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Lagged Effect of TV Advertising on Sales of an Intermittently Advertised Product

Cesar C. Rufino
De La Salle University
rufinoc@dlsu.edu.ph

This study is an empirical evaluation of the dynamic effect of intermittent television ad placements on the sales of a consumer product using three classes of distributed lag models. The study is also geared to analytically determine the duration of advertising effects and the dependability of the firm's pulsing type of advertising strategy. Empirical results support the soundness of the company's strategy. Maximum duration of advertising effect is estimated at six months, which is about the largest number of consecutive months the product was not seen on TV during the sample period.

Keywords: Television advertising, sales, distributed lag models, times series, econometrics

The dynamic impact of advertising on sales has been one of the most researched topics related to the effectiveness of various marketing instruments from the time Palda (1964) published his pioneering work on cumulative advertising effects. Since then, the marketing literature has seen a surge of articles on the topic of sales response model building. Most of these publications involve the use of distributed lag econometric models (e.g., Tull, 1965; Palda, 1965; Doyle, 1968; Bass & Clarke, 1972; Parsons, 1976; Clarke, 1976) highlighting the delayed response of sales to advertising expenditures of different products or product categories. Some of the more recent applications of distributed lag models on the effects of advertising to sales are the publications of Pieters and Bijmolt (1997); Schmit, Dong, Chung, Kaiser, and Gould (2002); and Mela, Gupta, and Lehmann (1997).

A majority of these studies employ the Koyck distributed lag specification, which describes a process in which the impact of advertising on sales declines geometrically with time (Bass & Clarke, 1972). The popularity of the Koyck model lies not only on its intuitively appealing notion that the effect of an intervention is highest at the time it is made and gradually diminishes over time, but also on its ready transformation into a simple autoregressive model. Other models commonly employed in examining the sales response to advertising are the Almon (or Polynomial distributed lag) model and the Truncated lag model.

This study is an empirical evaluation of the usefulness of the aforementioned distributed lag models in representing the dynamic effect of intermittent television ad placements on the sales of a consumer product. This study is also geared to analytically determine the duration of advertising effects as well as the soundness of the firm's pulsing type of advertising strategy.

This study is structured as follows: The next section briefly outlines the specification and characteristics of the models to be used. This will be followed by the presentation of the results of implementing the different alternative distributed lag models to the data set of the study, which consists of 12 years of monthly observations in sales and number of television spots bought by the firm. Immediately following will be the section on the results of the diagnostics procedures so as to help determine the most suited model relating sales of the product and the television advertising spots bought to promote the product. The final section provides conclusions of the study.

The General Distributed Lag Model

In the context of advertising-sales relationship, the general form of the distributed lag (DL) model may be presented as follows:

$$(1) \quad S_t = \alpha + \beta_0 A_t + \beta_1 A_{t-1} + \beta_2 A_{t-2} + \ldots + \beta_m A_{t-m} + u_t$$

where S_t = sales during time t, A_{t-i} = advertising intervention during the i^{th} period prior to time t, α = autonomous sales, and β_i = marginal effect of advertising during i^{th} time lag. The integer m is the maximum time the intervention is expected to have its effect, with the sum $\sum_{i=0}^{m} \beta_i = \beta$ which is the cumulative effect of the advertising. When m is finite, the model is said to be a finite distributed lag model; otherwise it is called an infinite distributed lag model. The last variable in the specification u_i is the random error term which is assumed to be normally distributed with mean zero and constant variance. Interest is focused on those model variants of (1) where all β_i's are non-negative and have a finite sum $\sum_{i=0}^{m} \beta_i = \beta$ even if $m \to \infty$. On the basis of this restriction, the general model (1) can be rewritten as:

$$(2) \quad S_t = \alpha + \beta[\omega_0 A_t + \omega_1 A_{t-1} + \omega_2 A_{t-2} + \ldots + \omega_m A_{t-m}] + u_t$$

where $\omega_i \geq 0$ and $\sum_{i=1}^{m} \omega_i = 1$, thus the ω's may be considered as probabilities defined over non-negative numbers β_i's. At the lag length m, the advertising is thought of to have its maximum duration, at which time 100 percent of its impact is already reflected in sales.

The Koyck Distributed Lag Model

Perhaps the best known distributed lag model associated with sales-advertising relationship is the Koyck distribution (Koyck, 1954). Popularized by Palda (1964) along this type of application, the Koyck specification describes a process where the impact of advertising decays continuously over time in a geometric fashion. Thus when $\omega_i = (1 - \lambda)\lambda^i$, and $m \to \infty$, the general model becomes:

$$(3) \quad S_t = \alpha + \beta(1-\lambda)[A_t + \lambda A_{t-1} + \lambda^2 A_{t-2} + \ldots] + u_t$$

The parameter λ is known as the rate of decay, with the restriction that it is within (0, 1) interval and its complement $1 - \lambda$ is referred to as the speed of adjustment. When the one period lag of (3) is multiplied by λ, with the product subtracted from (3), the following autoregressive form of the Koyck model will result:

$$(4) \quad S_t = \alpha(1-\lambda) + \lambda S_{t-1} + \beta_0 A_t + u_t - \lambda u_{t-1}$$

with $\beta_0 = \beta(1 - \lambda)$ known as the impact multiplier. This elegant transformation of the Koyck model from an infinite form (3) to a specification (4), involving only two explanatory variables, contributed immensely to its empirical appeal. The gain in degrees of freedom, however, may prove to be costly in terms of the precision of the OLS estimates of β_0 and λ in light of the endogeneity of S_{t-1} at the right-hand side of (4), rendering the estimates biased and inconsistent (Gujarati, 2003). Unless one is willing to make the strong assumption that $u_t = \lambda u_{t-1} + \varepsilon_t$, with $\varepsilon_t \sim$ iid normal and one empirically proves this contention, one should not risk using ordinary least squares in estimating the parameters of the transformed model. As an alternative, an instrumental variable technique can be used which involves the use of A_{t-1} as an instrument to proxy for S_{t-1} in a two staged regression implementation.

The Polynomial Distributed Lag (PDL) Model

When the idea behind the Koyck model – that the effect of advertising on sales exhibits a geometric decay towards the infinite time horizon – cannot be sustained, an attractive alternative model, attributed to Almon (1965), may be used. The essence of the PDL model is to estimate the general distributed lag model (1) subject to the restriction that the advertising lag effects β_i's lie along a polynomial of degree p. Explicitly, it is assumed that there exist parameters $\gamma_0, \gamma_1,...,\gamma_p$ such that $\beta_i = \gamma_0 + \gamma_1 i + \gamma_2 i^2 + ... + \gamma_p i^p$ for $i = 1,2,...,m$ (with $p < m$). This assumption essentially reduces the number of parameters of (1) from $m + 1$ to $p + 1$. Furthermore, imposing the restriction will lead to more efficient estimates and more powerful tests if the restriction is true; but it will lead to biased and inconsistent estimates and invalid tests if the restriction is false (Schmidt & Waud, 1973). The possibility of a false restriction is notable when there is no lag present; hence the PDL model should never be applied unless there is a strong *a priori* reason to believe that a lag structure in the relationship is present.

In implementing the PDL model, the challenge lies in the choice of the appropriate m (or the maximum duration of the intervention) and p (the degree of the polynomial). When reliable *a priori* information is available about the presence of lagged relationship, as in the sales response to advertising, establishing m is usually undertaken by using either the Akaike Information Criterion (AIC) or the Bayesian Information Criterion (BIC) of a series of alternative lag structures. The choice of p is also important, as an improperly chosen polynomial degree may lead to specification error; while a substantial gain in efficiency is made when it is appropriately chosen.

Another contentious issue in the use of the PDL model refers to the imposition of the so-called "end points" constraints $\beta_{-1} = 0$ or $\beta_{m+1} = 0$ or both. Unfortunately, no convincing reason has ever been advanced as to why these constraints are true (Schmidt & Waud, 1973), except perhaps to terminate the effect after the maximum duration, or to mathematically formalize the fact that the effect of the intervention will start to be felt at the contemporaneous period, not before it. Almon (1965) merely stated, without explanation, that we will "always" want to impose these constraints. At any rate, software (e.g., Eviews 5.1) automating the PDL routine provides an option for users to impose or forego both or any of the endpoint constraints. Godfrey and Poskitt (1975) proposed an LR test in verifying the validity of the endpoint restrictions; while Batten and Thornton (1983) implemented an innovative procedure to test the validity of the end point restrictions in their PDL estimation of the St. Louis Equation to obviate the potential breakdown of inference seen by Schmidt and Waud (1973). Andrews and Fair (1992) also proposed a novel way of addressing the end point constraints in the PDL model, which focuses on the estimation of the lag length, instead of fixing it.

Kmenta (1971) provided the procedural basis of estimating the lagged effects β_i's and their corresponding standard errors out of the OLS estimates of the γ_i's and their standard errors, permitting valid tests of significance for the β parameters.

The Truncated Distributed Lag Models

This group of models is actually nothing but the general finite distributed lag specification (1) successively generated for feasible m starting at $m = 1$, following the method proposed independently by Alt (1942) and Tinbergen (1949). The procedure calls for sequentially estimating the general model until the coefficient of the latest included lagged regressor becomes either insignificant or shifts algebraic sign. Many practitioners criticize this method on the following grounds: (1) it is an *ad hoc* trial and error procedure; (2) it decreases degrees of freedom that may compromise inference, which may become pronounced in small sample studies; and (3) successive lags of the regressor tend to be highly correlated, thus multicollinearity may set in.

The above points of criticism notwithstanding, distributed lag modelers employ the Alt-Tinbergen procedure as a convenient starting point in order to empirically identify the lag structure of the relationship implied by the available data. The result of the sequential estimation on (1) may also serve as the basis of judging the appropriateness of the use of either the Koyck lag distribution model or the Polynomial Distributed lag model suggested by the graph of the resulting β_i estimates across the different lags.

THE DATA AND THE RESULTS OF IMPLEMENTING THE MODELS

The well recognized fact that the effects of advertising persists beyond the period of expenditure makes this activity a valuable part of the firm's marketing mix (Palda, 1964). Doing a micro model of the sales response to advertising may open up a whole range of valuable insights on how to assess the effectiveness of the firm's advertising planning and how to improve this very important function of marketing management.

The setting of the study is a company whose main product enjoys a traditional following, such that it can sustain a market presence even with sparse advertising. The company's management would like to believe that they have attained a solid enough, albeit subjective, understanding of the market for them to take advantage of the momentum of previous advertising expenditures for sales to smoothly ride on. Thus, for a certain number of months, the company deliberately stays away from the advertising scene, only to return to develop an additional wave of momentum for a sustained market presence. Advertising presence of the product is confined only to the television medium.

This study will attempt to empirically analyze the soundness of the firm's advertising strategy by modeling its product's lagged response to TV advertisements using relevant distributed lag models. Twelve years of monthly sales (revenue in millions of Pesos) and advertising (number of TV spots bought) of the firm are the only data provided to implement the study. Anonymity of the company and the product is preserved in the study as requested by management.

Presented in Figure 1 is the line graph of the monthly sales of the product under study. It reflects an almost stationary behavior of the series over the 144 monthly period signifying a mature product whose sales does not exhibit any recognizable upward or downward trend. In Figure 2, the bar graph is a pictorial device more appropriate to present the advertising data, due to the proliferation of zero monthly observations. The gaps in the sequence of bars in the figure correspond to the months the firm has no placements in the TV ads market. Gaps may be noted in a number of prolonged periods.

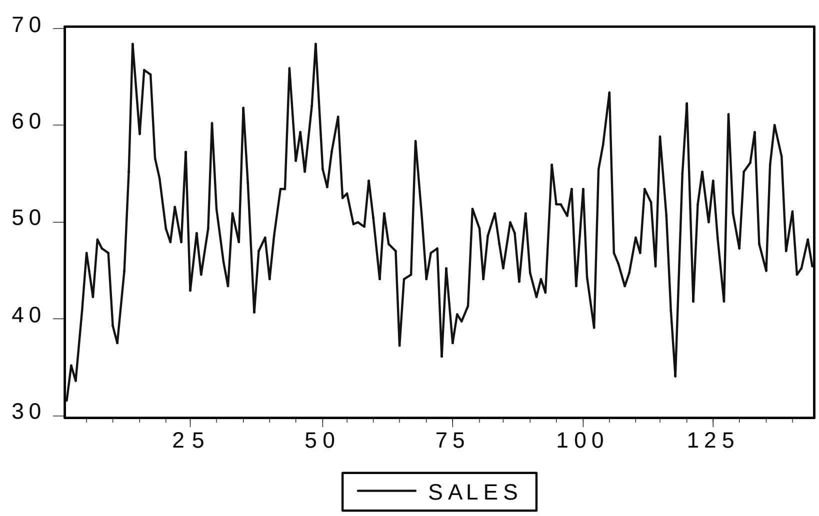

Figure 1
Monthly sales of the company (in millions of Pesos)

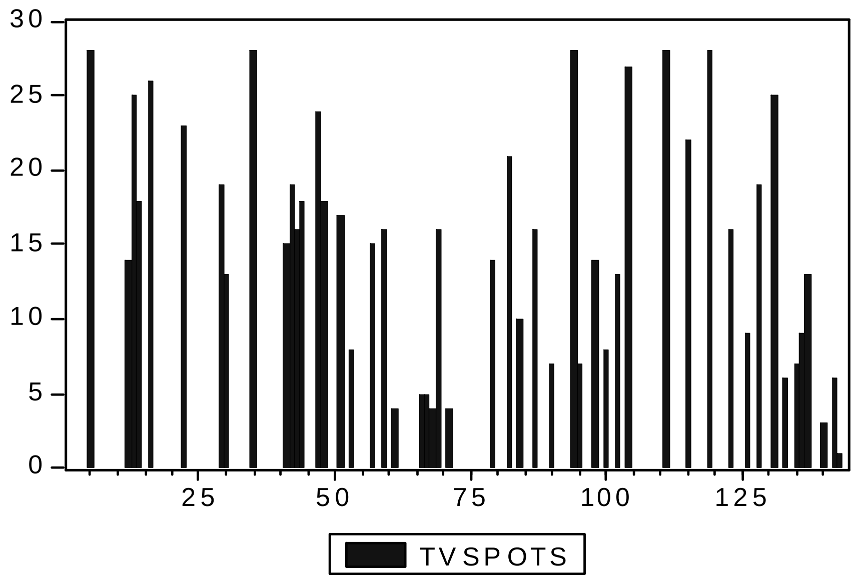

Figure 2
Monthly number of TV ads bought by the company

The sophisticated software known as Census X12 being used at the U.S. Bureau of Census detects any trace of seasonality (both stable and moving seasonality) in the monthly sales series. The results of the tests are presented in Appendix A, signifying the lack of any form of seasonality in the revenue time series. The perceived stationarity of the sales in Figure 1, when subjected to the formal unit root test of Augmented Dickey Fuller (ADF) (Dickey & Fuller, 1979), turned out to be a false impression, as a single unit root was significantly detected in the level sales series. This means that the monthly revenue of the product is a non-stationary series which can be converted into a stationary one by using the first difference transformation once.

Results of the Truncated Lag Modeling

Employing the Alt-Tinbergen technique to the general distributed lag model specification (1) resulted in a series of estimated truncated lag models shown in Appendix C. The sequential model search proceeded up to the inclusion of the sixth lagged TVSpots variable which yielded a two tailed p-value of 0.0833; this may still be considered significant at five percent level. However, when the seventh lagged TVSpots variable was entered, the estimated coefficient turned in a two tailed p-value of 0.3141 denoting lack of significance, thus signaling a stop to the sequential model search at lag six. Incidentally, all of the estimated β coefficients exhibit the correct positive sign.

Further analysis of the results presented in Appendix C reveals that the model with maximum effect duration of six periods also has the smallest Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), aside from its having the highest adjusted R^2. These desirable features of the model, in addition to intuitive signs and magnitudes of its estimated parameters, qualify the model with the maximum of six lagged TVSpots variables as the model of choice among the truncated distributed lag models. The maximum advertising duration of six months also approximates the maximum number of months the firm stayed away from the TV ads market over the whole 12-year data horizon. This model appears to empirically validate the subjective judgment of the firm's marketing management about the adoption of the intermittent type (or pulsing) advertising strategy. This tentative result of the study will have to be corroborated by the other two models to strengthen its policy importance.

The possible presence of multicollinearity in the best-fitting truncated DL model is addressed, using the Variance Inflation Factor (VIF) diagnostics, which are appended, in brackets, to the summary equation of winning truncated lag model.. All lagged advertising variables at the right hand side of the model registered VIFs figures that do not cross beyond the multicollinearity threshold of six, signifying the absence of this anomaly that may compromise inference.

Results of the Polynomial Distributed Lag (PDL) Modeling

The empirical lag structure suggested by the results of the truncated lag modeling is used as the convenient starting point in implementing the PDL modeling framework. By means of the established six-month maximum advertising duration, a total of eight alternative PDL models are estimated and subjected to a battery of diagnostic testing. These models are the following: PDL(6,2,0); PDL(6,2,1); PDL(6,2,2); PDL(6,2,3); PDL(6,3,0); PDL(6,3,1); PDL(6,3,2); PDL(6,3,3). The first number is the maximum lag length; the second is the lag polynomial degree; and the third is the code for end-point restriction (0 for no restriction, 1 for near-end restriction, 2 for far-end restriction and 3 for both-end restriction) being imposed. Summarized in Appendix D are the results of the estimation of the models from which we can draw our conclusions on their relative merits.

Among the fitted PDL models presented in the Appendix D, the PDL(6,2,2) – which assumes a maximum of six months for the advertising duration, with the monthly lagged effects lying in a second degree polynomial, which imposes a “far-end” parameter restriction, appears to be the best-fitting model among the eight alternatives tested. All of the goodness-of-fit indicators are unanimous in affirming the superiority of this model. Even the Durbin-Watson statistic indicates the absence of serial correlation.

Looking at the estimated PDL(6,2,2) featured in Table 1, it may be inferred that at the time the TV spot is aired, about 25 percent (24.9 percent) of its total impact on sales will be immediately felt. A little less than half of its total effect (46.2 percent) is reflected in company’s sales only after a period of one month; and about 90 percent (89.2 percent) after four months. Following a lapse of six months, all of the sales influence of that particular TV advertising is expected to be totally discounted. Almost similar conclusions can be gleaned from the best-fitting truncated lag model shown in Table 2.

The almost identical results of the ‘best in their respective classes’ models underscore the success of using distributed lag models in quantifying the sales response to advertising strategy adopted by the subject firm.

Table 1
Advertising Effects on Sales at Different Lags from the PDL(6,2,2) Model

Effect at:	Lag 0	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Total
β_i =	0.4305	0.3700	0.3093	0.2481	0.1866	0.1248	0.0626	1.7318
ω_i =	24.9%	21.4%	17.9%	14.3%	10.8%	7.2%	3.6%	
Cum ω_i =	24.9%	46.2%	64.1%	78.4%	89.2%	96.4%	100.0%	

Table 2
Advertising Effects on Sales at Different Lags from the Best Truncated DL Model

Effect at:	Lag 0	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Total
β_i =	0.4261	0.4150	0.2935	0.2027	0.1539	0.2029	0.0808	1.7748
ω_i =	24.0%	23.4%	16.5%	11.4%	8.7%	11.4%	4.6%	
Cum ω_i =	24.0%	47.4%	63.9%	75.3%	84.0%	95.4%	100.0%	

Table 3
Advertising Effects on Sales at Different Lags from the Results of Koyck Model

Effect at:	Lag 0	Lag 1	Lag 2	Lag 3	Lag 4	Lag 5	Lag 6	Total
β_i =	0.3275	0.2471	0.1865	0.1407	0.1062	0.0801	0.0605	1.3347
ω_i =	24.54%	18.51%	13.97%	10.54%	7.96%	6.00%	4.53%	
Cum ω_i =	24.54%	43.05%	57.02%	67.56%	75.52%	81.52%	86.05%	

Results of the Koyck Distributed Lag Modeling

When the Koyck distributed lag model is fitted to the sales-advertising data of the company, a different scenario emerged from the results. Through the instrumental variable estimation procedure, with the Cochrane-Orcutt correction for autocorrelation, the estimated Koyck model is summarized below (estimated t values are within parentheses).

(5) $\hat{S}_t = 10.6483 + 0.7546\, S_{t-1} + 0.330\, A_t$
$\qquad\quad (1.35) \qquad (4.83) \qquad (6.28)$

$R^2 = 0.3993 \qquad \bar{R}^2 = 0.3863$

AIC = 6.2597 BIC = 6.3430

This equation gives $\hat{\lambda} = 0.7546$ and $\hat{\beta}_0 = 0.330$. Using these parameter estimates, and the formulas $\omega_i = (1 - \lambda)\lambda^i$ for i ranging from 0 to ∞ and $\beta = \beta_0 (1 - \lambda)^{-1}$, the following estimates may be drawn.

The simulation presented in Table 3 reveals that after six months from the airing of the ads, only about 86 percent of the effect is transmitted to sales, and it is estimated that the 90 percent (89.46 percent) cumulative impact will be felt after seven months. Full impact is expected to be felt only after approximately 16 months as seen in the counter factual simulation chart shown in Figure 3.

The simulation results based on the estimated Koyck model do not concur with the outcomes of the two other distributed lag models. The implied maximum advertising duration of sixteen months for the Koyck model is almost three times as long as the empirically determined duration by the best-fitting PDL

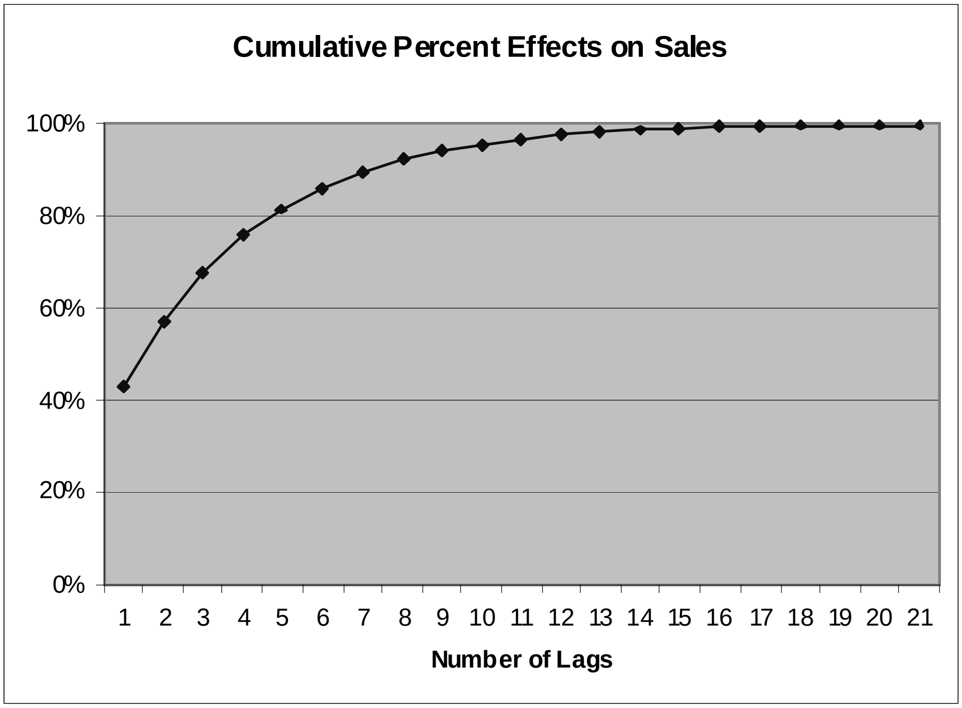

Figure 3
Cumulative effects of TV ads on sales, Koyck model

model and the best-fitting truncated DL model. This duration figure is simply not realistic in light of the company's experience. The latter two models also turned in almost identical intermediate advertising multipliers at various lags, further accentuating their empirical validity.

CONCLUDING REMARKS

The response of sales to advertising impulses is rarely instantaneous. Very often, this dependency relationship involves a time lapse or lag structure, the nature of which may be of strategic importance to the firm. As with mature products known to have attained traditional following, quantifying such dynamic connection may provide marketing managers the basis for developing an effective "pulsing" or intermittent type of strategy that could allow sales to ride on the momentum of previous expenditures.

This study is an attempt to analytically examine the soundness of a long-established "pulsing" policy adopted for a consumer product. The study not only confirms the presence of significant lag structure, but also substantiates the company planners' subjective judgment on the maximum duration of advertising effects on product sales. On the basis of the more detailed intermediate results of the study, the company may use this information to further improve their advertising planning. However, it has to be emphasized that the company should also look into the roles of other factors that may impact on the effectiveness of advertising expenditures. These may include, among others, the time slot bought (prime time or not), program rating, reputation or ranking of the TV station, etc.

REFERENCES

Almon, S. (1965). The distributed lag between capital appropriations and expenditures. *Econometrica, 33,* 178-196.

Alt, F. (1942). Distributed lags. *Econometrica, 10,* 113-128.

Andrews, D., & Fair, R. (1992). Estimation of polynomial lags and leads with end point constraints. *Journal of Econometrics, 53*, 123-139.

Bass, F., & Clarke, D. (1972). Testing distributed lag models of advertising effects. *Journal of Marketing Research, 9,* 298-308.

Batten, D., & Thornton, D. (1983). Polynomial distributed lags and the estimation of the St. Louis Equation. *Federal Reserve Bank of St. Louis*, April 1983, 13-25.

Clarke, D. (1976). Econometric measurement of the duration of advertising effect on sales. *Journal of Marketing Research, 13*, 345-357.

Dickey, D., & Fuller, W. (1979). Distribution of estimators for autoregressive time series with a unit root. *Journal of America Statistical Association, 74*, 427-431.

Doyle, P. (1968). Economic aspect of advertising: A survey. *The Economic Journal, 78*(311), 570-602.

Godfrey, L., & Poskitt, D. (1975). Testing the restrictions of the Almon lag technique. *Journal of American Statistical Association, 70*, 105-108.

Gujarati, D. (2003). *Basic econometrics* (4th ed.). Boston, MA: McGraw-Hill.

Kmenta, J. (1971). *Elements of econometrics*. New York, NY: McMillan Co.

Koyck, L. (1954). *Distributed lags and investment analysis*. Amsterdam: North Holland Publishing Co.

Mela, C., Gupta, S., & Lehmann, D. (1997). The long term impact of promotion and advertisement on consumer demand choice. *Journal of Marketing Research, 34*(2), 248-261.

Palda, K. (1964). *The measurement of the cumulative advertising effects*. Englewood Cliffs, NJ: Prentice-Hall.

Palda, K. (1965). On the measurement of advertising effectiveness. *Journal of Advertising Research, 4,* 12-16.

Parsons, L. (1976). A rachet model of advertising carry over effects. *Journal of Marketing Research*, *13*(1), 76-79.

Pieters, R., & Bijmolt, T. (1997). Consumer memory of TV advertising: A field study of durations, serial position and competition effects. *The Journal of Consumer Research, 23*(4), 362-372.

Schmidt, P., & Waud, R. (1973). The Almon lag technique and the monetary versus fiscal policy debate. *Journal of American Statistical Association, 68*(341), 11-19.

Schmit, T., Dong, D., Chung, C., Kaiser, H., & Gould, B. (2002). Identifying the effects of generic advertisement on the household demand for fluid milk and cheeses: A two-step panel data approach. *Journal of Agricultural and Resource Economics. 27*(1), 165-186.

Tinbergen, J. (1949). Long term foreign trade elasticities. *Macroeconomica, 1*, 174-185.

Tull, D. (1965). The Carry-over effect of advertisement. *Journal of Marketing, 29*(2), 46-53.

Appendix A
Test for Stable and Moving Seasonality of Sales Series

Null Hypothesis: No Seasonality in the Monthly Sales Data

Census X-12 Seasonality Tests	Test Stat	p-value
F-test for stable seasonality	1.106	0.3628
F-test for moving seasonality	1.028	0.4259

Appendix B
ADF Unit Root Test of Stationarity of Sales

Null Hypothesis: SALES has a unit root

Exogenous: None (Random Walk)

Lag Length: 4 (Automatic based on AIC, MAXLAG=13)

		τ-Statistic	p-value*
Augmented Dickey-Fuller test statistic		-0.187823	0.6169
Test critical values:	1% level	-2.581705	
	5% level	-1.943140	
	10% level	-1.615189	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: D(SALES) has a unit root

Exogenous: None (Random Walk)

Lag Length: 7 (Automatic based on AIC, MAXLAG=13)

		τ-Statistic	p-value*
Augmented Dickey-Fuller test statistic		-6.838440	0.0000
Test critical values:	-2.582204	-2.581705	
	-1.943210	-1.943140	
	-1.615145	-1.615189	

*MacKinnon (1996) one-sided p-values.

Appendix C
OLS Estimates of the Truncated Lag Models Using the Alt and Tinbergen Method

Lags Included	Estimated Model (t-values in Parentheses)	Adj. R Squared	AIC	BIC
0 – Period	$S_t = 47.82 + 0.338A_t$ (5.25) (74.15)	0.1568	6.625	6.666
1 – Period	$S_t = 45.97 + 0.367A_t + 0.342A_{t-1}$ (45.97) (6.45) (6.02)	0.3242	6.372	6.434
2 – Period	$S_t = 44.49 + 0.384A_t + 0.366A_{t-1} + 0.251A_{t-2}$ (63.57) (7.29) (6.93) (4.77)	0.4126	6.216	6.299
3 – Period	$S_t = 43.51 + 0.395A_t + 0.377A_{t-1} + 0.266A_{t-2} + 0.163A_{t-3}$ (56.75) (7.77) (7.40) (5.22) (3.22)	0.4455	6.134	6.238
4 – Period	$S_t = 42.66 + 0.390A_t + 0.390A_{t-1} + 0.278A_{t-2} + 0.178A_{t-3} + 0.129A_{t-4}$ (50.81) (7.79) (7.70) (5.51) (3.54) (2.59)	0.4620	6.105	6.231
5 – Period	$S_t = 41.13 + 0.428A_t + 0.384A_{t-1} + 0.299A_{t-2} + 0.198A_{t-3} + 0.149A_{t-4} + 0.200A_{t-5}$ (47.16) (8.86) (8.11) (6.25) (4.17) (3.15) (4.25)	0.5294	5.984	6.131
6 – Period	$S_t = 40.58 + 0.426A_t + 0.416A_{t-1} + 0.293A_{t-2} + 0.203A_{t-3} + 0.154A_{t-4} + 0.203A_{t-5} + 0.081A_{t-6}$ (43.29) (8.99) (8.69) (6.27) (4.31) (3.29) (4.34) (1.75) VIF = **[1.064] [1.065] [1.080] [1.088] [1.077] [1.110] [1.097]**	**0.5479**	**5.949**	**6.118**
7 – Period	$S_t = 40.18 + 0.427A_t + 0.417A_{t-1} + 0.301A_{t-2} + 0.203A_{t-3} + 0.160A_{t-4} + 0.209A_{t-5} + 0.087A_{t-6} + 0{,}047A_{t-7}$ (40.18) (8.97) (8.67) (6.23) (4.28) (3.37) (4.41) (1.84) (1.01)	0.5478	5.963	6.155

Appendix D
Estimated PDL Models of Sales Response to Advertising

Model Indicators	Alternative PDL Models PDL(6,2,0)		PDL(6,2,1)		PDL(6,2,2)		PDL(6,2,3)	
Effect at:	**Coefficient**	**t-value**	**Coefficient**	**t-value**	**Coefficient**	**t-value**	**Coefficient**	**t-value**
Lag 0	0.4435	10.38	0.1542	8.63	0.4305	**10.54**	0.1297	8.54
Lag 1	0.3668	12.61	0.2579	8.81	0.3700	**12.79**	0.2224	8.54
Lag 2	0.2981	10.11	0.3110	8.99	0.3093	**11.25**	0.2780	8.54
Lag 3	0.2374	7.56	0.3136	9.02	0.2481	**8.36**	0.2965	8.54
Lag 4	0.1847	6.22	0.2656	8.21	0.1866	**6.29**	0.2780	8.54
Lag 5	0.1400	4.82	0.1671	4.93	0.1248	**4.96**	0.2224	8.54
Lag 6	0.1033	2.47	0.0181	0.38	0.0626	**4.07**	0.1297	8.54
Total Effect	1.7738	11.15	1.4875	8.21	1.7318	**11.25**	1.5567	8.54
AIC	5.9323		6.2487		**5.9259**		6.2790	
BIC	6.0171		6.3123		**5.9896**		6.3214	
Adj. R Squared	0.5427		0.3681		**0.5428**		0.3441	
Durbin Watson	2.0203		1.6913		**1.9776**		1.6486	

Model Indicators	PDL(6,3,0)		PDL(6,3,1)		PDL(6,3,2)		PDL(6,3,3)	
Effect at:	**Coefficient**	**t-value**	**Coefficient**	**t-value**	**Coefficient**	**t-value**	**Coefficient**	**t-value**
Lag 0	0.4425	9.62	0.2961	9.96	0.4467	9.84	0.2490	9.50
Lag 1	0.3680	10.52	0.3978	11.02	0.3590	11.25	0.3586	10.31
Lag 2	0.2993	8.52	0.3663	11.21	0.2931	8.69	0.3629	10.79
Lag 3	0.2374	7.53	0.2627	8.06	0.2409	7.78	0.2960	9.35
Lag 4	0.1835	5.25	0.1482	4.15	0.1938	6.26	0.1922	5.70
Lag 5	0.1388	3.98	0.0839	2.48	0.1436	4.23	0.0855	2.45
Lag 6	0.1044	2.32	0.0310	2.79	0.0818	2.93	0.0100	0.38
Total Effect	1.7738	11.11	1.6859	10.10	1.7590	11.16	1.5542	9.35
AIC	5.9468		6.0474		5.9353		6.1006	
BIC	6.0528		6.1322		6.0202		6.1643	
Adj. R Squared	0.5393		0.4870		0.5414		0.4551	
Durbin Watson	2.0206		1.9717		2.0047		1.8170	