

RESEARCH ARTICLE

Towards a 21st Century Mathematics Classroom: Investigating the Effects of the Problem-Solving Approach Among Tertiary Education Students

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Abstract: The development of 21st-century skills that are deemed necessary for learners to excel in a knowledge-based and highly globalized society is at the core of today's education system. Towards this end, classroom practices focus on the delivery of instruction using various innovative instructional methodologies. One of the strategies used in Mathematics is the problem-solving approach. The problem-solving approach emphasizes that important mathematics concepts and procedures can be best taught through problem-solving tasks or activities, which engage students in thinking about the important mathematical concepts and skills they need to learn. This study utilized the problem-solving approach as supported by various collaborative strategies as an instructional intervention in teaching mathematics to first year college students and investigated its effects on the enhancement of their performance in and attitude towards College Algebra. The pretest-posttest control group design using two matched groups of respondents based on their intelligence quotient scores and mathematics test scores in the University Admission Test was utilized. Ten problem-solving tasks involving routine, non-routine, and real problems were developed and provided to the experimental group. On the other hand, the conventional approach in teaching and learning was employed in the control group. Necessary data to fulfill the objectives of the study were gathered through the attitude scale questionnaire and the researcher-made test and were analyzed using appropriate statistical tools. The results showed that the experimental group outperformed the control group on the bases of their posttest and mean gain scores. The experimental group also posted significant enhancement of their attitude towards college Algebra. Thus, the problem-solving approach, when applied to classroom instruction, can significantly improve students' cognitive and affective attributes in mathematics, hence indicating the effectiveness of the approach in teaching mathematics.

Keywords: Affective attributes, cognitive attributes, collaboration, collaborative strategies, mathematics education, problem-solving approach, teaching through problem-solving

The 21st Century Learning Framework formulated by the Partnership for 21st Century Skills (P21) “describes essential skills, knowledge, and expertise learners need to master” (Dass, 2014, p. 290) for them to “excel in today's globalized, knowledge-based society” (Fong et al., 2014, p.130). Among

these skills are life and career skills, learning and innovation skills, and information, media, and technology skills (Partnership for 21st Century Skills, 2009). Ongardwanich et al. (2015) reported that the development of these skills among learners is vital as only those who have relevant knowledge and skills in

dealing with continuous changes and adapting oneself to new situations can attain success in the new global economy. This necessitated countries around the world to infuse the 21st-century skills into education (Johnson, 2009), hence enabling the learners to develop broader and more sophisticated skills like evaluating and analyzing information and thinking creatively about how to solve real-world problems (Silva, 2009).

Developing the ability of individuals to solve problems is one of the fundamental goals of education (Yavuz et al., 2015; *Organisation for Economic Cooperation and Development [OECD]*, 2014). Problem-solving is an essential competence in both education and daily life (Greiff et al., 2012). It is regarded as a complex cognitive activity that utilizes an individual's intellectual faculties (memory, perception, reasoning, conceptualization, and language) and appetitive faculties (emotions, motivation, and self-confidence; Raynal & Rieunier, 1997).

Problem-solving is an important component of mathematics education (Căprioară, 2015). In mathematics, the term "problem-solving" refers to a systematic approach in conceptualizing and understanding a given problem, designing strategies to solve the problem, and evaluating the strategies implemented (Allen & Graden, 2002). It involves mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and to develop their ability to find meaningful solutions to solve problems using effective and timely strategies (Karabacak et al., 2015; Yavuz & Erbay, 2015).

Gokkurt et al. (2012) reported that mathematics remains a challenge for most countries. For this reason, different approaches that promote students' active participation in the teaching and learning process have emerged. In recent years, using problem-solving in teaching mathematics subjects was one of the methods used in teaching the subject (Yavuz & Erbay, 2015), and mathematics teachers have considered the shifting of emphasis from teaching problem-solving to teaching through problem-solving (Taplin, n.d.). It is believed that important mathematics concepts and procedures can be best taught through problem-solving tasks or activities, which engage students in thinking about the important mathematical concepts and skills they need to learn (South African Institute for Distance Education [SAIDE], 2008). It is in this context that this research was conceptualized to analyze the effects of using the

problem-solving approach in teaching mathematics to the academic achievement and attitude of first-year university students.

The problem-solving approach, also referred to as teaching through problem-solving, focuses on teaching mathematical topics through problem-solving contexts and inquiry-oriented environments, which are characterized by the teacher helping the students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying (Cai & Lester, 2010).

In a learning environment where the problem-solving approach in teaching mathematics is applied, the students with different performance levels work in pairs or in small groups to learn the mathematical concepts by engaging themselves in a problem-solving task or activity whose required solutions and processes are not clearly known (Kayan & Cakiroglu, 2008; SAIDE, 2008). As such, students need to explore the concepts, develop an understanding of the problem, make connections with mathematical knowledge previously learned, and select an appropriate mathematical skill that leads to the solution of the problem (Polya, 1962, as cited by Yavuz & Erbay, 2015).

In the problem-solving approach, learning occurs during the process of attempting to solve problems, in which relevant mathematics concepts and skills are embedded (Lester & Charles, 2003; Schoen & Charles, 2003). As students solve problems, they can use various approaches, utilize the knowledge that they have previously learned, and employ convincing strategies to justify their ideas. The learning environment of teaching through problem-solving provides a natural setting for students to present their solutions to their group or class in a way that is comfortable to them and learn mathematics through social interactions, meaning negotiation, and reaching a shared understanding. Consequently, students are provided with opportunities to clarify their ideas and acquire different perspectives on the concept or idea they are learning (Cai & Lester, 2010).

Cai and Lester (2010) reported the idea of Lambdin (2003) that teaching through problem-solving perceives a symbiotic connection between problem-solving and concept learning. Further, students can learn and understand mathematics through solving mathematically rich problems, and problem-solving skills are developed through learning and

understanding mathematics concepts and procedures. This view is parallel to SAIDE (2008) when it stressed that obtaining a full understanding of mathematics should be a result of solving problems, rather than teaching the students how to understand it.

The use of the problem-solving approach in teaching can facilitate the development and acquisition skills, knowledge, and expertise that are essential in the 21st-century society (Fong et al., 2014; Dass, 2014). Among these essential 21st-century skills are collaboration and problem-solving.

Collaboration has become a trend and a promising mode for human engagement in the 21st century (Laal, 2013). As a skill, collaboration emphasizes the ability of a person to work effectively with a diverse group of people (Pacific Policy Research Center, 2010) with flexibility and willingness to contribute in planning, decision making, and problem-solving towards accomplishing a common goal or a given task (Trilling & Fadel, 2009; Allen & Graden, 2002). Such skill is deemed necessary in working with critical issues in a knowledge-based society, hence leading to the transition from individual efforts to group work, and from independence to community (Laal et al., 2013; Mao & Woolley, 2016; Eikey et al., 2015; Dey et al., 2011). Engaging diverse stakeholders and decision-makers in a collaborative process facilitates knowledge integration, which is essential to making an effective decision (Sobandi & Sudarmadji, 2015; Brinkman et al., 2015; Sitas et al., 2016).

In education, collaboration skill is fostered through the implementation of collaborative learning in the teaching and learning process. Many studies in education involving various disciplines have found that the use of collaborative learning is beneficial in terms of social, psychological, academic, and assessment aspects (Laal & Ghodsi, 2012). Learners in a collaborative learning environment tend to develop higher-level thinking skills and retain information longer than those students who work silently as individuals (*Gokkurt et al., 2012*). The active exchange of ideas within small groups increases interest among the participants and promotes critical thinking (Gertrude, 2015). Srinivas (n.d.) reported that collaborative learning creates an environment of active, involved, exploratory learning, promotes a positive attitude toward the subject matter, develops oral communication and social interaction skills, uses a team approach to problem-solving while maintaining individual accountability, and develops the

ability to criticize ideas and not people. Also, Laal and Ghodsi (2012) emphasized that collaborative learning develops valuable problem-solving skills.

Advocates of the problem-solving approach suggested that the quality of mathematics education can be enhanced through a learning environment where learners are exposed to teaching via problem-solving (Taplin, n.d.). Learners can achieve higher levels of learning and retain more information when they work in a group as they are actively involved in the learning process. Moreover, the collaborative environment, which the problem-solving approach provides, emphasizes the value of cooperation and friendly interactions among the members of the group rather than competition. The approach promotes positive responses to challenges encountered by the members of the group and fosters a supportive environment (Laal et al., 2013), which enables the learners to develop mathematical power and confidence to do math (SAIDE, 2008).

Acquiring skills and understanding concepts through the process of individual learning can be tedious, boring, and overwhelming. When teachers allow learners to work together, the learning process becomes interesting and fun despite the nature of the subject (Laal & Ghodsi, 2012). Through the use of the problem-solving approach, the learners become interested and develop a more positive attitude towards mathematics (SAIDE, 2008).

However, a survey of the literature shows that most researches conducted and published in various journals are mainly concerned with evaluating how the problem-solving approach is implemented as an instructional methodology in teaching mathematics (Kuzle, 2018), teaching problem-solving to students (Pehkonen et al., 2013; Ernest, 1988), assessing and improving the problem-solving skills of students in various levels (Widada et al., 2019; Simamora, & Saragih, 2018; Krawec & Huang, 2017), evaluating students' thinking process in solving mathematical problems (Widodo et al., 2019), determining problem-solving skills and strategies (Gurat, 2018; Karabacak et al., 2015; Evans, & Swan, 2014), and evaluating the effectiveness of problem-based learning and problem-posing as strategies in teaching mathematics (Asfar et al., 2019; Căprioară, 2015; Abdullah et al., 2010; Duch et al., 2001).

As far as my knowledge and the conducted survey of the literature, there is no record found or evidence

of researches conducted with the problem-solving approach or teaching through problem-solving as an instructional intervention using a true experimental research design. Thus, this true experimental study using the pretest-posttest control group design was conducted to investigate the effects of the problem-solving approach as an alternative instructional strategy in teaching mathematics to the enhancement of performance and attitude of first-year college students in College Algebra. The problem-solving approach was employed to the experimental group, whereas the traditional method of teaching mathematics, characterized by the lecture and demonstration methods, was applied to the control group.

Specifically, this study determined the performance of the experimental group and the control group before and after the experiment using a validated researcher-made test. The attitudes of the groups towards the subject College Algebra before and after the experiment were also measured using an attitudinal questionnaire. In addition, the study compared the posttest scores and mean gain scores of the two groups of the respondents. The attitudes of the experimental and control groups before and after the experiment were also compared. The results of the comparative analyses of the respondents' posttest scores, mean gain scores, and attitudes towards College Algebra provided pieces of evidence about the effects of the use of the problem-solving approach to the identified dependent variables of this study.

This study both verified existing knowledge and generated new knowledge about the problem-solving approach. It offers scientific evidence about the potential of the problem-solving approach as an instructional strategy in teaching mathematics. The findings of this research study also provided evidence of enhancement of students' mathematical learning and development of a favorable attitude towards mathematics as a result of a true experiment. Because the primary goal of mathematics education is to develop students' problem-solving skills, teaching through problem-solving may foster the development of this important skill while engaging students in learning important mathematical concepts and processes. The three types of problems provided to the experimental group help them generate various strategies and approaches in problem-solving, which are vital in dealing with real-life societal and community problems. Also, the interactive learning environment, which is developed

by the problem-solving approach, enables effective communication and collaboration between and among students to prosper and facilitate the development of a more positive attitude towards mathematics in general.

Conceptual Framework

The conduct of this study was anchored on the following concepts and theories—problem-solving as defined by various authors, six stages of the problem-solving process (OECD, 2005), three types of word problems (SAIDE, 2008; Becker & Shimada, 1997; Pehkonen et al., 2013), and the theories on constructivism, experiential learning, and the zone of proximal development.

Problem-solving plays a very important role in the study of mathematics as mathematics education primarily aims to develop students' problem-solving abilities (Wilson et al., 1993). The OECD defined problem-solving as the cognitive process of devising or selecting a strategic plan to use mathematics in order to arrive at the solutions of a problem-solving task (Krawec & Huang, 2017). The National Council of Teachers of Mathematics, as stated by Sharpe et al. (2014), described problem-solving as a task wherein the solution is not known or immediately obvious. Further, problem competencies is defined as “an individual's capacity to use cognitive processes to confront and resolve real, cross-disciplinary situations where the solution path is not immediately obvious and where the content areas or curricular areas that might be applicable are not within a single subject area of mathematics, science or reading” (OECD, 2003, p. 26). This definition leads to the three important characteristics of problems (OECD, 2005). First, problems should be real, that is, they should involve situations that are familiar to the students and situations that are considered important to society. Second, the solutions are not obviously known and identified using some defined process; hence, they should challenge students' flexibility in managing, evaluating, and reflecting on the information given. Lastly, the problems should involve numerous content areas.

Kuzle (2018) claimed that problem-solving is both an instructional goal and an instructional method. As an instructional method, problem-solving can be utilized to develop new knowledge in mathematics, to derive solutions to mathematical problems involving mathematics and other contexts, to generate a myriad

of relevant strategies, and to evaluate and reflect on mathematical processes employed. According to Schoen and Charles (2003), as students endeavor to find the solutions of rich problem tasks, they develop an understanding of the mathematical concepts and processes involved, utilize concepts that are previously learned, employ strategies to justify their solutions, and develop mathematical power that is very much helpful in dealing with various mathematical situations.

According to OECD (2005), problem-solving tasks must assess students' ability to confront, structure, represent, and solve problems effectively. Towards this goal, it enumerated the six stages of the problem-solving process, which students should effectively demonstrate:

- (1) Understanding the problem: This process involves demonstrating a full understanding of the text, diagrams, concepts, formulas, tables, making inferences from the given information, establishing connections between the information provided and concepts from other sources, and using previously learned concepts to understand the given information.
- (2) Characterizing the problem: This involves identifying the variables involved and their interrelationships, identifying relevant and irrelevant variables, hypothesizing, organizing, and critically evaluating contextual information.
- (3) Representing the problem: This includes illustrating the problem through tables, graphs, symbols, drawings, and verbal representations.
- (4) Solving the problem: This involves creating a viable solution plan and implementing the solution to meet the requirements of the problem-solving tasks.
- (5) Reflecting on the solution: This includes critically examining the completeness and correctness of the solutions, evaluating solutions and restructuring them into a more socially or technically acceptable, and justifying the solutions.
- (6) Communicating the solution: This includes the effective use of tools to comprehensively communicate the solutions to an audience.

Meanwhile, there are three types of problems to which students should be exposed (SAIDE, 2008). One is routine problems where the concept is embedded in a real-world situation, and the students are required to recognize and apply the appropriate rule. This type of problem helps prepare the students for life challenges. On the other hand, the non-routine problems require a higher degree of interpretation and organization of the information in the problem, rather than simply applying the rule. Non-routine problems encourage the development of general knowledge and common sense.

According to Autor et al. (2003), working on routine and non-routine tasks help students develop one of the five important skills necessary to secure or to stay in a job. Although routine tasks foster the development of procedural skills that are vital in both cognitive and manual operations, non-routine tasks emphasize the ability to perform one's functions effectively even though their rules are not explicitly stated. In the study of Arslan and Altun (2007), they confirmed that the use of non-routine problems in the mathematics classroom could develop students' ability to devise appropriate problem-solving strategies.

The third type of problem is called real problems, which are concerned with investigating a problem that involves real-life situations. Real problems refer to the open approach to mathematics teaching using open problems in Japan, which was developed in the 1970s (Pehkonen et al., 2013). Primarily, the said approach is aimed at developing students' creativity and fostering meaningful classroom discussion (Becker & Shimada, 1997).

Real problems do not necessarily require a fixed solution and use mathematics as a tool to find the solution. In working with real problems, the situation and the tasks are given, but students can generate various approaches to come up with various correct and reasonable solutions. The nature of the real problems allows students to produce different correct solutions through the implementation of a myriad of approaches (Fuchs & Fuchs, 2007), to share and discuss their thoughts, and to make logical justifications of their decisions (Kuzle, 2018). Most importantly, it facilitates the development of students' "habit of mind," which Cuoco et al. (1996) referred to as the ways of thinking that enable them to devise varied strategies and viable solution plans that can be applied to solving various academic challenges and those that occur in real-

life situations. Some of these habits of mind include recognizing patterns, exploring, communicating, explaining, conjecturing, refuting, and generalizing. Bernard and Chotimah (2018) verified in their study that the use of an open-ended approach in teaching mathematics could improve students' reasoning ability, particularly in justifying their created output relative to the given task. Also, the implementation of the open approach in the classroom can enhance students learning outcomes, especially in mathematics (Islamiyah, 2014; Irawan & Surya, 2017).

The problem-solving approach also reflects the ideas of the experiential learning theory, constructivism, and the zone of proximal development.

The experiential learning theory by David Kolb (1984) stressed that learning is influenced by experiences, including cognition, environmental factors, and emotions. The theory emphasizes that knowledge is generated through the transformation of experience. Grasping experience can be done through concrete experience and abstract conceptualization, whereas transforming experience can be done by reflective observation and active experimentation (Kolb et al., 2001). The experiential learning theory highlights the importance of providing students with real experiences to help them gain relevant learning and help them transform their experiences into reliable knowledge (Kolb, 2014). He further mentioned that one way of employing the theory in the classroom is through problem-based learning.

Constructivism points out that knowledge must be constructed by the learner and should not be supplied by the teacher. Its main proposition is that knowledge is created as a result of personal experience, interaction with the environment, making errors, and looking for solutions. In a constructivist classroom, the emphasis is given to learning in a meaningful context rather than on directly teaching specific skills (Büyükduman & Şirin, 2010).

In his article, Cobb (1994) described that in constructivism, students construct their mathematical and scientific ways of knowing, particularly in classroom tasks that involve investigations and explorations. This is reflective of the idea of Magoon, as stated by Noddings (1990), that humans have a highly-developed capacity to construct and organize knowledge. In mathematics education, the theory of constructivism had provided mathematics educators with useful ways of understanding learners and their

ways of learning, and thus provided a framework in the effort to reform classroom mathematics teaching (Simon, 1995). One of these reforms was the use of an open approach or the use of open problems in mathematics instruction, also described as problem-centered teaching (Pehkonen et al., 2013). It was emphasized by Dubinsky and McDonald (2001) that data on how students approach a problem-solving task based on the idea of constructivism can provide mathematics educators relevant information about how students learn mathematical concepts, which can lead to identifying their difficulties. The implementation of the idea of constructivism in mathematics teaching had shifted the focus of teachers to measuring students' reasoning and understanding the richness of student strategies and approaches in problem-solving (Confrey & Kazak, 2006).

Lev Vygotsky's (1978) concept of learning called the zone of proximal development (ZPD) articulates that there are categories of things that learners can learn but with the guidance of others. Fani and Ghaemi (2011) cited Vygotsky's definition of ZPD, which is "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 1551). This definition emphasized that learning is more meaningful through communication and social interactions than just through independent work. This is where the ideas of collaborative learning were rooted.

Verenikina (2003) described in her paper the role of teachers in a ZPD-activated classroom. She stated Vygotsky's idea that testing should be based not only on the current level of a child's achievements but also on the child's potential development. Thus, teachers must "reach and meet the level of the child's understanding and then leads the child from there to a higher, culturally mediated level of development" (para. 23). Verenikina (2003) further mentioned that effective social interaction is contributory to the successful implementation of ZPD in the classroom. In Vygotsky's developmental psychology, as stated by Langari et al. (2017), "higher mental functions are developed only with the assistance of a more capable other" (p. 399). One important component of ZPD is scaffolding, which provides a learning situation where novice learners are provided assistance and support in

their language to carry out a task that is beyond their ability (McDevitt & Ormrod, 2010). Then, the support given gradually diminishes as the students become more independent and become capable of performing the task on their own (Schumm, 2017).

In this study, the problem-solving approach was utilized as an instructional methodology in teaching College Algebra among first-year college students. Ten problem-solving tasks, which include three routine problems, three non-routine problems, and four real or open problems, were provided to the experimental group. The word problems followed the key features of a problem by the OECD (2005)—involve real-life situations, solutions are not obviously known, and involve numerous content areas. The solutions to these problems followed OECD's six stages of the problem-solving process—understanding the problem, characterizing the problem, representing the problem, solving the problem, reflecting on the solution, and communicating the solution.

The experiential learning theory, the theory of constructivism, and the ZPD were emphasized in this study through the different processes of the problem-solving tasks, including planning, organizing, implementing, evaluating the solutions of the given problems, and reflecting on the strategies implemented and decisions made by the students. Throughout the problem-solving tasks, the students identified knowledge previously learned, determined mathematical skills and concepts that they need to learn, selected appropriate strategies, exchanged ideas, and evaluated their work. In an attempt to provide solutions to the problem-solving tasks, the students were prompted to construct their own knowledge based on the mathematical concepts they have previously learned and consult references and materials to determine other useful information that they can utilize in the effective generation of viable strategies and approaches.

Also, Vygotsky's concept of zone of proximal development was evident as the problem-solving tasks enabled effective collaboration and communication among the students in the experimental group. Because the students involved in a group have different performance levels, the collaborative learning environment allowed them to support one another, to share their difficulties to the group, and to address these through the expertise of the other members. The various collaborative strategies

employed in the problem-solving tasks delivered opportunities to establish different modes of working in groups, to increase commitment to be responsible for their own learning, and to the learning of the other members in their group. As the group worked towards the end goal of the tasks, every member was equipped with the necessary understanding and skills to explain the group's product. This process provided the members of each small group of the experimental group to achieve the expected learning competencies.

Moreover, the role of the researcher throughout the problem-solving tasks was that of a facilitator guiding the students to construct a deep understanding of mathematical ideas and processes by engaging them in problem-solving tasks. Students were allowed to ask questions to the researcher. However, the researcher's responses did not provide immediate answers to the questions. Instead, the researcher asked the students with relevant questions which, when correctly answered by the students, determine the right direction towards the accomplishment of the goal of the problem-solving tasks.

With the discussions provided above, this study tested the hypothesis that the problem-solving approach can improve the academic performance in and attitude towards mathematics of the respondents by employing the true experimental research design, specifically the pretest-posttest control group design. True experimental research design, or randomized experiment, is a term used to describe research studies that involve the experimental manipulation of at least one independent variable and with at least one dependent variable (Salkind, 2010). The subjects of study are randomly assigned to the treatment group and the control group (Boudah, 2019). In this study, the treatment variable was the use of the problem-solving approach in teaching the experimental group. The dependent variables were the respondents' performance and attitude in College Algebra.

Data about the performance of the control and experimental groups as identified by their pretest, post-test, and mean gain scores were gathered. Their level of attitude towards Mathematics before and after the experimentation was also determined. Data about the pretest and posttest scores, mean gain scores, and attitude towards mathematics were pre-processed to ensure normality and to satisfy relevant assumptions. Then, the data were compared using appropriate

statistical tools to determine whether or not there exist significant differences in the performance and attitude between the experimental and control groups.

Methods

Participant Characteristics

Two of the competencies-based outcomes indicated in the Commission on Higher Education (2013) Memorandum Order, known as the General Education Curriculum, are to develop tertiary students' critical, analytical, and creative thinking skills, and effective application of different analytical modes in tackling problems methodically. These provided bases for the selection of tertiary education students as respondents of this study.

This study involved two classes of first-year tertiary education students from two distinct curricular programs of the Don Mariano Marcos Memorial State University, Philippines, who are enrolled in College Algebra and who are under the direct supervision of the researcher. For the first-year level, heterogeneous grouping was employed to create a relatively even distribution of students. The distribution of first-year students of approximately the same age was done on the basis of their academic standing in secondary level education and the results of the university admission test.

Sampling Procedures and Sample Size

The students from the two classes were paired on the basis of their Intelligence Quotient (IQ) scores and mathematics test scores in the University Admission Test, which are available at the University Guidance Office.

Twenty pairs of students with equivalent IQ and mathematics test scores formed the respondents of the study. The respondents have varied levels of performances, categorized as low, average, and high, in the university admission test to ensure even distribution of respondents for each class in terms of cognitive abilities. The assignment of the experimental group and the control group was done randomly through the tossing of a coin. The students who were not selected as respondents underwent the same teaching and learning process as those considered respondents following the teaching methodology assigned to their class.

Research Design

The true experimental research using pretest-posttest control group design was employed to examine and compare the performance in and attitude towards College Algebra of first-year college students, thus the use of two groups of respondents only, which were randomly assigned to the experimental group and control group. Data were gathered from 20 pairs of identified respondents through the results of the validated researcher-made test and the attitudinal questionnaire adopted from Baldemor and Albay's (2012) study, which were administered before and after the experiment.

Experimental Manipulations or Interventions

Mathematics education primarily aims to develop an individual's ability to solve complex mathematical problems (Wilson et al., 1993). The National Council of Teachers of Mathematics (2000) recommended that the focus of mathematics education should be on problem-solving. Therefore, the problem-solving approach in teaching College Algebra was utilized as the treatment or intervention of this study.

The problem-solving approach was implemented to the experimental group, together with collaborative strategies such as simple jigsaw, structured problem-solving, guided collaboration, focused listing, and paired annotations.

To fulfill the objectives of the study, 10 mathematical problem-solving tasks involving routine, non-routine, and real problems in various topics in the approved course syllabus in College Algebra were developed by the researcher. The mathematical problems provided to the experimental group satisfied the following criteria (Lappan & Phillips, 1998): involve useful mathematics concepts and processes, require higher-level thinking and problem solving, have various solutions, allow different strategies and decisions to be taken and defended by the students, encourage students' active engagement and discourse, have a connection with other important mathematical ideas, and promote skillful use of mathematics. The problem-solving tasks provided a platform for collaborative work for the group.

To accomplish the task, the respondents in the experimental group were engaged in discussion of the question/s under investigation; exploring the concepts and mathematical processes needed to answer the questions; identifying appropriate strategies; creating

a plan of action, implementing the plan; and making, testing, and verifying conjectures. The final goal is to develop a presentation highlighting the solutions of the groups to the tasks provided.

Moreover, the 20 identified respondents were distributed to different groups to work with the non-respondents in the class. The role of the researcher was to provide just enough information to establish the background of the tasks, to accept correct or wrong answers in a non-evaluative way, and to intervene in the problem-solving process when appropriate and necessary.

The control group, on the other hand, was taught using the conventional method of teaching where the presentation of concepts and mathematical processes was done through lecture and demonstration methods. The role of the students was to listen to the discussion and to answer questions raised by the researcher. The classroom instruction started with the presentation of the lesson, discussion of relevant concepts, demonstration of mathematical procedures, practicing the concepts and skills learned through board activities and oral recitations, and assessing students' learning by providing them with written activities. Most of the evaluative activities provided were accomplished individually. Group activities were also implemented, but such did not follow a definite structure for group work, unlike the collaborative strategies applied to the experimental group.

The experimental study was conducted for a period of 10 weeks, with three hours of meeting each week. This provided the experimental group with a sufficient

amount of time to facilitate exploration, discourse, development of strategies, and formulation, testing, and verification of conjectures using the problem-solving approach following the structures of the identified collaborative strategies. Both the experimental and control groups do not have any knowledge as to the conduct of the experimental study.

Results

Performance in College Algebra

The performance of the respondents in College Algebra includes their pretest and posttest scores in the administered test. The pretest determined the respondents' prior knowledge of the mathematical concepts and skills related to the topics in College Algebra involved in the study. The posttest, on the other hand, determined how much of these skills and concepts were mastered by the respondents. Table 1 indicates the performances in College Algebra of the two groups of respondents before and after the conduct of this study based on the results of the pretest and posttest.

It is reflected from the Table 1 that 65% of the experimental group and 70% of the control group obtained pretest scores that cluster around the 0 to 20 score range. Also, it is observed that 35% and 30% of the experimental group and control group, respectively, scored more than 20 in the pretest.

Moreover, there are slight differences in the pretest scores of the two groups. However, there are notable comparisons between the performances in College

Table 1
Performance of the Respondents in the Pretest and Posttest

Score Range	Experimental Group				Control Group			
	Pretest		Posttest		Pretest		Posttest	
	Frequency	Percentage	Frequency	Percentage	Frequency	Percentage	Frequency	Percentage
0-10	1	5.00	0	0.00	1	5.00	0	0.00
11-20	12	60.00	5	25.00	13	65.00	12	60.00
21-30	5	25.00	3	15.00	4	20.00	5	25.00
31-40	2	10.00	11	55.00	2	10.00	2	10.00
41-50	0	0.00	1	5.00	0	0.00	1	5.00
Mean Gain Score	7.70				2.75			

Algebra of the two groups that can be observed in their posttest scores.

For the experimental group, the percentage of respondents who got a score from 0 to 20 decreased by 40%, from 65% in the pretest to 25% in the posttest. In the same score range, the control group posted a reduction rate of only 10 %, from 70 % to 60 percent, in the pretest and posttest, respectively.

The experimental group recorded an increase of 40% in the percentage of respondents, which obtained scores of at least 20, that is, from 35% in the pretest to 75% in the posttest. For the control group, an increase of only 10% in the percentage of respondents who obtained such scores was noted.

In terms of mean gain score, which refers to the average of the differences between the posttest and pretest scores, the experimental group garnered an average increase of 7.70 in their posttest scores as compared to the 2.75 average gain scores obtained by the control group.

When the posttest and mean gain scores of the two groups were compared, the results indicated p -values equivalent to 0.000 and 0.011, respectively. These statistics show that the performances of the two groups are significantly different. Although both groups improved in the posttest, the tests of difference employed to the posttest and mean gain scores confirmed that the experimental group obtained higher scores and performed better than the control group.

Attitude towards College Algebra

In Table 2, it can be observed that both the control group and the experimental group conveyed a neutral attitude towards College Algebra before the experimentation, with weighted means of their responses equal to 3.19 and 3.12, respectively.

Further, the table reflects that the experimental group indicated a neutral attitude to 20 indicators. These include the indicators that convey ideas about the importance of the subject College Algebra, the topics and their applications to real life, the rules involved in the subject, and being comfortable, happy, and excited about College Algebra. They also rated with a neutral attitude the indicators that state that College Algebra helps them think logically and reason out accurately, and being interested and confident in working with exercises and assignments.

Also, the experimental group rated with an unfavorable attitude the indicators “I do well in this

subject,” “The subject makes me feel I am an expert,” and “I enjoy analyzing College Algebra problems.” Seven of the indicators garnered a favorable attitude from the experimental group, and these include the ideas that College Algebra is stimulating, interesting, fun; it makes them think clearly and encourages them to try harder; and the activities and the explanation of the teacher make them understand the subject better.

For the control group, 17 indicators were rated with neutral attitude, including those that state that the subject makes them feel comfortable and excited to learn; they are able to think clearly when working with the subject; the teacher is strict; College Algebra is worth studying; and the rules are easy to follow. The group also indicated a neutral attitude on the indicators stating that the topics and their applications to real-life are fascinating; they do well in the subject; and College Algebra is easy to understand. In addition, 10 indicators were rated with a favorable attitude by the control group. Like the experimental group, the control group also favored the idea that the subject is stimulating, fun, interesting, and extraordinary; and it encourages them to think and try harder. The group also conveyed a favorable attitude on the indicators, which indicate that they learn to think logically in this subject; they are confident and they love to solve math problems; and they feel a positive reaction towards College Algebra.

After the experiment, it can be noted that the attitude of the experimental group towards College Algebra improved from neutral to favorable, as evident in the average weighted mean of their responses equal to 4.07. On the other hand, the control group remained consistent with its neutral attitude towards College Algebra after the experiment.

The study also looked into how the respondents rated the different indicators. From the 30 indicators in the questionnaire, the experimental group rated two indicators with a neutral attitude, 15 with a favorable attitude, and 13 with a very favorable attitude. After the experiment, the group highly agreed that College Algebra is interesting, stimulating, and fun. Although the group stated that they highly favor the idea that the subject is challenging, they highly recognize the importance of the subject in acquiring the ability to think logically and to reason out accurately. The subject also allowed them to think critically by pushing them to work and think harder. Also, the competence of the teacher helped them gain a positive attitude towards

Table 2
Attitude of the Respondents Towards College Algebra

Indicators	Experimental Group				Control Group			
	Pre		Post		Pre		Post	
	WM	VD	WM	VD	WM	VD	WM	VD
College Algebra is stimulating and interesting.	3.55	F	4.45	VF	3.8	F	4.00	F
College Algebra is enjoyable, and fun.	3.90	F	4.25	VF	3.85	F	4.15	F
The topics and their applications to real life are fascinating.	2.90	N	4.05	F	2.50	U	1.85	U
This subject makes me feel comfortable.	3.15	N	3.90	F	2.90	N	3.60	F
I am able to think clearly when working with this subject.	3.60	F	4.05	F	3.20	N	3.30	N
I enjoy this subject very much.	2.90	N	4.00	F	3.35	N	3.45	F
It encourages me to think and try harder.	3.60	F	4.40	VF	3.80	F	4.00	F
I understand this subject because the teacher explains well.	3.60	F	4.55	VF	3.30	N	3.70	F
I do well in this subject.	2.40	U	3.95	F	2.50	U	2.25	U
I feel excited to work with College Algebra.	2.85	N	3.45	N	2.80	N	3.05	N
I feel at ease in this subject.	3.15	N	3.90	F	3.15	N	3.15	N
I learn to think logically in this subject.	3.05	N	4.55	VF	3.50	F	3.40	F
This subject makes me feel I am an expert.	2.55	U	3.25	N	2.85	N	2.40	U
I find this subject easy to understand.	2.95	N	4.05	F	2.55	U	2.30	U
This Mathematics subject is important to me.	2.90	N	3.90	F	2.85	N	2.05	U
I am happy in my College Algebra class.	2.95	N	3.80	F	3.55	F	3.80	F
I am confident that I can solve problems in College Algebra.	3.05	N	4.20	VF	3.5	F	3.60	F
This subject develops my ability to think and reason out accurately.	2.95	N	4.35	VF	3.15	N	3.95	F
College Algebra is my most loved subject.	3.15	N	4.35	VF	3.20	N	3.80	F
I love this subject even if my teacher is strict.	2.90	N	4.40	VF	3.15	N	3.90	F
College Algebra is challenging.	3.25	N	4.40	VF	2.95	N	3.10	N
I do like the rules used in this subject.	3.15	N	3.85	F	3.35	N	4.05	F
I like working on exercises and assignments.	2.95	N	4.25	VF	3.60	F	3.85	F
This is my favorite subject.	2.95	N	3.55	F	2.75	N	3.55	F
The activities in this subject make me understand the lesson better.	3.90	F	4.20	VF	3.35	N	4.05	F
This subject is not an ordinary one.	3.65	F	3.95	F	3.55	F	3.80	F
College Algebra is worth studying.	2.95	N	3.65	F	3.00	N	3.10	N
The rules and processes are easy to follow.	3.20	N	4.15	F	2.65	N	2.75	N
I enjoy analyzing College Algebra problems.	2.45	U	4.25	VF	3.40	F	3.50	F
I feel a definite positive reaction towards College Algebra.	3.15	N	4.05	F	3.70	F	3.95	F
Average weighted mean	3.12	N	4.07	F	3.19	N	3.38	N

Legend:

WM = Average weighted Mean

VD = Verbal Description 5.00

Scale

4.20 – 5.00

3.40 – 4.19

2.60 – 3.39

1.80 – 2.59

Verbal Description

Very Favorable (VF)

Favorable (F)

Neutral (N)

Unfavorable (U)

College Algebra. Further, the experimental group agreed that the subject has fascinating applications to real life.

For the control group, five indicators were rated with an unfavorable attitude, six with a neutral attitude, and 19 with a favorable attitude after the conduct of the experiment. The control group still favorably viewed College Algebra as a fun, interesting, and stimulating subject, and they positively recognized the potential of the subject to push them to work and try harder. They agreed that College Algebra enabled them to develop skills like reasoning and logical thinking. However, the respondents in the control group do not view College Algebra as an important subject as they do not recognize its relevant applications to real life.

The study also compared the attitudes of the groups of respondents before and after the experiment. Test results showed a significant difference between each group's attitude towards mathematics before and after the experiment and indicated that only the experimental group posted a significant improvement or change in their attitude with a p -value equal to 0.000. The improvement in the attitude of the control group towards College Algebra, on the other hand, was not sufficient to claim a significant change or enhancement in their attitude as viewed in the obtained p -value of 0.195.

Discussion

The data and the results of the analyses conducted on the performance and attitude of the respondents provide scientific evidence about the potential of the problem-solving approach to cause a significant improvement in the cognitive and affective attributes of students in College Algebra. It has been shown in this study that teaching through problem-solving develops the skills of students to prosper in a self-regulatory and problem-centered learning environment. The use of routine, non-routine, and real problems delivered learning opportunities and provided direct experiences for the experimental group to learn and understand mathematics by applying both structured and non-structured methods of solving real-life problems. This fostered the development of the students' skills in utilizing previously learned concepts, critically evaluating the given variables and their interrelationships, constructing new knowledge by sourcing out information from various content areas,

selecting appropriate ways of representing or modeling the given problems, generating and devising a myriad of viable strategies and approaches towards achieving the goals of the tasks, evaluating these planned solutions, reflecting on their applied cognitive and metacognitive strategies, and effectively communicating their solutions using relevant media and tools. True to the ideas of Gur and Korkmaz (2003) and Altun (2008, as reported by Yavuz and Erbay, 2015), allowing students to solve mathematically rich problems enable them to gain meaningful experiences in solving mathematics problems and in selecting appropriate strategies to solve the problems. Therefore, posing mathematically rich problems can be an effective means of learning and understanding mathematics, hence resulting in a better performance in the subject (Căprioară, 2015).

The six stages of the problem-solving process discussed in the framework laid down a definitive method for presenting the experimental group's solutions to the problem-solving tasks. By correctly identifying the given variables and the unknown, the students were able to determine further knowledge that they need to acquire by carefully considering relevant contents from numerous resources and through social interactions. A connection between previously learned concepts and newly acquired knowledge was then established, thereby making them proficient in constructing visual representations or models for the problems. By exploration and manipulation of the models developed, the students were deeply and actively engaged in generating, testing, refining, and justifying mathematical conjectures, which guided their planned strategies and solutions. The collaborative examination and reflection on the solutions generated allowed the members of each group in the experimental group to scrutinize the correctness, completeness, and accuracy of their solutions. In cases where inconsistencies are noted, the group refined their solutions and presented them in ways that are comprehensible to the rest of the experimental group. Because every member was involved in the entire process of the problem-solving tasks, each member gained the necessary confidence in explaining their approaches or strategies and in justifying their solutions. Although the control group was given the same problem-solving tasks to accomplish, the learning environment characterized by the conventional method of mathematics teaching did not provide essential conditions, allowing for

deeper exploration and investigation of the problem tasks at hand.

As teaching through problem-solving promotes a collaborative nature of the learning environment, the experimental group achieved a higher level of conceptual understanding, critical thinking skills, and problem-solving skills through their active engagement in the discourse and exploration process (Gokkurt *et al.*, 2012). This feature of the problem-solving approach was highly significant, especially when the experimental group dealt with the open problems that prompted them to demonstrate effective communication, logical thinking, reasoning skills, and handling diverse and varied perspectives. Although the members of each group were provided with opportunities to listen to different perspectives and to challenge the ideas of the other members, each was given a chance to present and defend their conceptual frameworks. All the members in the experimental group were compelled to explain how their ideas work based on the given information in the problems and to justify what prompted them to generate a particular conclusion. In short, they were encouraged to explain their thinking to their group. Thus, the problem-solving approach provided opportunities for the students to hone valuable problem-solving skills by formulating their ideas, discussing them with their group members, receiving immediate feedback, and responding to questions and comments (Laal & Ghodsi, 2012) through which they were able to refine, combine, and modify knowledge they already learned (Cai & Lester, 2010). Further, the approach highlighted the importance of working as a team and demonstrating effective communication to solve a problem-solving task (Abdullah *et al.*, 2010); Duchet *et al.*, 2001). Such an active exchange of ideas did not occur in the class of the control group as the conventional method of teaching capitalized on individual learning and seldom provided platforms for group work.

Meanwhile, the support and assistance a member received from the other members of the group also play a crucial role in the implementation of the problem-solving approach in mathematics teaching. It is essential to create a learning environment where asking for necessary support from advance members or providing the needed assistance to challenged members is an integral component of learning and understanding mathematics. Therefore, Vygotsky's idea of zone of proximal development and scaffolding

also contributed to the results of the study based on the performance of the experimental group as compared to the control group's performance. The different collaborative strategies implemented along with the approach required the members of the group to assess each other's learning and mastery of the relevant concepts and mathematical processes. The scaffolding strategies employed by each group provided the needed support and assistance to the struggling members. In their attempt to solve the problem tasks, the students in the experimental group made sure that they meet the level of their members, they provided opportunities for clarifications, and they maintained open communication to address any difficulties that the members encounter in the course of accomplishing the goal of the tasks. This kind of supportive learning environment prevailed and dominated the classroom of the experimental group in the entire duration of the experiment, producing students who can effectively implement the six stages of the problem-solving process. As anyone from each group can be chosen to present their respective outputs with or without help from the other members, this necessitated all the members to acquire the necessary understanding and mastery to comprehensively discuss their group's output and to substantially answer questions from the class. Indeed, the high level of interaction and interdependence assisted the students in acquiring deep understanding and mastery of the concepts and mathematical processes. Thus, the experimental group developed mathematical power that enabled them to solve mathematical problems on their own during the administration of the posttest. The active support provided through scaffolding ensured that all students of the experimental group had gained the necessary competencies to solve mathematical problems independently. This enabled the group to record higher scores in the posttest and, thus, outperformed the control group.

Moreover, the problem-solving approach developed the students' positive attitude towards mathematics as a result of the supportive learning environment of the approach where collaboration is valued rather than competition, which is most likely to occur in an individualistic approach to teaching mathematics.

Considering that the respondents in the experimental group had to work collaboratively since the start of the experiment, the interactions happened regularly; hence they came to know each other better, and they were able

to build a healthy, friendly, and helping relationship. They are free to seek help from their peers by asking questions or requesting explanations. Although the low performers gained conceptual understanding and acquired problem-solving skills from the high performing members, the latter also became more committed to discuss and explain concepts, processes, and mathematical ideas in order to address the difficulties encountered by the former. The approach also highlighted the role of the high performing members, particularly on taking responsibility not only of their own learning but also the learning of their low performing members. Moreover, the active exchange of ideas and supportive environment created by the students nurtured positive feelings that consequently increased their interests towards learning.

Being engaged in collaborative problem-solving tasks enabled the experimental group to develop patience, perseverance, disposition, and motivation to learn and understand mathematics. The strong support of the high performing members led the low performing members to develop a better understanding of the problems and to explore and focus on the meaning of the problems. One of the observed behaviors of the experimental group was the satisfaction and happiness they derived from discovering the solutions of the problem-solving tasks.

Additionally, the respondents of the experimental group were observed to be more confident to present their output to the class, thinking that it was a product of the collective efforts of the group. This, in turn, helped the students to eliminate their fear towards the subject and their anxiety in working with College Algebra problems. By working as a group, it transformed them from passive listeners and information receivers to active, well-engaged learners, and problem solvers (Ali et al., 2010). As a result, the experimental group gradually built confidence in working with the problem-solving tasks, exercises, and other activities; hence the development of mathematical power that contributed to the attainment of a more positive attitude towards College Algebra.

The role of the researcher in the problem-solving process also played a vital role in the improvement of the experimental group's performance and attitude. The presence of the researcher enabled the students to instantly acquire guidance in situations where they were stuck. Although instructions as to how the students should work on the problems cannot be given,

the insightful questions that the researcher asked served as a springboard for students' reflection about their work or the process they applied. This allowed them to integrate essential modifications to their strategies, hence leading them to the solutions of the tasks.

To sum up, the problem-solving approach can cause significant improvement to the cognitive and affective attributes of students in tertiary education. When effectively implemented, the approach can enhance students' academic achievement in mathematics. Equally important, it can also strengthen or reinforce a more positive attitude towards the subject. The use of the approach in teaching mathematics also provides effective learning conditions that allow students to develop essential 21st-century skills.

Conclusion

Developing students' ability to solve problems is an integral part of mathematics learning in all grade levels. To help students become effective problem solvers, teachers must develop a learning environment where problem-solving activities are essential components of classroom practices. Students should be taught in a way that obtaining a full understanding of mathematical concepts and processes is a result of solving problems, rather than teaching them how to understand it. Through the implementation of the problem-solving approach, this aspect of mathematics education can be developed. This study utilized the problem-solving approach as an alternative method of instruction in mathematics. The use of such an approach provided platforms for the students to brainstorm in understanding the problem, assessing, analyzing and utilizing data, developing strategies, implementing the agreed steps of the solutions systematically, and evaluating and justifying solutions. These activities often require different cognitive and metacognitive processes, such as analytical reasoning, quantitative reasoning, analogical reasoning, and combinatorial reasoning skills.

Central to the effectiveness of the approach is the ability of the students to collaborate, communicate, and actively engage in the entire process of problem-solving. The competence of the teachers to provide well-planned and well-developed problem tasks also play a crucial role in the successful implementation of the problem-solving approach in mathematics teaching. Nevertheless, the results ascertained that

the problem-solving approach did not only impact the enhancement of the students' performance but also the reinforcement of a positive attitude towards College Algebra. Also, the use of the problem-solving approach in mathematics instruction contributed to the development and acquisition of various 21st-century skills, such as critical thinking, collaboration, communication, and creativity.

Although this study provided pieces of evidence of the effectiveness of the problem-solving approach, for any instructional methodology to effectively work, it should always involve appropriate teacher support and careful selection of problem-solving tasks and learning experiences that engage students actively and enable them to work collaboratively.

Although the results of the study contribute to the existing body of knowledge on the use of a problem-solving approach in mathematics education, more extensive research studies involving the use of the approach need to be conducted. Future researchers may explore on the skills and competencies of mathematics teachers that are considered vital for the effective implementation of the problem-solving approach. As this study only considered the results of the test and the attitudinal questionnaire, researchers may consider analyzing students' interaction and communication strategies while attempting to solve the problem-solving tasks and investigate their effects on the performance of the students. They may also explore the approaches and strategies devised by the students in accomplishing the problem-solving tasks, which may lead to the creation of a problem-solving model that is distinct in a problem-solving approach mathematics classroom.

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Declaration of ownership:

This report is my original work.

Conflict of interest

None.

Ethical clearance

This study was approved by the institution.

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