reduction of risk. Risk is measured by the standard deviation of portfolio returns. The costs are transaction and holding costs. Statman (2004) illustrated that expected standard deviation declines as portfolios becomes increasingly diversified.

Statman (2004) argued that the benefits of diversification as measured by the rules of mean-variance portfolio theory have increased in recent years, but the level of diversification in investor portfolios has not changed. It remains below the optimal level currently prescribed by mean-variance optimization, which exceeds 300 stocks.

Further, Campbell, Lettau, Malkiel, and Xu (2001, as cited in Statman, 2004), in their study of U.S. stocks, concluded that there is a clear tendency for correlations among individual stocks to decline over time as evidenced by correlations based on five years of monthly data from 0.28 in the early 1960s to 0.08 in 1997. Accordingly, the declining correlations among stocks imply that the benefits of diversifying stock portfolio are increasing over time. A conventional rule of thumb supported by the results of Bloomfield, Leftwich, and Long (1977, as cited in Statman, 2004) indicated that the total benefits of equity diversification are obtained with a portfolio of 20 stocks.

Contrary to these findings, Statman (2004) noted that actual levels of equity diversification were much lower than 20 stocks in 1977 and remains so today. Studies revealed that the mean number of stocks in a portfolio in the 1991-1996 period was four and the median number was three (Goetzmann & Kumar, 2001, as cited in Statman, 2004). Additional survey of 14 million households conducted by Polkovnichenko (1998, as cited in Statman, 2004) yielded portfolio holdings of one to five stocks.

How Diversification Reduces Risks

The question of predicting future stock performance for possible recommendation to buy or sell cannot be found in the newspaper; newspapers seem to avoid definite statements about prospects for securities. Most financial analysts start by observing past variability. In Brealey and Myers (1996), it was surmised that it is safe to assume that there is no risk in hindsight, but it is also reasonable to say that portfolios with histories of high variability also have the least predictable future performance.

Measures of variability can be equally calculated for individual securities and portfolio of securities. The Philippine Stock Index (PHISIX) has 32 stocks and Appendix A presents the estimated standard deviations for each of the stocks for the period 1996 to 1998.

This raises an important question. The market portfolio (PHISIX) is made up of individual stocks, so why doesn't its variability reflect the average variability of its component? The answer, which is the focus of this study, is that diversification reduces variability.

Even a little diversification can provide a substantial reduction in variability. Calculating and comparing the standard deviations of randomly selected one-stock portfolios, two-stock portfolios, five-stock portfolios, and so forth, would yield the result that diversification can cut the variability of returns by about half. However, most of the resulting benefits can be attained with relatively few stocks. Brealey and Myers (1996) concluded that the improvement is slight when the securities are increased beyond 20 or 30. (See Figure 1.)

Diversification works because prices of different stocks do not move exactly together. Most statisticians would agree that stock price changes are less than perfectly correlated. An investment in one stock could be very variable, but on many occasions, a decline in the value of one stock was canceled out by a rise in the price of another stock. Therefore, there was an opportunity to reduce risk by diversification, that is, if funds were divided evenly between two or more stocks, the variability of the portfolio would have been substantially less than the average variability of each of the stock.

The risk that can be potentially eliminated by diversification is called unique risk (Brealey & Myers, 1996). Unique risk stems from the fact that many of the perils that surround an individual

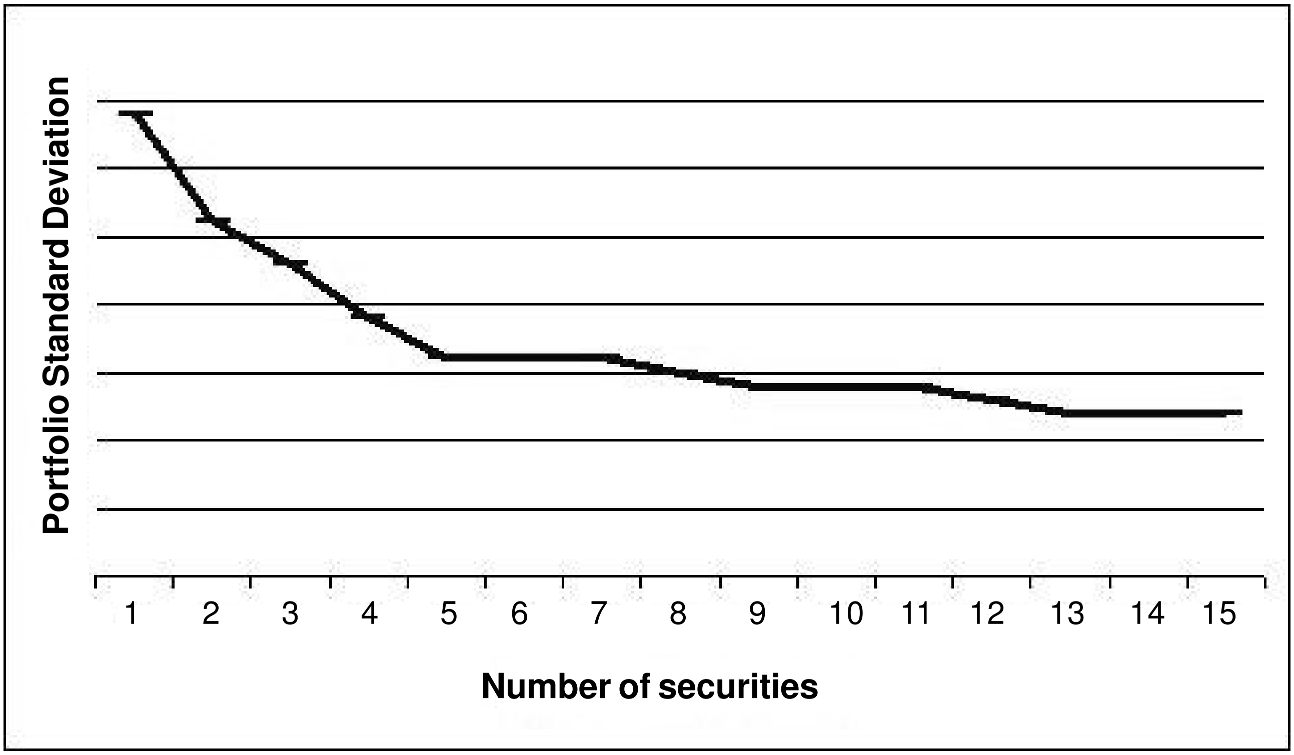


Figure 1. *Diversification reduces risk (standard deviation) rapidly at first, then more slowly*

company are peculiar to that company and perhaps its immediate competitors. But there are some risks that cannot be avoided regardless of how much diversification is made. This is generally known as market risk. Market risk stems from the fact that there are economy-wide perils which threaten all businesses. That is why stocks have a tendency to move together. And that is why investors are exposed to "market uncertainties" no matter how many stocks they hold. Thus, if an investor has only a single stock, unique risk is very important; but once a 20 or more stock portfolio is held, diversification has done most of its work where only market risk matters. Therefore, the predominant source of uncertainty for a diversified investor is that the market will rise or plummet, carrying the investor's portfolio with it.

Integer Programming

An integer programming model, as defined in Render, Stair, and Hannah (2008), is a model that has constraints and an objective function identical to that formulated by linear programming. The only difference is that one or more of the decision variables has to take on an integer value in the final solution. There are three types of integer programming models:

1. Pure integer programming involves cases in which all variables are required to have integer values;
2. Mixed-integer programming is suitable when some, but not all, of the decision variables are required to have integer values; and
3. Zero-one integer programming is used for special cases in which all decision variables must have integer solution values of 0 or 1.

RESEARCH OBJECTIVES

This study aims to present a basic method of determining the portfolio of stocks that would yield optimal reward-to-volatility (RTV) ratio by calculating fluctuations in the prices of stock issues that might indicate variability or spread of year-to-year returns. A histogram or frequency distribution of stock prices could graphically, but crudely, illustrate these data. More sophisticated and accurate measures such as variance and standard deviation would be suitable in calculating these data needed as inputs to the integer programming model.

To meet these objectives, the following assumptions were made in this study: (1) The model does not encompass all factors affecting choice of stocks to be included in the portfolio such as political and economic uncertainties. Thus, market risk or volatility may not be totally accounted for by the model. (2) The model also assumes that the 32 stocks comprising the PHISIX are representative of the most viable stocks in the Philippine Stock Exchange. (3) The coverage of the model can be extended up to a definite period of time only, specifically for the most recent five-year period since the level of variability or volatility for a longer period of time may not be as interesting and realistic for listed companies. It is a rare for a company to face the same business risk today as it did 10 or 50 years ago. (4) The Microsoft Excel function used to compute the RTV is assumed to be similar to the variance and standard deviation formulas found in textbooks.

FRAMEWORK

Developing the Model and the Solution

The integer programming model is intended to maximize the value of the objective function, which is the ratio of reward-to-volatility ratio, by selecting a portfolio composed of five stocks from the 32 stocks comprising the PHISIX. Using zero-one integer programming, decision variables with solution value of "1" will be selected. At the same time, sensitivity analysis can be performed.

The portfolio's reward-to-volatility ratio is the sum of the individual stock's reward-to-volatility ratio (R) gained from each of the five stocks to be included in the portfolio. R for each of the stocks is determined by dividing the reward or average change in a stock's monthly closing price by the standard deviation of the same stock's average change in monthly closing prices. Total reward-to-volatility ratio will be defined as Z, and the objective function can be expressed mathematically as:

$$Maximize\ Z = \sum R_i X_i$$

where:

$$R = \frac{average\ change\ in\ the\ monthly\ closing\ price\ of\ a\ stock}{standard\ deviation\ of\ the\ change\ in\ the\ montly\ closing\ price\ of\ a\ stock}$$

X = solution value of the decision variable as either "0" or "1"

i = stocks that can be included in the five-stock portfolio (1, 2, 3, …, 32)

In this model, the decision variables (stocks) can have a solution value of either "0" or "1". If the decision variable representing a stock will have a value of "0", that stock will not be selected to be included in the five-stock portfolio. On the other hand, if the decision variable representing a stock will have a value of "1", that stock will be selected to be included in the five-stock portfolio.

The model is subject to the following constraints:

1. $X_i = 0\ or\ 1$
2. $\sum X_i = 5$

There would be two constraints per decision variable, one constraint for a solution value of "1" and another constraint for a value of "0". Since there are 32 stocks with two constraints each, a total of 64 zero-one constraints are included in the model. Meanwhile, the imposition of the constraint reflects the contingency that only five out of the 32 stocks can be selected to maximize reward-to-volatility ratio. A variation of the multiple choice constraint, according to Brealey and Myers (1996), can be used to formulate this constraint where some specific number of stocks out of the total must be selected.

Computation of Reward-To-Volatility Ratio

The standard statistical measures of spread of outcomes are variance and standard deviation. The choice between the two is a matter of convenience. Since standard deviation is in the same units as the rate of return (i.e., changes in monthly average stock closing price), it is generally more convenient to use than variance. However, if proportion of risk due to some factor is considered, it is usually less confusing to work in terms of variance.

In principle, the variability of any portfolio of stocks or bonds can be estimated by getting the average changes in stock prices, then computing for its standard deviation. Thus:

$$\frac{P_1 - P_0}{P_0} \tag{1}$$

refers to the change in stock price or return or reward (if positive), where P_1 is the average monthly closing price of the succeeding period and P_0 is the average monthly closing price of the preceding period.

$$\frac{\left(\dfrac{P_1 - P_0}{P_0}\right)}{\textit{total number of periods}} \tag{2}$$

refers to the average change in stock price or return or reward (also called as the mean of the changes in stock prices for the period under study).

Using Microsoft Excel function:

= VAR (range of values of changes in stock price for all periods) (3)

refers to the variance of changes in stock prices for the period under study.

Using Microsoft Excel function:

= SQRT (variance) (4)

refers to the standard deviation of a specific issue.

$$R = \frac{\textit{average change in the monthly closing price of a stock}}{\textit{standard deviation of the change in the montly closing price of a stock}}$$

refers to reward-to-volatility ratio.

RESULTS OF THE MODEL

The reward-to-volatility ratios for each of the 32 stocks were inputted in the WinQSB program. Using relevant data computed for all stocks, which include average change in the monthly closing prices of each of the 32 stocks, variance of all the monthly changes in a stock's closing price, and the standard deviation for each of the stock, reward-to-volatility ratios were generated. Microsoft Excel was used to compute for these data.

Objective Function Coefficient

Appendix B illustrates the integer programming results. The efficient portfolio combination yielded an optimal value of reward-to-volatility ratio of 0.14. This means that the composition of the five-stock portfolio (see the highlighted rows in Appendix B) would yield maximum reward-to-volatility ratio 0.14 units with every one unit of volatility or wide fluctuation in their prices. These stocks include: (1) Ayala Corporation (AC, X_2); (2) Filinvest Land, Inc. (FLI, X_{11}); (3) Jollibee Foods Corporation (JFC, X_{14}); (4) San Miguel Corporation A (SMC; X_{29}); and (5) Philippine Long Distance Telephone Company (PLDT; X_{30}).

Sensitivity Analysis

Appendix C illustrates the shadow prices and right hand side (RHS) ranges of all 32 stocks comprising the PHISIX for the period 1996-1998. The shadow prices column indicated that out of the 32 stocks included in the study, only four out of the five-stock portfolio that yielded solution values of one had non-zero shadow prices.

Allowable number of stocks in the portfolio. Another conclusion from Appendix C (Row 65) is the RHS range for the five-stock portfolio indicating that the number of stocks in the portfolio can be decreased to as low as four or increased to as much as five without changing the optimal solution mix of attaining maximum reward-to-volatility ratio of 0.14.

The lowest number of the stocks in the portfolio should be four (i.e., any number lower than this will not maximize reward-to-volatility ratio). This means that the portfolio can contain four stocks out of the five stocks or all of the stocks yielded by the optimal solution mix. Thus, an investor could have a combination of TEL, JFC, AC, and SMC that will yield RTV ratio of 0.14. Meanwhile, the allowable maximum RHS value for this constraint is "5" meaning, an addition of one more stock in the portfolio will not maximize the reward-to-volatility ratio.

The five-stock portfolio analysis. Out of the five stocks in the portfolio, TEL yielded the highest reward-to-volatility ratio at 0.09, followed by JFC and AC at 0.04 and 0.02, respectively. FLI and SMC had -0.01 and 0.00 reward-to-volatility ratios, respectively. The combined five-stock portfolio yielded the optimal reward-to-volatility ratio, any other combination will yield lower RTV ratios.

Shadow prices for the five-stock portfolio. Row 65 in Appendix D indicates that for every unit of a stock in the portfolio added, reward-to-volatility ratio increases by 0.01. Also in Appendix D, it can be observed that, of the five stocks selected to be part of the portfolio that would maximize reward-to-volatility ratio, FLI had zero shadow price, which means that even if FLI is added to the portfolio, reward to volatility ratio will not increase at all.

The rest of the stocks in the portfolio in Appendix D yielded positive contributions to the objective function whenever one unit of that stock is added to the portfolio. AC's shadow price of 0.03 means that if this stock is added to the portfolio, the reward-to-volatility ratio will be increased by 0.03. JFC's shadow price of 0.05 means that if this stock is added to the portfolio, the reward-to-volatility ratio will be increased by 0.05. SMC's shadow price of 0.01 means that if this stock is added to the portfolio, the reward-to-volatility ratio will be increased by 0.01. TEL's shadow price of 0.10 means that if this stock is added to the portfolio, the reward-to-volatility ratio will be increased by 0.10.

CONCLUSION

Comparison of Five-stock Portfolio and Individual Stock returns

From the discussions above, the portfolio diversification theory showed decreased volatility when a five-stock portfolio is implemented as against the individual stocks. Figure 2 below showed the level of returns for the five-stock portfolio which attained an optimal reward-to-volatility ratio of 0.14.

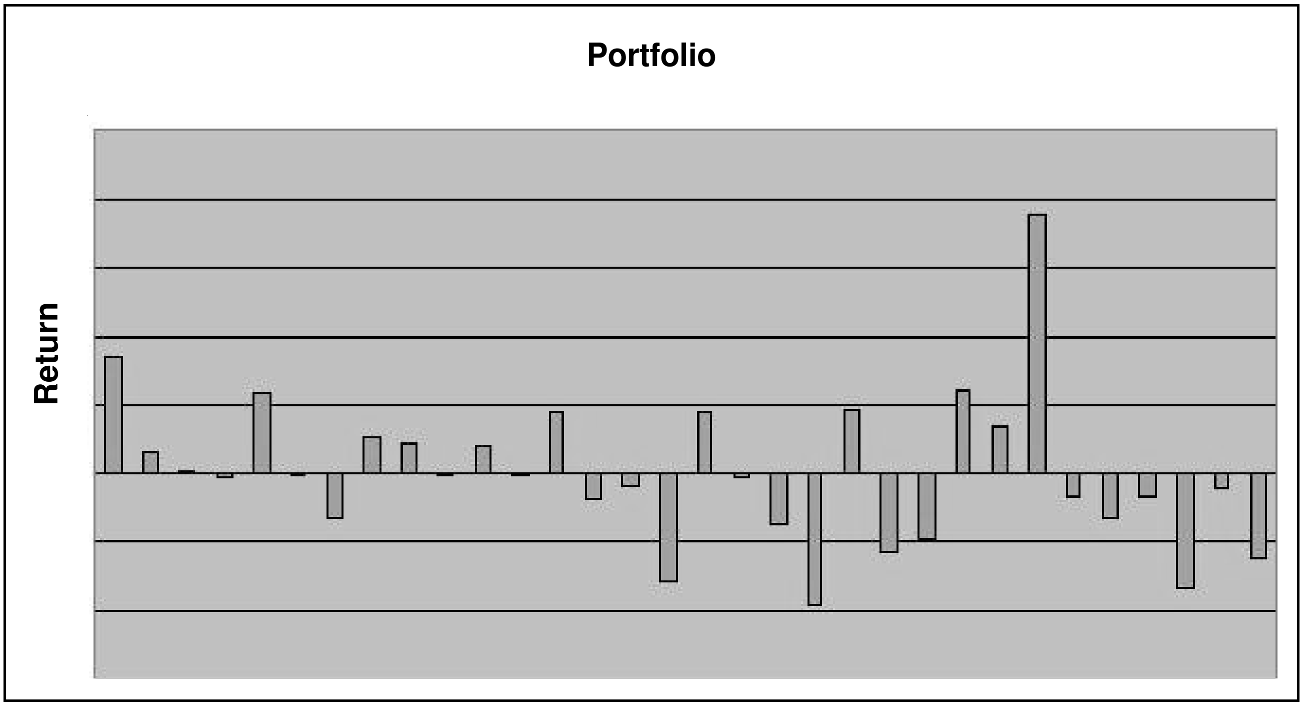
Wide fluctuations in changes in closing prices in Figure 2 proved to be less in a portfolio (in this case average returns of the five-stock portfolio of AC, FLI, JFC, SMC, and TEL) than in individual stocks (see Figures 3 to 6).

Future Research Direction

It is evident that the integer programming approach can be used to determine which stocks to include in a portfolio of equity investment. Furthermore, the model can be extended to other periods such as from 1998 to the most recent closing prices.

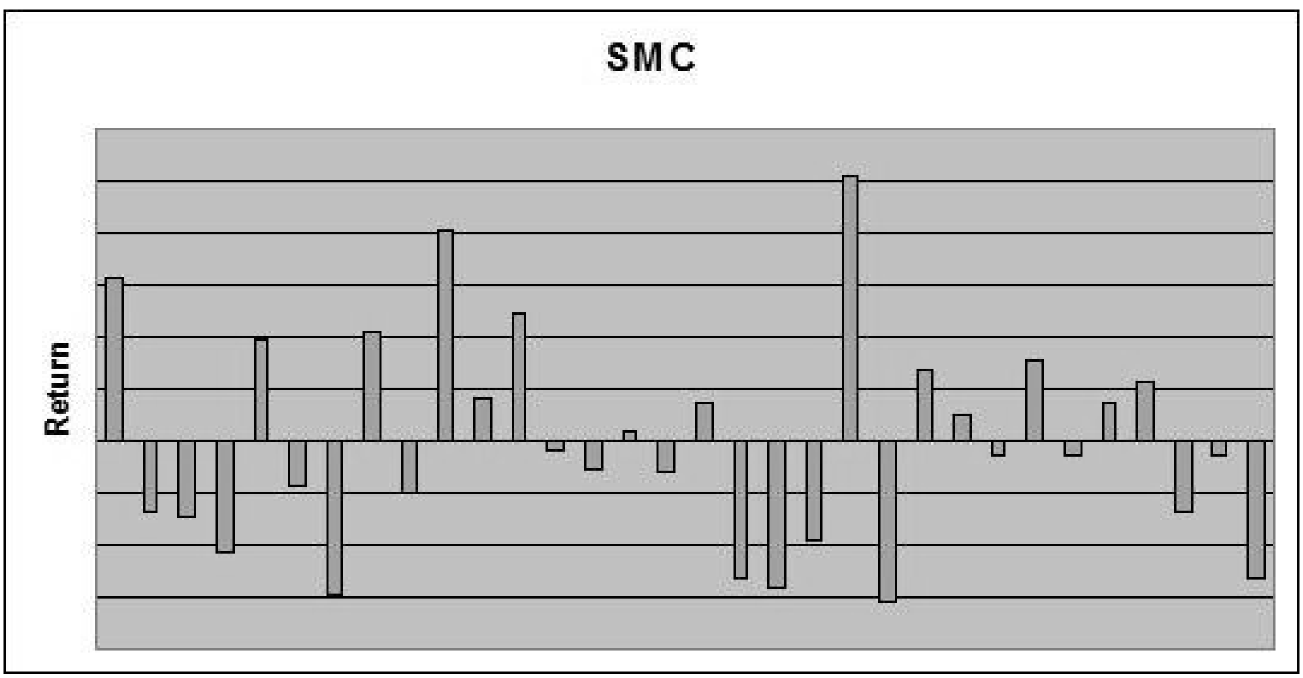
According to Bertsimas, Darnell, and Soucy (1999), mixed-integer programming (MIP) is a more sophisticated method to construct portfolios that are close (in terms of sector and security exposure) to target portfolios, and have the same liquidity, turnover, and expected returns.

However, the shortcomings of the portfolio theory cited in the previous sections necessitated the testing of the behavioral portfolio theory of Shefrin and Statman (2000) in this study. Behavioral portfolio theory does not consider portfolios as a whole since behavioral investors divide their money into two layers of a portfolio pyramid, a downside protection layer designed to protect them from poverty, and an upside potential layer designed to make them rich. Unlike the



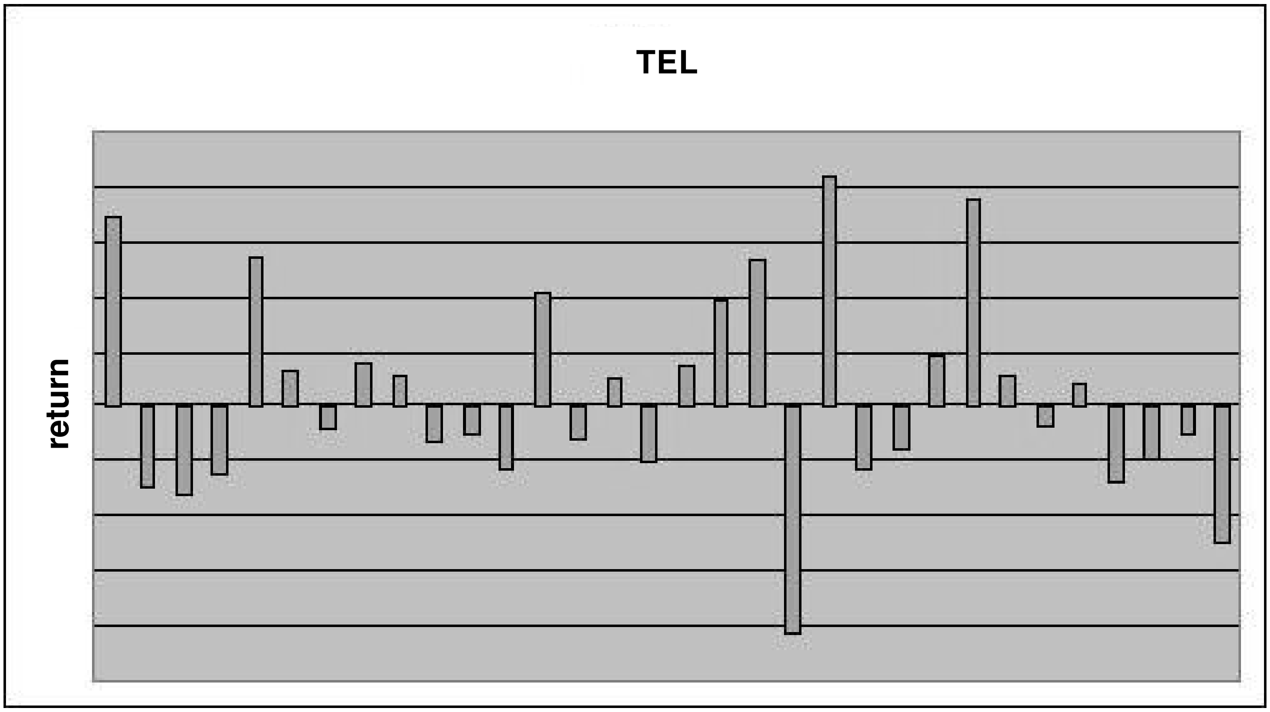
Note: Horizontal (x) axis refers to risk while vertical (y) axis refers to return.

Figure 2. *Returns of Five-stock Portfolio*



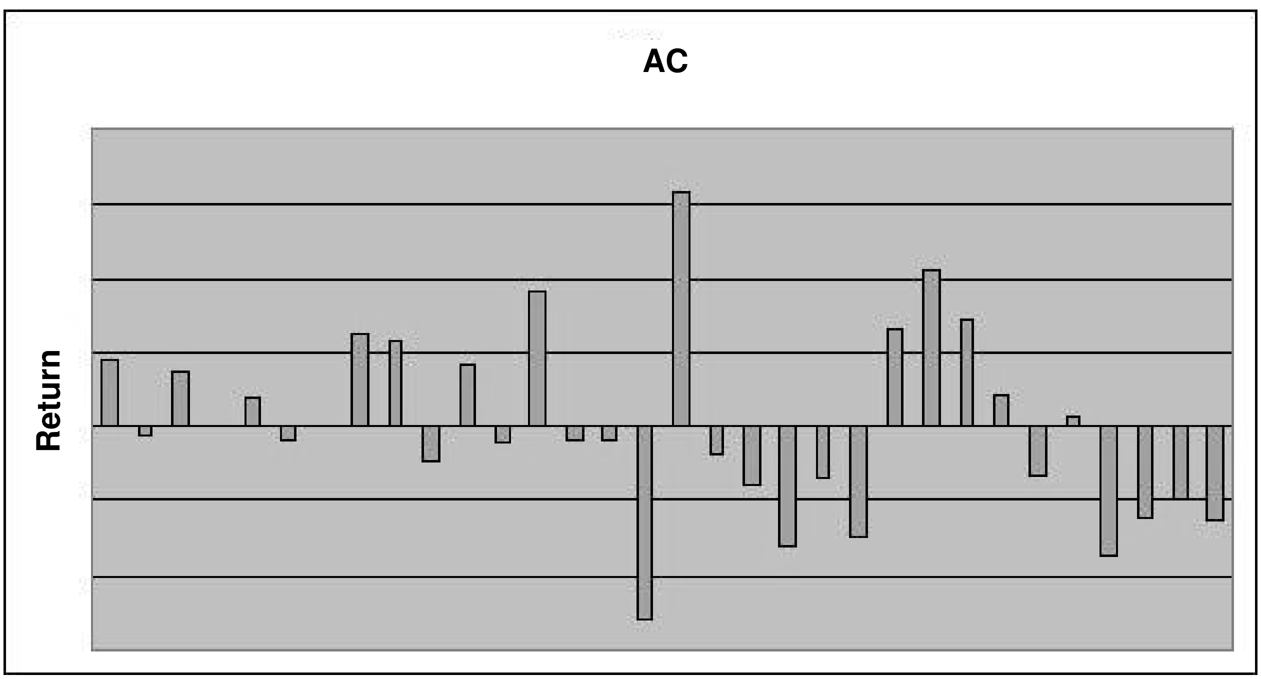
Note: Horizontal (x) axis refers to risk while vertical (y) axis refers to return.

Figure 3. *Stock Returns of SMC*



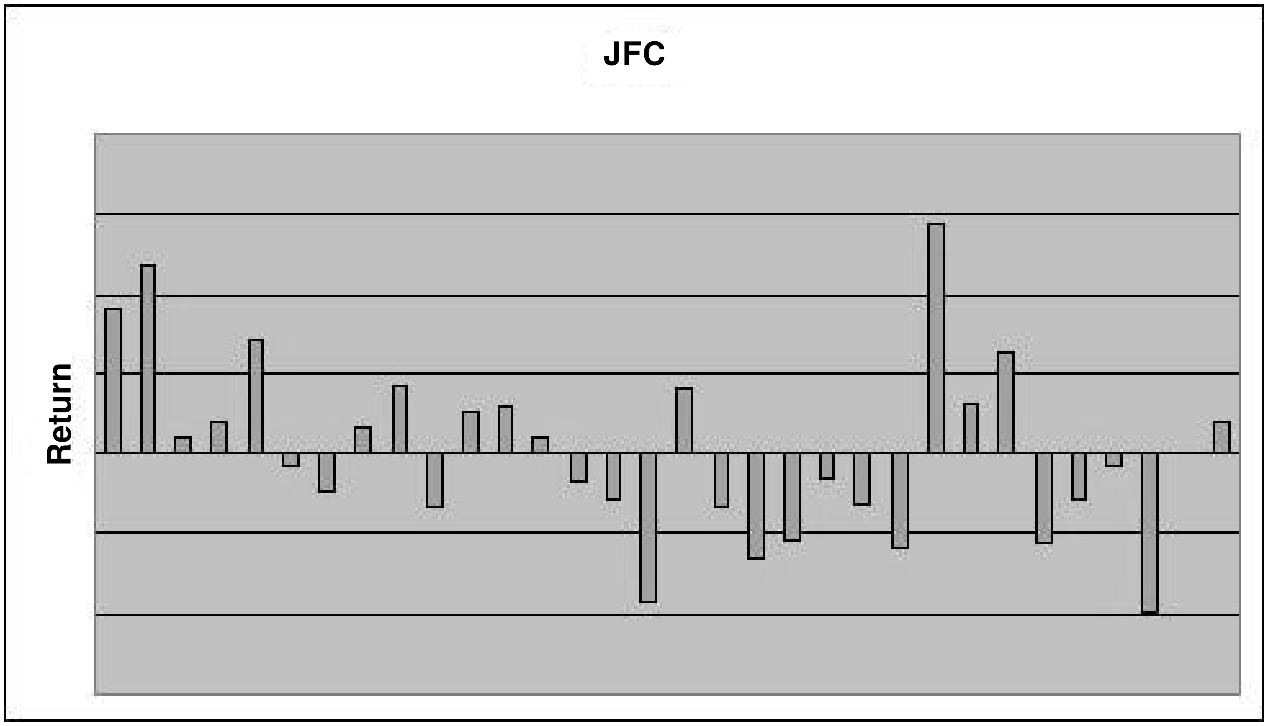
Note: Horizontal (x) axis refers to risk while vertical (y) axis refers to return.

Figure 4. *Stock Returns of TEL*



Note: Horizontal (x) axis refers to risk while vertical (y) axis refers to return.

Figure 5. *Stock Returns of AC*



Note: Horizontal (x) axis refers to risk while vertical (y) axis refers to return.

Figure 6. *Stock Returns of JFC*

reward-to-volatility portfolio, the behavior investors' outlook for risk assumes a segmentation of portfolio into layers depending on where investors are willing to take risks with some of their money; reward-to-volatility investors have a single attitude toward risk with all their money.

In short, although the rules of diversification in behavioral portfolio are not as precise as the rules in reward-to-volatility portfolio theory, they are clear enough as guidelines for investors, financial advisors, and companies who should draw the line between upside potential and downside protection. In the future, integer programming should not only include changes in stock prices but also risk and aspiration profiles of the investors in the behavioral portfolio theory.

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Appendix A
Estimated Variability of the 32 Stocks Comprising the PHISIX

	Decision Variable	Mean Change in Price	Variance	Standard Deviation	Reward-to-Volatility Ratio
1	ABS	-0.01	.01157	0.11	-0.10
2	AC	0.00	.01531	0.12	0.02
3	AEV	-0.03	.0352	0.19	1.12
4	ALI	-0.01	.01786	0.13	-0.04
5	BPC	-0.00	.0038	0.06	-0.07
6	BEL	-0.02	.03782	0.19	-0.13
7	CMP	-0.06	.03656	0.19	-0.32
8	CMT	-0.05	.02618	0.16	-0.31
9	DMC	-0.02	.06566	0.26	-0.09
10	FDC	-0.04	.02749	0.17	-0.25
11	FLI	-0.00	.01172	0.34	-0.01
12	ICT	-0.02	.05127	0.23	-0.09
13	ION	-0.00	.02035	0.14	-0.02
14	JFC	0.00	.01265	0.11	0.04
15	JGS	-0.03	.01974	0.14	-0.25
16	LND	-0.07	.05198	0.23	-0.29
17	MBT	-0.01	.0179	0.13	-0.08
18	MEG	-0.04	.0631	0.25	-0.16
19	MER	-0.01	.00843	0.09	-0.08
20	MERB	-0.01	.01034	0.10	-0.08
21	MPC	-0.03	.03736	0.19	-0.17
22	PCI	-0.01	.02534	0.16	-0.09
23	PCOR	-0.03	.02751	0.17	-0.17
24	PLTL	-0.05	.05271	0.23	-0.23
25	PNB	-0.04	.02285	0.15	-0.27
26	PX	-0.03	.01485	0.12	-0.25
27	PXB	-0.03	.01485	0.12	-0.25
28	SMCB	-0.00	.01683	0.13	-0.03
29	SMC	-0.00	.01052	0.10	-0.00
30	TEL	0.01	.00865	0.09	0.09
31	UBP	-0.01	.00675	0.08	-0.13
32	URC	-0.04	.0281	0.17	-0.25

Appendix B
Combined Report for Stock Selection

	Decision Variable	Solution Value	Unit Cost or Profit	Total Contribution	Reduced Cost	Basis Status	Allowable Min	Allowable Max
1	ABS	0	-0.1000	0	-0.0900	at bound	-M	-0.0100
2	**AC**	**1.0000**	**0.0200**	**0.0200**	**0**	**basic**	**-0.0100**	**M**
3	AEV	0	-0.1500	0	-0.1400	at bound	-M	-0.0100
4	ALI	0	-0.0400	0	-0.0300	at bound	-M	-0.0100
5	BPC	0	-0.0700	0	-0.0600	at bound	-M	-0.0100
6	BEL	0	-0.1300	0	-0.1200	at bound	-M	-0.0100
7	CMP	0	-0.3200	0	-0.3100	at bound	-M	-0.0100
8	CMT	0	-0.3100	0	-0.3000	at bound	-M	-0.0100
9	DMC	0	-0.0900	0	-0.0800	at bound	-M	-0.0100
10	FDC	0	-0.2500	0	-0.2400	at bound	-M	-0.0100
11	**FLI**	**1.0000**	**-0.0100**	**-0.0100**	**0**	**basic**	**-0.0200**	**0**
12	ICT	0	-0.0900	0	-0.0800	at bound	-M	-0.0100
13	ION	0	-0.0200	0	-0.0100	at bound	-M	-0.0100
14	**JFC**	**1.0000**	**0.0400**	**0.0400**	**0**	**basic**	**-0.0100**	**M**
15	JGS	0	-0.2500	0	-0.2400	at bound	-M	-0.0100
16	LND	0	-0.2900	0	-0.2800	at bound	-M	-0.0100
17	MBT	0	-0.0800	0	-0.0700	at bound	-M	-0.0100
18	MEG	0	-0.1600	0	-0.1500	at bound	-M	-0.0100
19	MER	0	-0.0800	0	-0.0700	at bound	-M	-0.0100
20	MERB	0	-0.0800	0	-0.0700	at bound	-M	-0.0100
21	MPC	0	-0.1700	0	-0.1600	at bound	-M	-0.0100
22	PCI	0	-0.0900	0	-0.0800	at bound	-M	-0.0100
23	PCOR	0	-0.1700	0	-0.1600	at bound	-M	-0.0100
24	PLTL	0	-0.2300	0	-0.2200	at bound	-M	-0.0100
25	PNB	0	-0.2700	0	-0.2600	at bound	-M	-0.0100
26	PX	0	-0.2500	0	-0.2400	at bound	-M	-0.0100
27	PXB	0	-0.2500	0	-0.2400	at bound	-M	-0.0100
28	SMCB	0	-0.0300	0	-0.0200	at bound	-M	-0.0100
29	**SMC**	**1.0000**	**0**	**0**	**0**	**basic**	**-0.0100**	**M**
30	**TEL**	**1.0000**	**0.0900**	**0.0900**	**0**	**basic**	**-0.0100**	**M**
31	UBP	0	-0.1300	0	-0.1200	at bound	-M	-0.0100
32	URC	0	-0.2500	0	-0.2400	at bound	-M	-0.0100

Appendix C
Sensitivity Analysis of 32 PHISIX Stocks

	Constrait	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	ABS	0	<=	1.0000	1.0000	0	0	M
2	ABS	0	>=	0	0	0	-M	0
3	**AC**	**1.0000**	**<=**	**1.0000**	**0**	**0.0300**	**1.0000**	**1.0000**
4	**AC**	**1.0000**	**>=**	**0**	**1.0000**	**0**	**-M**	**1.0000**
5	AEV	0	<=	1.0000	1.0000	0	0	M
6	AEV	0	>=	0	0	0	-M	0
7	ALI	0	<=	1.0000	1.0000	0	0	M
8	ALI	0	>=	0	0	0	-M	0
9	BPC	0	<=	1.0000	1.0000	0	0	M
10	BPC	0	>=	0	0	0	-M	0
11	BEL	0	<=	1.0000	1.0000	0	0	M
12	BEL	0	>=	0	0	0	-M	0
13	CMP	0	<=	1.0000	1.0000	0	0	M
14	CMP	0	>=	0	0	0	-M	0
15	CMT	0	<=	1.0000	1.0000	0	0	M
16	CMT	0	>=	0	0	0	-M	0
17	DMC	0	<=	1.0000	1.0000	0	0	M
18	DMC	0	>=	0	0	0	-M	0
19	FDC	0	<=	1.0000	1.0000	0	0	M
20	FDC	0	>=	0	0	0	-M	0
21	**FLI**	**1.0000**	**<=**	**1.0000**	**0**	**0**	**1.0000**	**M**
22	**FLI**	**1.0000**	**>=**	**0**	**1.0000**	**0**	**-M**	**1.0000**
23	ICT	0	<=	1.0000	1.0000	0	0	M
24	ICT	0	>=	0	0	0	-M	0
25	ION	0	<=	1.0000	1.0000	0	0	M
26	ION	0	>=	0	0	0	-M	0
27	**JFC**	**1.0000**	**<=**	**1.0000**	**0**	**0.0500**	**1.0000**	**1.0000**
28	**JFC**	**1.0000**	**>=**	**0**	**1.0000**	**0**	**-M**	**1.0000**
29	JGS	0	<=	1.0000	1.0000	0	0	M
30	JGS	0	>=	0	0	0	-M	0
31	LND	0	<=	1.0000	1.0000	0	0	M
32	LND	0	>=	0	0	0	-M	0

	Constrait	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
33	MBT	0	<=	1.0000	1.0000	0	0	M
34	MBT	0	>=	0	0	0	-M	0
35	MEG	0	<=	1.0000	1.0000	0	0	M
36	MEG	0	>=	0	0	0	-M	0
37	MER	0	<=	1.0000	1.0000	0	0	M
38	MER	0	>=	0	0	0	-M	0
39	MERB	0	<=	1.0000	1.0000	0	0	M
40	MERB	0	>=	0	0	0	-M	0
41	MPC	0	<=	1.0000	1.0000	0	0	M
42	MPC	0	>=	0	0	0	-M	0
43	PCI	0	<=	1.0000	1.0000	0	0	M
44	PCI	0	>=	0	0	0	-M	0
45	PCOR	0	<=	1.0000	1.0000	0	0	M
46	PCOR	0	>=	0	0	0	-M	0
47	PLTL	0	<=	1.0000	1.0000	0	0	M
48	PLTL	0	>=	0	0	0	-M	0
49	PNB	0	<=	1.0000	1.0000	0	0	M
50	PNB	0	>=	0	0	0	-M	0
51	PX	0	<=	1.0000	1.0000	0	0	M
52	PX	0	>=	0	0	0	-M	0
53	PXB	0	<=	1.0000	1.0000	0	0	M
54	PXB	0	>=	0	0	0	-M	0
55	SMCB	0	<=	1.0000	1.0000	0	0	M
56	SMCB	0	>=	0	0	0	-M	0
57	**SMC**	**1.0000**	**<=**	**1.0000**	**0**	**0.0100**	**1.0000**	**1.0000**
58	**SMC**	**1.0000**	**>=**	**0**	**1.0000**	**0**	**-M**	**1.0000**
59	**TEL**	**1.0000**	**<=**	**1.0000**	**0**	**0.1000**	**1.0000**	**1.0000**
60	**TEL**	**1.0000**	**>=**	**0**	**1.0000**	**0**	**-M**	**1.0000**
61	UBP	0	<=	1.0000	1.0000	0	0	M
62	UBP	0	>=	0	0	0	-M	0
63	URC	0	<=	1.0000	1.0000	0	0	M
64	URC	0	>=	0	0	0	-M	0
65	**5-stock portfolio**	**5.0000**	**=**	**5.0000**	**0**	**-0.0100**	**4.0000**	**5.0000**

Appendix D
Shadow Prices for Five-Stock Portfolio

	Constrait	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
3	AC	1.0000	<=	1.0000	0	0.0300	1.0000	1.0000
21	**FLI**	**1.0000**	**<=**	**1.0000**	**0**	**0**	**1.0000**	**M**
27	JFC	1.0000	<=	1.0000	0	0.0500	1.0000	1.0000
57	SMC	1.0000	<=	1.0000	0	0.0100	1.0000	1.0000
59	TEL	1.0000	<=	1.0000	0	0.1000	1.0000	1.0000
65	5-stock portfolio	5.0000	=	5.0000	0	-0.0100	4.0000	5.0000