The Effect of Noise Traders on The Distribution of Returns: Evidence from an Ising-based Model of Financial Markets

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Abstract: Financial markets serve as prime examples of complex systems due to the emergence of statistical signatures in the form of power laws as a result of the behaviors and interactions between traders. The self-organizing Ising model of financial markets, a physics-inspired agent-based model, took traders' intricate behaviors into account and successfully retrieved statistical properties observed in real markets. This model has been revisited and extended in this study by introducing varying proportions of noise traders, defined as traders who decide to buy or sell an asset randomly with equal probabilities. Upon scanning different model parameter sets, it was found that the general effect of noise traders on the market dynamics, specifically, on the distribution of absolute logarithmic returns, is the reduction of fat-tails and the smooth evolution to the Gaussian distribution. The distributions of the artificial markets also reveal that a mix of noise and informed traders is sufficient to simulate efficient markets with Gaussian distributions of absolute logarithmic returns. Power law distributions with exponents $\alpha \approx 3$ were also observed in the artificial markets for certain parameter sets. These markets were identified as realistic since power law exponents $\alpha \approx 3$ are typical of real-world markets. It was observed that the Ising models that have retrieved the real-market distributions have a small percentage of noise traders, which is consistent with what is observed in reality since uninformed traders are also allowed to participate in markets.

Key Words: complex systems; power laws; financial markets; Ising model

1. INTRODUCTION

Complex systems are characterized by emergent statistical signatures, often in the form of power laws, due to heterogeneous interactions between many agents. (Clauset et al., 2009; Sornette, 2009). Such power laws are observed in various physical and socio-economic systems. A few examples to note are the observance of power laws in earthquake magnitudes (Newman et al., 2005), city sizes (Bettencourt et al., 2007), internet traffic (Mislove et al., 2007), social network structures (Le.skovec et al., 2007), and species abundance in ecosystems (Ulrich et al., 2010). In finance, two power laws may be observed: fat-tails in the distribution of returns, also known as the inverse cubic law of returns, and volatility clustering (Lux, 2009). The latter describes the tendency of financial markets to undergo periods of high volatility followed by low volatility periods and is described statistically by the hyperbolic decay in the autocorrelation function of the absolute logarithmic returns, an indicator of long-memory in financial time series. The former, emphasized in this study, describes the presence of thicker tails in the distribution of returns, indicating a higher probability of extreme returns than a Gaussian distribution. The literature on the investigation of power laws in finance is extensive. Power law exponents of $\alpha \simeq 3$ and $\gamma \simeq 0.3$ have been found in the tails of the distribution of absolute logarithmic returns and autocorrelation functions, respectively (Gopikrishnan et al., 1998; Liu et al., 1999; Gabaix 2009).

The fact that power laws are observed in finance is evidence of the complex nature of financial markets. Thus, it is necessary to formulate a model of financial markets using a bottom-up, complex systems-based approach. The self-organizing Ising model of financial markets (Zhou & Sornette, 2007), based on interactions among spins in a lattice in Physics, was thus formulated with this approach in mind. In this model, each agent decides to either buy or sell a financial asset based on various factors that define market conditions, such as the external news, behavior of neighbors/fellow traders, and personal idiosyncrasies. The aggregate behavior of all agents' trading decisions results in complex market dynamics. This model successfully retrieved the statistical stylized facts of real-world financial markets: fat tails in the distribution of large returns, volatility clustering, the existence of bubbles and crashes, and the evolution of the distribution of returns at the largest time scale. In a recent study, the analysis of the Ising model of financial markets was extended with efficiency considerations (Antenorcruz & Batac, 2023). Different parameter configurations were scanned to identify the following statistical stylized facts: fat tails in the distribution of large returns, volatility clustering, and multifractality, measured using the multifractal detrended fluctuation analysis and used as a quantification for market efficiency. The current study builds upon the foundation of the original model by incorporating noise traders, individuals who decide to buy/sell randomly. By introducing noise traders into the Ising model of financial markets, we seek to explore how irrational behavior influences market dynamics and the statistical properties of returns. Specifically, we investigate how varying proportions of noise traders from 0% to 100% affect the distribution of absolute logarithmic returns within Ising-based financial markets.

The rest of this paper is organized as follows: the methodology is discussed in the next section. The description of the Ising model of financial markets with the incorporation of noise traders, along with the specific parameter sets investigated will be discussed in the methodology. The third section, the results and discussion, will illustrate and discuss the effect of noise traders on the distribution of absolute logarithmic returns for the parameter sets considered. Finally, key findings and recommendations for further study are discussed in the conclusions in the fourth section.

2. METHODOLOGY

2.1 Description of the Ising-based Model of Financial Markets with Noise Traders

The Ising model in physics involves interactions between spins in an $n \times n$ two-dimensional lattice. The state of a spin in the lattice is affected by factors such as the states of its nearest neighbors and external magnetic fields applied to the system. In financial markets, the spins are considered traders making up the market. These traders are placed on a two-dimensional lattice like the original Ising model. In this study, we have investigated traders placed on a 20×20 square lattice with periodic boundary conditions. At the time $t = 0$, each trader is assigned a random initial state. $s_i = +1$ denotes a buyer whereas $s_i = -1$ denotes a seller. For future time steps t, a trader decides to buy or sell an asset based on the following equation:

$$s_i(t) = sgn\left[\sum_{j\in N} K_{ij}(t)E[s_j](t) + \sigma_i(t)G(t) + \epsilon_i(t)\right]$$

(Eq. 1)

where:

$s_i(t)$ = the state of trader i at time t; $s_i = +1$ denotes a buyer whereas $s_i = -1$ denotes a seller.

$E[s_j](t) = s_j(t-1)$ = trader i's expectation on their neighbor j's state at time t, wherein trader i expects that their neighbor will have the same state at time t based on the previous time step $t-1$.

$G(t)$ = the external news, assumed to be a white Gaussian noise with unit variance.

$\sigma_i(t)$ = the relative sensitivity of the trader i to the news, assumed to be uniformly distributed in $(0, \sigma_{max})$

$\epsilon_i(t)$ = idiosyncratic judgment associated with private information, normally distributed around zero with standard deviation $s_{\epsilon,i} = CV + \varepsilon$ where ε is uniformly distributed in $(0, 0.1)$ and CV is a common constant across all traders.

Finally, the variable $K_{ij}(t)$ is the relative propensity of trader i to be influenced by their neighbor j, which is determined by the following equation:

$$K_i(t) = b_i + \alpha K_i(t-1) + \beta r(t-1)G(t-1)$$

(Eq. 2)

where:

b_i = trader i's idiosyncratic imitation tendency, uniformly distributed in $(0, b_{max})$

α = persistence of past influence on the present, fixed at 0.2

β = propensity of agent i to imitate based on the role of the external news in determining the returns at time t, set to 1.

$r(t-1)$ = the market logarithmic returns at the previous time step.

The market logarithmic returns were calculated as follows:

$$r(t) = \frac{\sum_{i \in N} s_i(t)}{\lambda N} \quad \text{(Eq. 3)}$$

where:

λ = the market depth, set to 40.
N = the market (grid) size, set to 20.

With all fixed and stochastic parameters defined, the Ising model of financial markets is specified by the parameter space (b_{max}, σ_{max}, CV). This study investigated values of $b_{max} \in [0.2, 0.3]$ with an increment of 0.1, $\sigma_{max} \in [0.15, 0.45]$, with increments of 0.15, and $CV = 0.5$. The motivation behind choosing these ranges for these parameters is because these values for b_{max}, σ_{max} and CV have admitted logarithmic return statistics comparable to real-world markets (Antenorcruz & Batac, 2023).

To incorporate noise traders into the Ising model, noise traders were defined as agents that decide to buy or sell at time t in a purely random manner:

$$s_{i,Noise}(t) = \pm 1 \text{ with probability } ½. \quad \text{(Eq. 4)}$$

These noise traders were randomly determined on the grid at the initial time and will remain as noise traders until the final time step. Well-informed traders who make decisions based on Eq. 1 shall remain as such up to the final time step. This study has generated artificial time series of length $t_{max} = 2520$ in units of trading days, equivalent to ten years of data. To investigate the effect of noise traders on the distribution, the proportion of noise traders were varied from 0% to 100% with increments of 20%. Therefore, the Ising model of financial markets with noise traders was explored in the parameter space (b_{max}, σ_{max}, CV, NT) where NT is the proportion of noise traders in the grid. The locations of noise traders for different proportions are illustrated in Fig. 1.

2.2 Investigation of the Effect of Noise Traders on the Distribution of Absolute Logarithmic Returns

For each logarithmic return series generated by a parameter set (b_{max}, σ_{max}, CV, NT), the distribution of the absolute logarithmic returns, $|r(t)|$ were obtained. Specifically, the empirical complementary cumulative distributions were calculated, written as:

$$P(|r(t)| > x) = 1 - P(|r(t)| \le x) \quad \text{(Eq. 5)}$$

For a given parameter set (b_{max}, σ_{max}, CV), distributions for different proportions of NT are placed on a single double logarithmic plot of the distribution of logarithmic returns to visually inspect the effect of NT. Each distribution was fitted against the Gaussian distribution to inspect any deviations or similarities with the Gaussian visually. Parameter sets with distributions closely matching the Gaussian were identified as efficient markets.

2.3 Estimation of Power Law Exponents

In real-world markets, the distributions of absolute logarithmic returns are observed to have fat tails, described by the following power law (Gopikrishnan et al., 1998):

$$P(|r(t)| > x) \sim x^{-\alpha} \quad \text{(Eq. 6)}$$

Where the power law exponent $\alpha \approx 3$ is found in empirical studies of financial markets. Parameter sets with distributions exhibiting power laws $\alpha \approx 3$ were identified as realistic markets. The power law exponents for a given parameter were estimated using the *powerlaw* package in *Python* (Alstott et al., 2014).

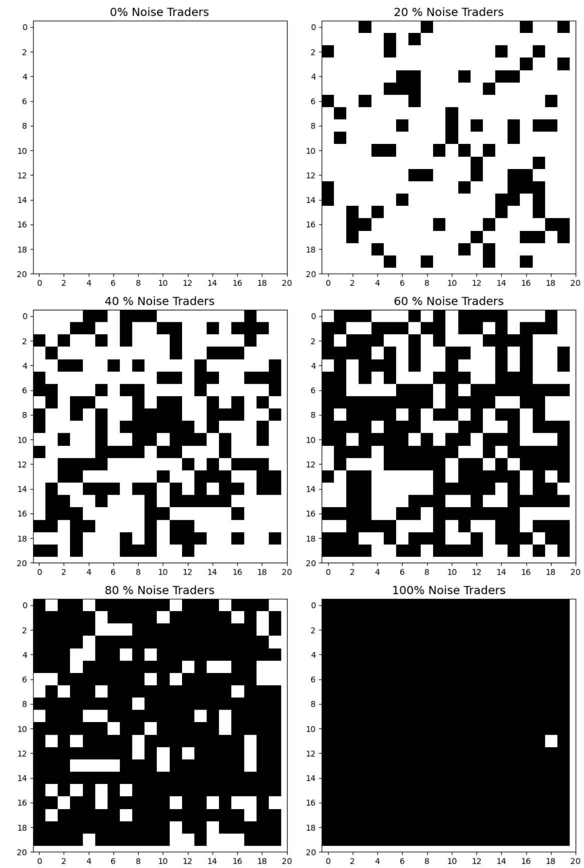

Fig. 1. Locations of noise traders for different proportions. Each subplot represents a 20 x 20 Ising grid. The black cells correspond to noise traders whereas the white cells correspond to informed traders.

3. RESULTS AND DISCUSSION

The distributions of the absolute logarithmic returns for the parameter sets (0.2,0.15,0.5), (0.2,0.3,0.5), (0.2,0.45,0.5),(0.3,0.15,0.5),(0.3,0.3,0.5) and (0.3,0.45,0.5) are shown in Figs. 2-7. The proportion of noise traders, *NT,* range from 0% to 100% with increments of 20%. In Figs. 2-7, the distributions with shades closer to red represent those which deviate from the Gaussian distribution, whereas the distributions with shades closer to green represent those that resemble the Gaussian distribution. The broken, colored lines represent the Gaussian distribution for a given proportion of noise traders. In Figs. 2, 4, and 5, the dotted line represents a power law with $\alpha = 3$.

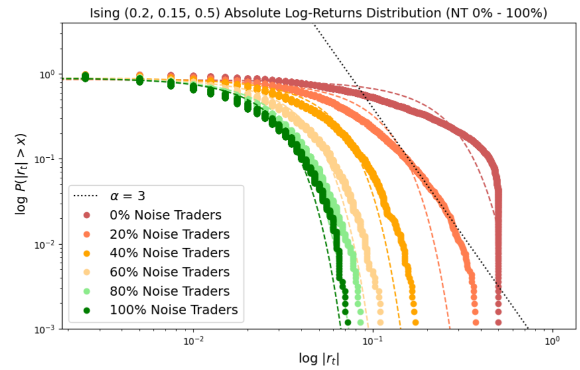

Fig. 2. The effect of noise traders on the distribution of absolute log-returns for the parameter sets $(b_{max} = 0.2, \sigma_{max} = 0.15, CV = 0.5, NT)$

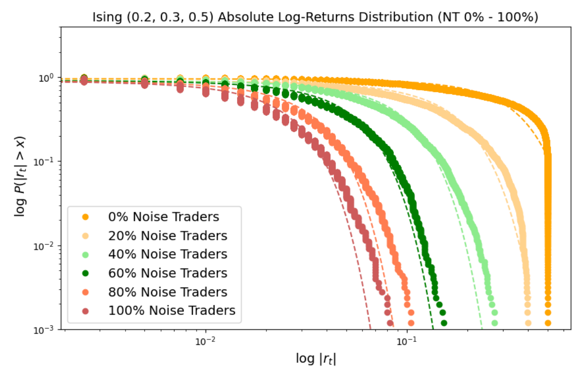

Fig. 3. The effect of noise traders on the distribution of absolute log-returns for the parameter sets $(b_{max} = 0.2, \sigma_{max} = 0.3, CV = 0.5, NT)$

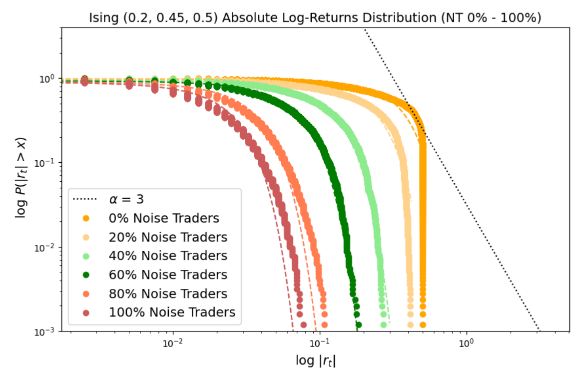

Fig. 4. The effect of noise traders on the distribution of absolute log-returns for the parameter sets $(b_{max} = 0.2, \sigma_{max} = 0.45, CV = 0.5, NT)$

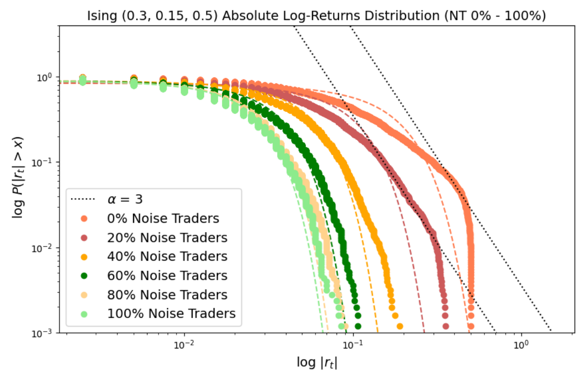

Fig. 5. The effect of noise traders on the distribution of absolute log-returns for the parameter sets ($b_{max} = 0.3$, $\sigma_{max} = 0.15$, $CV = 0.5$, NT)

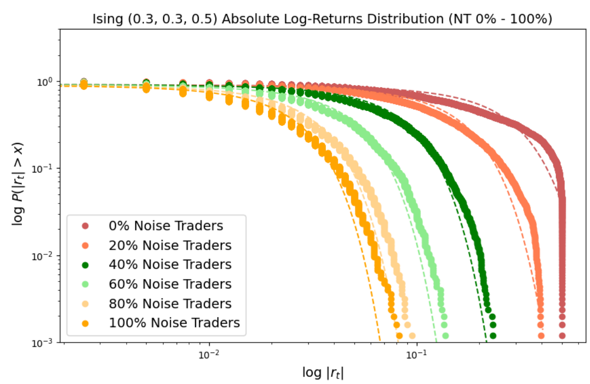

Fig. 6. The effect of noise traders on the distribution of absolute log-returns for the parameter sets ($b_{max} = 0.3$, $\sigma_{max} = 0.3$, $CV = 0.5$, NT)

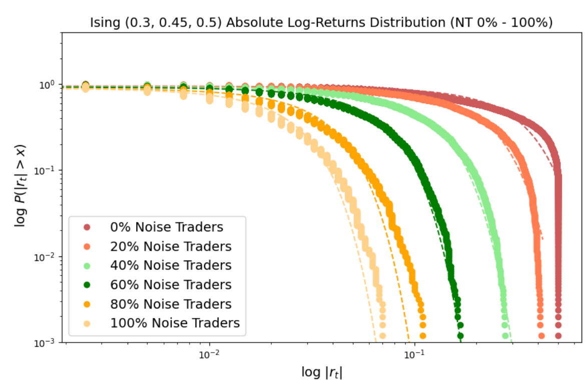

Fig. 7. The effect of noise traders on the distribution of absolute log-returns for the parameter sets ($b_{max} = 0.3$, $\sigma_{max} = 0.45$, $CV = 0.5$, NT)

Upon observation of Figs. 2-7, at 0% noise traders, the distributions of the absolute logarithmic returns were found to be generally fat. This makes sense: if a market is entirely composed of intelligent traders who make decisions based on their neighbors' behavior, the external news, and any private information they may personally hold, the result is the emergence of a market with a higher likelihood of extreme return events, shown in the fat tails of the distribution of returns. Now, as more noise traders were introduced into the market, a reduction of the fatness in the tails and smooth transitions to the Gaussian were observed. It is worth noting, however, that not all parameter sets have fully transitioned to the Gaussian at 100% proportion of noise traders. In fact, this may only be observed with the parameter set (0.2,0.15,0.5). For the parameter sets (0.2,0.3,0.5), (0.2,0.45,0.5), (0.3,0.15,0.5), (0.3,0.45,0.5), a Gaussian distribution of returns was observed at 60% proportion of noise traders, whereas for the parameter set (0.3,0.3,0.5), the Gaussian was observed at 40% proportion of noise traders. These results are counterintuitive: if the dynamics of a market are determined by the collective behavior of traders, then a market that consists purely of noise traders must, therefore lead to the emergence of a Gaussian distribution of returns. Our findings, however, contradict this. The simulated markets imply that a mix of noise and informed traders leads to a Gaussian distribution of returns, and therefore, an efficient market.

In addition to observing the effect of noise traders on the distribution of returns, several parameter sets that have retrieved return distributions akin to real-life markets have been identified: (0.2,0.15,0.5,20%), (0.2,0.45,0.5,0%), (0.3,0.15,0.5,0%), and (0.3,0.15,0.5,20%). These artificial markets are recognized as the most 'realistic' markets since the power-law exponents obtained were comparable to what is observed in data on real-world markets. The estimated power law exponents for these parameter sets are summarized in Table 1.

Table 1. Power law exponents

Parameter set (b_{max}, σ_{max}, CV, NT)	Minimum value for fitting x_{min}	Estimated power law exponent α
(0.2,0.15,0.5,20%)	0.0925	3.2448
(0.2,0.45,0.5,0%)	0.1550	2.1517
(0.3,0.15,0.5,0%)	0.1200	2.6147
(0.3,0.15,0.5,20%)	0.1325	3.9649

It can be observed based on the estimated power law exponents summarized in Table 1 that only two parameter sets retrieved power law exponents greater than $\alpha = 3$, namely: (0.2,0.15,0.5,20%) and (0.3,0.15,0.5,20%). The similarity between these two parameter sets is the low percentage of noise traders. Therefore, only low percentages of noise traders must be explored to retrieve realistic markets from the Ising model. Realistically, this makes sense: a few traders are not well informed in real markets and therefore make decisions similar to a noise trader.

4. CONCLUSIONS

In summary, this study extended the Ising model of financial markets by introducing noise traders with varying proportions, from 0% to 100%. Upon scanning various parameter sets, it was found that the effect of increasing noise traders in a market is generally a decrease in the fatness of tails in the distribution of absolute logarithmic returns. The parameter sets (0.2,0.15,0.5, 100%), (0.2,0.3,0.5, 60%), (0.2,0.45,0.5, 60%), (0.3,0.15,0.5,60%), (0.3,0.45,0.5,60%), and (0.3,0.3,0.5,40%) were identified as efficient markets with a Gaussian distribution; whereas the parameter sets (0.2,0.15,0.5,20%), (0.2,0.45,0.5,0%), (0.3,0.15,0.5,0%), and (0.3,0.15,0.5,20%) were identified as realistic markets with power law distributions of exponent $\alpha \approx 3$. These results reveal that a mixture of noise and informed traders leads to the emergence of efficient markets, whereas a small percentage of noise traders mixed with informed traders give rise to realistic markets.

This study has only explored parameter sets $(b_{max}, \sigma_{max}, CV, NT)$ where, $b_{max} \in [0.2, 0.3]$ with increments of 0.1, $\sigma_{max} \in [0.15, 0.45]$, with increments of 0.15, $CV = 0.5$ and $NT \in [0\%, 100\%]$ with increments of 20%. Although a total of thirty-six parameter sets have been investigated, a much broader range of parameters must be explored in future studies. Additionally, it is recommended that future studies explore the effect of noise traders on other statistical properties of markets such as: the autocorrelation functions, distributions at different time scales, identification of bubbles and crashes, and multifractality. In doing so, the creation of an agent-based model in finance that can accurately predict market dynamics of real-world markets can be made possible.

5. ACKNOWLEDGMENTS

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