Random Walk Model on Philippine Inflation Rate: An estimation of time varying parameters using Kalman filter

Maria Francesca Tomaliwan
Assistant Professorial Lecturer
Economics Department, De La Salle University
**Corresponding Author: maria.francesca.tomaliwan@dlsu.edu.ph*

Abstract: The paper estimates the time-varying parameters of the random walk model of inflation in the Philippines. Through the Kalman filter applied on the model using monthly inflation rates from January 2008- December 2019, we find that there had been non-constant parameters of the model through time. The instability of the parameters could be due to the changes in the inflation expectation of economic agents. The findings show that the agents had adopted a forward-looking inflation expectation around the years late 2008 to early 2016 then changed to backward-looking expectations from late 2016 to early 2018 when the Philippines experienced regular low inflation rates. The revision of the agents' expectation could be attributed to their response to lower-than-expected inflation. These significant results can contribute to the increased role of inflation expectations in the inflation dynamic process.

Key Words: Inflation Rate, Kalman Filter, Random Walk Model, the Philippines

1. INTRODUCTION

It is basic economic knowledge that both monetary and fiscal policies' interaction would jointly determine a country's macroeconomic health including that of a nation's inflation rate. Specifically, most modern central banks aim to target the rate of change of the price level (Leeper, 2018). Hence, while a policy is either in its formulation or early implementation, its efficacy and creditability can come into question if any macroeconomic indicators like the GDP or unemployment rate misses its target. The event of doubt on a policy's commitment may result from the rational economic agents' change in expectation. This agent's shift could lead to non-constant time-varying economic relationship. The exposure of a policy over time can lead to parameters of the model to be non-structural (Koirala, 2013).

In the Philippines, ever since the Bangko Sentral ng Pilipinas (BSP) had introduced inflation targeting in 2002, the early years had reported economic agents to be more backward-looking with lagged inflation weighing more on the determination of current inflation. By the late 2011 and onwards, inflation expectations started to weigh more on the assessment of current inflation. Ironically, when the Philippines had experienced consistently low inflation rates during the latter part of 2014 until 2016, it was noted by Guinigundo (2016) through Ehrmann (2015) who observed that under persistently low inflation, inflation expectations are not as stabilized as they are more dependent on lagged inflation and disagreement between forecasters ensues (Guinigundo, 2016).

Interestingly when Guo, Karam, & Vlcek (2019) noticed that the Philippine inflation rates rose sharply in 2018 from an anticipated 2%-4% target inflation to as high as 6.7% in actual rate, they reported that the central bank's response of a contractionary monetary policy casted a shadow of uncertainty which may affected the central bank's commitment towards the policy's formulation in the said time period. Thus, any change in policy commitment through time may mean a change in expectation for the economy agents. Eventually, this may result to a time-varying relationship and a model's parameters being time-varying too.

State space models can involve a dynamic time series that could result to an estimation of coefficients that are inherently time-varying in nature and make economic relationships potentially unstable. In these types of models, an observed time series is being explained by a state or unobserved variable which are driven by a stochastic process. One way to solve state space models in the linear case is through the Kalman Filter (Koirala, 2013). Application of the state space models have been done previously by Kim and Nelson (1989) which gave insights how agents update their estimates of a model's parameters when new information becomes available.

Therefore, by utilizing a state space model through the Kalman filter, the objective of this paper is to estimate the time-varying parameters of both the constant and AR(1) parameters of random walk inflation model in the Philippine setting. Through state space modelling via the Kalman filter, the study would be able to integrate expectation into the aforementioned model and determine what kind of inflation expectation can be found in Philippine inflation dynamics, whether its backward or forward looking. In Section 2, the paper discusses the data and methodology. In Section 3, the report includes the analysis while a conclusion is in Section 4.

2. METHODOLOGY

2.1 Data

The raw time series data are obtained from the Bangko Sentral ng Pilipinas. To be able to measure monthly inflation rate, the study uses monthly 2012-based inflation rate. The data ranges from January 2008- December 2019 which encompassed important Philippine monetary and fiscal policy changes in respond and/or due to presidential administration changes like Global Financial Crisis and Tax Reform for Acceleration and Inclusion Act (TRAIN). We use a total of 144 observations.

2.2 Econometric Methodology

2.2.1. State Space Modelling

State Space modelling deals with a dynamic time series that involve unobserved variables. It has a linear state space representation of the dynamics of the (n x 1) vector that is represented by 2 equations namely: measurement/observed/output equation and state/unobserved/dynamic/transition equation (Mapa, 2018 and Nadal-De Simone, 2000). The measurement equation relates the set of observed variables to the set of dynamic equation. The following five equations describe what can be called a representative state-space model:

$$y_t = H_t\beta_t + d_t + \varepsilon_t \quad (1)$$

$$\beta_t = F\beta_{t-1} + c_t + \nu_t \quad (2)$$

$$\varepsilon_t \sim N(0, R) \quad (3)$$

$$\nu_t \sim N(0, Q) \quad (4)$$

$$E(\varepsilon_t, \nu_t) = 0 \quad (5)$$

where: y_t is a *n*x1 vector of measured/observed variables,
β_t is a *p*x1 state/dynamic vector of unobserved variables,
H_t is a *n*x*p* matrix that links the observed y_t vector and the unobserved β_t
d_t is a *n*x1vector of predetermined variables
c_t is a *m*x1vector of deterministic part of the dynamic equation
ε_t is a *n*x1 vector of white noise processes that perturb the measurement equation,
ν_t is a *k*x1 vector of (unknown) white noise processes that perturb the state/dynamic equation,
F_t is a *p*x*p* matrix of parameters (state/dynamic)
R and Q are hyper-parameters of the model

Equations (1) and (2) are the measurement and state/dynamic equations, respectively. Equations (3), (4) and (5) state that the sequences of ε_t and ν_t follow normal processes with zero means and variances of H and Q, and are uncorrelated (Nadal-De Simone, 2000 and Koirala, 2013). Once a model is put into state space form, the Kalman filter can be used to estimate state vector by filtering.

2.2.2. Kalman Filter

The Kalman filter is a recursive procedure that will provide estimates of the unobserved or state variable which plays a defining role in estimating changes. The reason for filtering is to revise the dynamic vector once a new data of y_t becomes available. Hence, Kalman filter becomes a recursive algorithm since "depending on the information set used, the basic filter or smoothing are obtained (Nadal-De Simone, 2000).

The following notations and equations for the Kalman Filter are utilized in Nadal-De Simone (2000):

ψ:	the information set
$\beta_{t\|t-1} = E[\beta_t\|\psi_{t-1}]$:	expectation (estimate) of β_t conditional on information up to *t-1*
$P_{t\|t-1} = E[(\beta_t\beta_{t\|t-1})(\beta_t - \beta_{t\|t-1})']$:	covariance matrix of β_t conditional on information up to *t-1**

$\beta_{t\|t} = E[\beta_t \| \psi_t]$:	expectation (estimate) of β_t conditional on information up to t
$P_{t\|t} = E[(\beta_t - \beta_{t\|t})(\beta_t - \beta_{t\|t}))']$:	covariance matrix of β_t conditional on information up to t*
$y_{t\|t-1} = E[y_t\|\psi_{t-1}] = x_t\beta_{t\|t-1}$:	forecast of y_t given information up to *t-1*
$\eta_{t\|t-1} = y_t - y_{t\|t-1}$:	prediction error
$f_{t\|t-1} = E\,[\eta^2_{t\|t-1}]$:	conditional covariance of the prediction error
$\beta_{t\|T} = E[\beta_t\|\psi_T]$:	expectation (estimate) of β_t conditional on information up to T (whole sample)
$P_{t\|T} = E[(\beta_t - \beta_{t\|T})(\beta_t - \beta_{t\|T})']$:	covariance matrix of β_t conditional on information up to T (whole sample)*

* The apostrophe means transposition.

To be able to infer the value of vector β_t, at the beginning of period t, the process will first have the estimated value of the previous period $\beta_{t-1|t-1}$ with some covariance matrix of $P_{t-1|t-1}$ and Equation (2) as the prior information. Thereafter, a forecasted value conditional on information set at period *t-1* can be obtained as $\beta_{t|t-1} = E[\beta_t|\psi_{t-1}]$. This meant that the basic filter refers to predicting the estimate of $\beta_{t|t-1}$ based on information available up to time *t-1* only (Forero, 2012).

Afterwards, new information related with β_t arrives in period t in the form of y_t according to Equation (1). As a result, an update towards the estimate of y_t combines the two sources of information as follows: $\beta_{t|t} = \beta_{t|t-1} + K_t\eta_{t|t-1}$, where K_t is the weight assigned to new information about β_t contained in the prediction error. Now, the basic filter refers to an estimate of $\beta_{t|t}$ that is based on information available up to time t (Forero, 2012).

Lastly, smoothing to an estimate of β_t is possible based on all the available information in the sample or dataset through time T or the whole dataset. There is no need to update as the whole sample is being used so the smoothened value would be generated as $\beta_{t|T}$ (Nadal-De Simone, 2000).
Summarizing the processes earlier, the basic Kalman filter has the following steps on which it is assumed that x_t is available at the beginning of time t and the latest information of y_t is made at the end of time t:

1. Setting Initial State

All are set to zero.

2. Prediction

At the beginning of time t, an optimal predictor of y_t is formed based on all the available information up to time *t-1*: $y_{t|t-1}$. To do this, $\beta_{t|t-1}$ has to be calculated.

3. Updating

Once y_t is realized at the end of time t, the prediction error can be calculated: $\eta_{t|t-1} = y_t - y_{t|t-1}$. This prediction error contains new information about β_t beyond that contained in $\beta_{t|t-1}$. Thus, after observing y_t, a more accurate inference can be made of β_t being multiplied with $\beta_{t|t}$, an inference of β_t based on information up to time t, may be of the following form: $\beta_{t|t} = \beta_{t|t-1} + K_t\eta_{t|t-1}$, where K_t is the weight assigned to new information about β_t contained in the prediction error.

4. Smoothing

This process will now use all the data available up to time T to re-estimate the prediction, which is $\beta_{t|T}$. It will utilize $\beta_{t|T}$'s as optimal estimate of state at time t and use $P_{t|T}$ as a measure of the noise. This smoothing recursion consists of the backward recursion that uses the filtered values of the aforementioned β and P. Such a smoothing that involves a backward recursive process is called "Rauch-Tung-Striebel algorithm".

Below are the equations used for each process:
Initial States:

$$\beta_{0|0} \tag{6}$$
$$P_{0|0} \tag{7}$$

Prediction:

$$\beta_{t|t-1} = \mu_t + F\beta_{t-1|t-1} \tag{8}$$
$$P_{t|t-1} = FP_{t-1|t-1}F' + Q \tag{9}$$
$$\eta_{t|t-1} = y_t - y_{t|t-1} = y_t - H_t\beta_{t|t-1} - d_t \tag{10}$$
$$f_{t|t-1} = H_tP_{t|t-1}H_t' - R \tag{11}$$

Updating:

$$\beta_{t|t} = \beta_{t|t-1} + K_t\eta_{t|t-1} \tag{12}$$
$$P_{t|t} = P_{t|t-1} - K_tH_tP_{t|t-1} \tag{13}$$
$$\text{where } K_t = P_{t|t-1}H_t{}'f^{-1}_{t|t-1}$$

Smoothing (*t= T-1, T-2,…,1*)

$$\beta_{t|T} = \beta_{t|t} + P_{t|t}F'P^{-1}_{t+1|t}\,(\beta_{t+1|T} - F\beta_{t|t} - \mu_t) \tag{14}$$
$$P_{t|T} = P_{t|t} + P_{t|t}F'P^{-1}_{t+1|t}\,(P_{t+1|T} - P_{t+1|t})P^{-1}_{t+1|t}{}'FP^{t}_{t|t} \tag{15}$$

The Kalman gain is an inverse function of R, the variance of the measurement equation (ε_t) and, given

x_t, it is a direct function of the uncertainty underlying $\beta_{t|t-1}$(or K_t). For example, as uncertainty associated with $\beta_{t|t-1}$(or K_t) falls, relatively less weight is given to new information in the prediction error $\eta_{t|t-1}$and the shock is said to be less informative.

2.2.3. Random Walk Model for Inflation Rate using Kalman Filter

To estimate the time-varying parameters of the random walk model for inflation rate, the following model was adopted from Koirala (2013) as follows:

$$\pi_t = c_t + b_t\pi_{t-1} + \varepsilon_t, \quad VAR\,(\varepsilon_t) = R \qquad (16)$$

Assuming π_t to be a stochastic process generated (inflation data from the Consumer Price Index or CPI) based on unobserved process of π_{t-1} with c_t and b_t respectively the time-varying coefficients of constant and autoregressive AR(1) coefficient. Equation (16) can be represented in a state space form as:

$$\underbrace{\begin{pmatrix} c_t \\ b_t \end{pmatrix}}_{\beta_t} = \underbrace{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{\mu} + \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{F} * \underbrace{\begin{pmatrix} c_{t-1} \\ b_{t-1} \end{pmatrix}}_{\beta_{t-1}} + \begin{pmatrix} \nu_{1t} \\ \nu_{2t} \end{pmatrix}, \quad VAR\begin{pmatrix} \nu_{1t} \\ \nu_{2t} \end{pmatrix} = Q \qquad (17)$$

note: F = 1, π = 0

The Matlab software was used to estimate the time-varying parameters of the random walk model using the Kalman filter.

3. RESULTS AND DISCUSSION

To be able to see the movement of the Philippine inflation from the years 2008-2019, Figure 1 shows an upward and downward trend through the years. One observation would be an upward peak during the Global Financial Crisis period on the year 2008 and the onset of the implementation of Duterte administration's new tax system (Tax Reform for Acceleration and Inclusion Act or TRAIN Law) around the year 2018 which related to an increase of fuel prices, sin products and sugar-sweetened beverages. These two increasing trends may result from expectation of various economic agents on inflation from policy/ event changes.

As the paper's objective is to estimate the time-varying parameters of the random walk inflation model in the Philippines, the data generating process is represented by Equation 16 and both the constant and AR(1) parameters of the model are analyzed if they are changing over time. The cause of the change might be related to the change in expectation of economic agents due to policy or event changes. Contrary to structural estimation like regression models or even static models that treats coefficients as constants, the parameters are estimated through a recursive procedure that involves a filtering process that uses latest data available in the form of observable information.

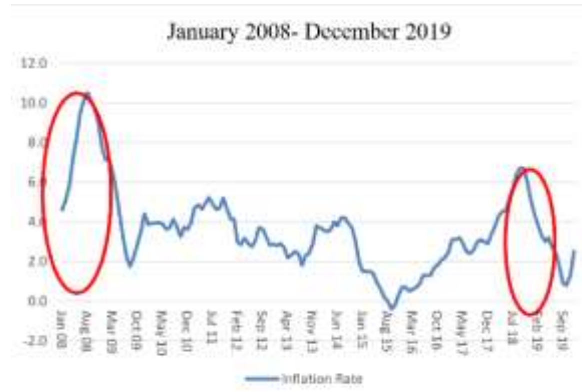

Fig. 1. Philippine Inflation Rate (Year 2008-2019)

The filtering starts by stating the states having a value of zero. The diagonal of the variance of the coefficients have been set to 100 as represented by $\begin{pmatrix} 100 & 0 \\ 0 & 100 \end{pmatrix} = Q$ since those parameters are assumed to unknown. The diagonal elements of Q signify the dynamic/system noise while the R represents the measurement noise of the system. Based on the initial Kalman filter, it resulted $VAR\,(\varepsilon_t) = R$ to be R=0.6 while the estimated state-space form for the random walk model for inflation are as follows:

$$\underbrace{\begin{pmatrix} 0.2322 \\ 0.8767 \end{pmatrix}}_{\beta_t} = \underbrace{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{\mu} + \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{F} * \underbrace{\begin{pmatrix} 0.0530 \\ 0.8866 \end{pmatrix}}_{\beta_{t-1}} + \begin{pmatrix} 0.0060 \\ 0.001 \end{pmatrix}, \quad VAR\begin{pmatrix} 0.0060 \\ 0.001 \end{pmatrix} = Q$$

With the aforementioned result for R and Q, along with the assumption of zero vector of μ and an identity matrix of F, the graphical representation of the estimated parameters of the time-varying constant and AR(1) coefficients are seen in Figure 2 and Figure 3 for the whole time period.

As seen on both figures, both the parameters of constant and AR(1) are seen to have convergence on the part where there are green circles. On the other hand, much emphasis should be given on the divergence of the constant and AR(1) coefficients as seen on the red circles. This divergence signifies the lack of consistent time-varying parameters of the model which lasted for 3 to 6 months. Such results imply that any static model on inflation modelling would result to a poor performance.

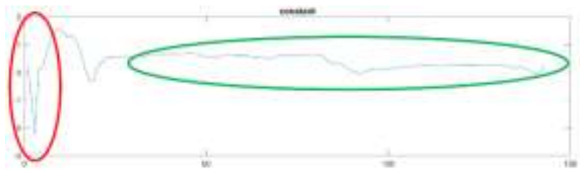

Fig. 2. Kalman-Filtered Time Varying Constant

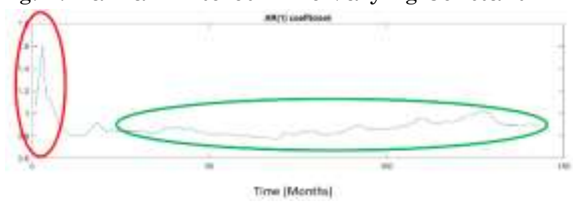

Fig. 3. Kalman-Filtered Time Varying AR(1) Coefficient

Additionally, we also examine the smoothened parameters of this model as these could show the long run behavior of the coefficients. As seen on Figure 4 and Figure 5, the filtered coefficients found previously on Figure 2 and Figure 3 were graphically matched against the smoothed coefficients. The figures could see a sharp contrast between the filtered and smoothed estimates since the smoothed coefficients uses more observations than the filtered. To see the difference between the minimum and maximum values for both the filtered and smoothed coefficients, the estimates could be found on Table 1.

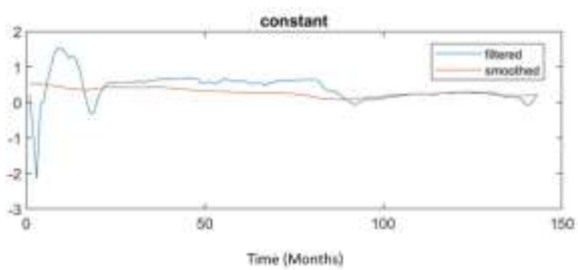

Fig. 4. Kalman-Filtered and Smoothened Time Varying Constant

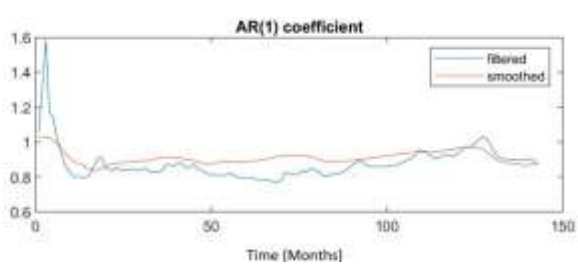

Fig. 5. Kalman-Filtered and Smoothed Time Varying AR(1) Coefficient

Table 1. Minimum and Maximum Values of Constant and AR(1) Estimates

	Filtered Estimates	Smoothed Estimates
Constant Min	-2.15325	0.081391
Constant Max	1.539392	0.52813
AR(1) Min	0.770285	0.837449
AR(1) Max	1.584001	1.027939

To be able to foresee the Philippine inflation expectation from the years 2008 to 2019, one could see the trend found in Figure 6 and Figure 7. Similar to what Guinigundo reported back in 2016, economic agents seemed to be going more forward-looking as coefficient of lagged inflation or AR(1) began to steadily decline even from the late 2008 as seen on the red circle. This could mean inflation expectations started to weigh more on the estimation of current inflation. Since this study had now compassed data beyond 2016, the trend shows an upward movement from 2016 until late 2018. This could mean that for a short period of time, inflation expectations could be again backward-looking or based on lagged inflation which is consistent with Ehrmann (2015) as seen in the green circles. Ehrmann (2015) had reported that during the period of low inflation rates, inflation expectations are more dependent on lagged inflation. In this event, economic agents revised down their expectations in response to lower-than-expected inflation.

Fig. 6. Inflation Coefficient Estimates- Constant Trend

Fig. 7. Inflation Coefficient Estimates- AR (1) Trend

4. CONCLUSIONS

We determine that time-varying parameters of the random walk model of inflation are unstable over time. Through the Kalman filter applied on the model using monthly inflation rate from January 2008- December 2019, there had been a divergence which signifies the lack of consistent time-varying parameters of the model which lasted for 3 to 6 months for both constant and AR(1) coefficients. Some reasons that may contribute to this finding is due to the changes in the expectation of economic agents. This was found true when the study also analyzes the inflation expectation of the economic agents. Consistent with Guinigundo's finding (2016), the agents had adopted a forward-looking inflation expectation from the years 2008 onwards. Since the Philippines had experienced regularly low inflation rates from 2016 to 2018, there had been a switch in the expectation from forward to backward-looking trend which denotes expectation anchored from lagged inflation which is consistent with Ehrmaan's study (2015).

For future studies and due to the circumstances when the study was made, we would like to suggest further expansion on the said model like incorporating expectations-augmented Phillips curve or the New Keynesian Philips curve as the present report showed forward-looking inflation expectations on some years in the Philippine inflation dynamic modelling.

5. ACKNOWLEDGMENTS

We would like to acknowledge the Almighty God for His blessings and my family for their support during the COVID-19 pandemic when this paper was made.

6. REFERENCES

Choy, K., (2016). The inflation process and expectations in Singapore. *BIS Papers chapters, in: Bank for International Settlements (ed.), Inflation mechanisms, expectations and monetary policy*, volume 89, pages 335-343, Bank for International Settlements.

Ehrmann, M (2015): Targeting inflation from below: how do inflation expectations behave?. *International Journal of Central Banking*, no 11, pp 213–49, September

Forero, F. (2012). Estimating State-Space models through Maximum Likelihood. *Handout: ECONOMETRIC METHODS II TA session 4.* Retrieved on May 1, 2020 from http://www.kris-nimark.net/pdf/Handout_S4.pdf

Guinigundo, D., (2009). Measurement of Inflation and the Philippine Monetary Policy Framework. *BSP Economic Newsletter No.09-04* August 2009.

Guinigundo, D., (2016). The Role of Expectations in Inflation Dynamics in the Philippines - Has it Changed Following the Global Financial Crisis? (November 2016). *BIS Paper No. 89s*. Available at SSRN: https://ssrn.com/abstract=2872130

Guo, Karam, & Vlcek (2019). Decomposing the Inflation Dynamics in the Philippines. *IMF Working Papers. 19. 10.5089/9781498319324.001.*

Kim, C., & Nelson, C. (1989). The Time-Varying-Parameter Model for Modeling Changing Conditional Variance: The Case of the Lucas Hypothesis. *Journal of Business & Economic Statistics, 7(4), 433-440.* doi:10.2307/1391644

Koirala, T.P. (2013). Time-varying Parameters of Inflation Model in Nepal: State Space Modeling. *Nepal Rastra Bank Working Paper No. 16.* Retrieved on March 13, 2020 at https://www.nrb.org.np/ecorev/pdffiles/nrbwp201316.pdf

Leeper, Eric Michael (2018). Sweden's Fiscal Framework and Monetary Policy. *NBER Working Paper No. w24743.* Retrieved on March 12, 2020 at https://ssrn.com/abstract=3202046

Mapa, Dennis (2018). Nowcasting Inflation Rate in the Philippines using Mixed-Frequency Models. *BSP-UP Professorial Chair Lecture Series Presentation.* Retrieved on March 16, 2020 at http://www.bsp.gov.ph/events/pcls/papers_2018.htm

Nadal-De Simone, Francisco (2000). Forecasting Inflation in Chile Using State-Space and Regime-Switching Models. *IMF Working Paper,* Vol. , pp. 1-55, 2000.