Activity 3:
Length Measurements with the Four-Sided Meter Stick

OBJECTIVE:

The purpose of this experiment is to study errors and the propagation of errors when experimental data derived using a four-sided meter stick are arithmetically manipulated.

THEORY:

Error of measurement refers to the uncertainty of a reading. It carries no implication of mistake or blunder on the part of the observer. *Error* means the uncertainty between the measured value and the standard value. It can either be a *positive error*, that is the error that tends to make a reading too high, or a *negative error*, one in which reading tends to be too low.

Errors may be grouped into two classes, *systematic* and *random*. A *systematic error* is one that always produces an error of the same sign, for example, one that would tend to make all the observations too small. A *random error*, also called *erratic error*, is one in which positive and negative errors are equally probable.

Systematic errors can be subdivided into three groups depending on the sources of errors. These are:

1. Instrumental Errors

   This type of errors is caused by faulty or inaccurate apparatus. For example, an undetected zero error in a scale, an incorrectly adjusted or a meter with undue friction.

2. Personal Errors

   These errors are due to some peculiarity or bias of the observer. Probably the most common source of personal error is the tendency to assume that the first reading taken is correct and to look with suspicion on any variation from this reading. Other personal errors may be due to fatigue, eyestrain, or the position of the eye relative to a scale.

3. External Errors

   Errors of this type are caused by external conditions such as wind, temperature, humidity, vibration, etc. Examples of external errors are the expansion of scales as the temperature rises and the swelling of meterstick as humidity increases.

Corrections may be made for systematic errors when they are known to be present.

Random or erratic errors occur as variations that are due to a large number of factors, each of which adds its own contribution to the total error. Inasmuch as these factors are unknown and variables, it is assumed that the resulting error is a *matter of chance* and therefore positive and negative errors are equally probable. Because random errors are subject to the laws of chance, taking a large number of observations may lessen their effect in the experiment.

Due to the fact that variations in the observations only follow chance, the laws of statistics may be used to arrive at certain definite conclusions about the magnitude of the errors. The following results of statistical laws can be utilized to quantify error of measurement:

1. Arithmetic Mean, *a.m.*

   The arithmetic mean *a.m.* represents the best value obtainable from a series of observations. It is the value having the highest probability of being correct. It is obtained by dividing the sum of the individual readings by the total number n of observations. In symbol,

$$a.m. = \frac{\Sigma N}{n}.$$

2. Average Deviation, *a.d.*

   The average deviation *a.d.* is a measure of the accuracy of the observation. It shows the average value of the divergence of the observations from the arithmetic mean. The average deviation is calculated by dividing the sum of the deviations (without regard to sign) by the number n of observations, where deviation d refers to the difference between an observation and the arithmetic mean. In symbol,

$$a.d. = \frac{\Sigma d}{n}.$$

3. Average Deviation of the Mean, *A.D.*

   The average deviation of the mean *A.D.* measures the heterogeneity or evenness within a set of observations. If the average deviation of the mean is small, then it can be said that the set of observations is homogeneous. For it is known from theory of probability that an arithmetic mean computed from n equally reliable observations is on the average more accurate than any one observation by a factor of $\sqrt{n}$, the average deviation A.D. of the mean of n observations is given by

$$A.D. = \frac{a.d.}{\sqrt{n}}.$$

The importance of an error in an experimental value is not in its absolute value but in its *relative value*. By *percentage error* is meant the number of parts out of each 100 parts that a number is in error. The equation for solving the percentage error is

$$Percentage\ Error = \frac{Experimental\ Value - Standard\ or\ Accepted\ Value}{Standard\ or\ Accepted\ Value}\ x\ 100$$

When two values of the same quantity are obtained experimentally, the *percentage difference* is used to compare the two values. The equation is

$$Percentage\ Difference = \frac{Difference\ between\ the\ two\ values}{Average\ of\ the\ two\ values}\ x\ 100$$

In addition, when experimental data are arithmetically manipulated, errors may be propagated. The following rules are safe to apply in making computations:

1. In addition and subtraction, carry the result only through the first column that contains a doubtful figure.

2. In multiplication and division, carry the result to the same number of significant figures that are in the factor with the least number of significant figures.

One must learn therefore how to determine significant figures in a measurement. The significant figures of a number specify its *accuracy*, the number of digits which are *meaningful*. For example, if you have a ruler marked off in millimeters and you use it to measure the length of a line, you are able to measure accurately only to the nearest millimeter. Suppose you found the line to be 1351 mm or 1.351 m long. The number of significant figures in this case is 4 (1, 3, 5 and 1). It would not be correct to write 1351.00 mm or 1.3514500 m, since this implies that you know the length to a greater accuracy than you actually do.

The trickiest aspect of significant figures is knowing how to treat zero. For example, the number of significant figures in the number 7300 is two, while the number of significant figures in the number 7301 is four. This is because the zeros in 7300 are simply *place holders*, giving the magnitude or power of 10 for the number. Thus 7300 implies that this number is known only to the nearest power of 2 (100's). On the other hand, 7301 implies that the number is known to the nearest unit.

The same situation holds for numbers less than one. For example, the number 0.00345 has three significant digits, while the number 0.003 has only one. The zeros to the right of the decimal do not specify accuracy but, rather, are *place holders,* specifying the magnitude (powers of 10) of the number.

MATERIALS:

The materials needed for the experiment are:

- ☯ wooden block
- ☯ four-sided meter stick

PROCEDURE:

1. Use the four-sided meter stick to measure the block, making the same measurement successively by means of scales of progressively finer graduations. First, measure the length using the face with no graduation. This measurement is only an *estimate* of a fraction of a meter. Record this length and the three following sets of data in rows as follows: (a) observed length, including the estimated portion of the smallest scale division; (b) the value of the smallest scale division being read; (c) the fractional part of the scale division that can be estimated by the eye with reasonable certainty (for example, 0.1 of a scale division); (d) the numerical uncertainty of the observation, which will simply be (c) expressed as an actual length, for example 0.1 m; (e) the percentage uncertainty involved in the estimation, which will be obtained by dividing (d) by (a) and multiplying by 100 per cent.

2. Repeat the measurement of the length of the block, this time using the face of the stick calibrated in decimeters. Estimate the uncertainty, and record all the data, exactly as in the previous step. Repeat, using the face graduated in centimeters. Repeat, using the millimeter scale.

3. Measure and record the length of the block using the millimeter scale. Take six observations at different places on the block in order that a fair average may be obtained. In each reading estimate fractional parts of millimeter divisions. (Remember to include the proper number of zeros when the observed length seems to fall exactly on a scale division.) Calculate the mean of the observations of the length. Find the deviation from the mean and calculate the average of the deviations. Compute the A.D. of the mean length. Attach this A.D. to the mean length with a (±) sign and call this final result the observed length of the block.

4. Repeat Procedure 3 for the width and thickness of the block and determine the A.D. of each. Calculate the volume of the block, recording the result to the proper number of significant figures. Determine the numerical and percentage errors of this volume, using for the respective errors of the length, width, and thickness the A.D.'s found above.

DATA

Table 1. Measurement of Length Using a Meterstick with No Graduation

Trial	1	2	3
Observed length			
Smallest scale division			
Smallest fractional part of the scale division that can be reasonably estimated			
Numerical uncertainty			
Percentage uncertainty			

Table 2. Measurement of Length Using a Meterstick Calibrated in Decimeters

Trial	1	2	3
Observed length			
Smallest scale division			
Smallest fractional part of the scale division that can be reasonably estimated			
Numerical uncertainty			
Percentage uncertainty			

Table 3. Measurement of Length Using a Meterstick Calibrated in Centimeters

Trial	1	2	3
Observed length			
Smallest scale division			
Smallest fractional part of the scale division that can be reasonably estimated			
Numerical uncertainty			
Percentage uncertainty			

Table 4. Measurement of Length Using a Meterstick Calibrated in Millimeters

Trial	1	2	3
Observed length			
Smallest scale division			
Smallest fractional part of the scale division that can be reasonably estimated			
Numerical uncertainty			
Percentage uncertainty			

Table 5. Measurement of Length of a Block

Trials	Observed Length	Deviation from the Mean
1		
2		
3		
4		
5		
6		
Average	a.m. =	a.d. =
Average Deviation of the Mean	A.D. =	

Table 6. Measurement of Width of a Block

Trials	Observed Length	Deviation from the Mean
1		
2		
3		
4		
5		
6		
Average	a.m. =	a.d. =
Average Deviation of the Mean	A.D. =	

Table 7. Measurement of Thickness of a Block

Trials	Observed Length	Deviation from the Mean
1		
2		
3		
4		
5		
6		
Average	a.m. =	a.d. =
Average Deviation of the Mean	A.D. =	

Table 8. Propagation of errors during arithmetic manipulations

Volume (accepted value) = _________________ mm^3

Volume	Calculated Value	Numerical Error	Percentage Error
Minimum			
Maximum			

GUIDE QUESTIONS FOR ANALYSIS:

1. How does the side of the four-sided meter stick influence the precision of your measurement? How about the accuracy?

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2. Five different students take the following measurements of the same object:
   1.0 m 1.45 m 1.5 m 1.4530 m 1.46 m
   (A) Why are the measurements different? (B) Which one is correct? (trick question)

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3. Discuss the relative accuracy obtained and the variation in the error caused by estimating fractional parts of scale divisions as the same length is measured with scales of progressively decreasing lengths of scale divisions (refer to the data gathered in Procedure 2 and Procedure 3).

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4. What does the average of the deviations signify?

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5. What is the significance of average deviation of the mean?

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