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An analysis of the risk-return profile of the daily Bitcoin prices using different variants of the GARCH Model

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Abstract: The introduction of the Bitcoin ushered in a new era – the era of virtual or cryptocurrencies. Since its maiden appearance in 2009, the price of Bitcoin has realized spectacular growth and wild swings, from a little less than US$70 per coin during the mid-2013 to about US$20,000 in the late 2017; with current price (March 2019) hovering around US$4000. Bitcoin is actively being traded against more than 30 hard currencies on well-organized virtual exchanges, and its exposures can now be hedged in its own futures market. Hailed as the currency of the future, the phenomenal success of Bitcoin spawned the entry of more than 1,600 virtual currencies with esoteric names such as quackcoin, etherium, anoncoin, Zcash, etc. Bitcoin however, account for more than the combined shares of all other cryptocurrencies, making it the overwhelming market leader. Analyzing the risk-return profile of this exotic investment instrument might be worthwhile to a whole range of stakeholders – investors, financial analysts, managers and speculators. This study aims to analyze the statistical properties of the daily bitcoin prices, focusing on its risk-return characteristics using the different variants of the GARCH (Generalized AutoRegressive Conditional Heteroscedasticity) model. Two distinct families of GARCH models are employed in the study namely (1) APARCH (Asymmetric Power ARCH) which includes the following: classical ARCH, vanilla GARCH, GJR (Glosten Jaganathan and Runkle), TARCH (Threshold ARCH) and the NARCH (Nonlinear ARCH) models, and (2) The EGARCH (Exponential GARCH) variants. Stylized facts analysis and modeling results confirm the high-level volatility structure, absence of leverage effects and significantly positive long run average return. Possibility of bubbles however is seen.

Key Words: Bitcoin; Cryptocurrency; Risk-return Profile; GARCH Models

1. INTRODUCTION

The successive episodes of calamitous financial crises in recent years have prompted investors to explore exotic, non-traditional, profitable but safe investment opportunities. The introduction of the Bitcoin at the height of the 2009 financial crisis presented much needed alternative that harnesses the seemingly endless potentials of the high technology era. It ushered-in the age of the so-called "cryptocurrencies". Being the pioneer, Bitcoin is the "King" of the virtual currencies and has become the standard means of payment over the internet (ECB 2012). The recent launch of Bitcoin as an exchange traded fund (http://www.nasdaq.com/markets/ipos/company/winklevoss-bitcoin-trust-909930-72927) and a futures market listing (https://www.cmegroup.com/trading/bitcoin-futures.html) made it a highly credible investment instrument.

Investors are attracted to Bitcoin as an investment vehicle due to its perceived desirable features: exceptionally high average return, extreme volatility, accessible even during weekends, and low correlation with traditional assets – features that offer significant diversification benefits (Briere, et. al, 2015)

This paper aims to empirically validate the perceptions of investors on the desirability of Bitcoin as an investment alternative. Employing the cutting-edge variants of the GARCH model and stylized facts statistical analyses and testing, the study attempts to provide stakeholders with empirically sound bases in examining Bitcoin as an attractive investment alternative.

2. METHODOLOGY

2.1 Risk-Return Tradeoff

One important characteristic of financial assets is the immutable trade-off between return from financial asset and the associated risk in holding it. Mainstream financial and economic theories predict a negative non-linear relationship. In formulating sound investment strategies for Bitcoin, this trade-off must be taken into consideration. The following techniques are employed in the study in analyzing the risk-return profile of Bitcoin:

2.2 Stylized Facts Analysis and Testing for the ARCH Effect

Daily closing prices (P) and Returns (rr) of Bitcoin within the sample horizon are subjected to a battery of graphical and descriptive analyses of their first four moments (Central tendency, Variability, Symmetry and Tail Density). In quantifying the returns series, the following formula is used in this study:

$$rr_t = 100 * \ln(P_t / P_{t-1}) \qquad (1)$$

To confirm the susceptibility of the return series to econometric modeling, a battery of Unit Root tests are implemented. These tests determine the order of integration of price series, and if its natural logarithm is shown to be *I(1)*, the first difference (identical to rr per formula (1)), is deemed to be *I(0)*. The following Unit Root Tests are used: Augmented Dickey-Fuller (ADF), Philips-Perron (PP) and the KPSS tests.

To verify the presence of volatility clustering or the so-called ARCH Effect, the Lagrange Multiplier (LM) test is implemented for alternative lag structures on the return series. Normality testing of the residual series is undertaken by the Jarque-Bera test, and three alternative tests.

2.2 The ARCH/GARCH Models

The ARCH Effect (Engle 1982) is an almost unique phenomenon associated with modeling returns to financial assets. ARCH stands for AutoRegressive Conditional Heteroscedasticity. In classical regression analysis, the presence of ARCH is a complete anathema to all the classical model stands for. Hence, instead of just modeling the mean return equation (or the population regression function (PRF) of the average return), the conditional variance equation is likewise specified owing to the presence of the time varying second moment. The basic ARCH(q) model is specified as follows:

Mean Equation:

$$E_{t-1}(rr_t) = \pi' x_t + u_t \qquad (2)$$

Conditional Variance Equation:

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 \qquad (3)$$

Bollerslev (1986) saw the need to generalize the ARCH effect to augment the current conditional variance with its past values, up to lag p The conditional variance equation for the classic GARCH(q,p) is now:

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \qquad (4)$$

2.3 The GARCH Variants

To check for the presence of certain special volatility effects (e.g. leverage effect, asymmetric effects, etc.), two different families of GARCH models are introduced in the literature: the APARCH (Asymmetric Power ARCH) and the EGARCH (Exponential GARCH) models.

The APARCH Family (Ding, et. al., 1993)

This family of GARCH models can accommodate various asymmetric effects and power transformations of the conditional variance. The general specification of the conditional volatility equation of the APARCH family is as follows:

$$\sigma_t^\delta = \omega' z_t + \sum_{i=1}^{q} \alpha_i (|u_{t-i}| - \gamma_i u_{t-i})^\delta + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^\delta \quad (5)$$

where $\sigma_t^2 = \sqrt{h_t}$

The parameter δ ranges between 1 and 2 performs a Box-Cox transformation and γ captures the asymmetric effects. Specific values for δ's and γ's give rise to different variants of the APARCH models:

ARCH (Engle 1982) - β's, γ's = 0, δ =2

GARCH (Bollerslev 1986) γ's = 0, δ =2

GARCH (Taylor 1986 and Schwert 1990)
γ's = 0, δ =1

GJR (Glosten, et.al. 1993) δ =2

TARCH (Zakoian 1994) δ =1

NARCH (Higgins and Bera 1992) β's, γ's = 0

The EGARCH Variants (Nelson 1991)

The Exponential GARCH, with the variance equation expressed in terms of log volatility captures the asymmetric effect as a function of standardized innovations. Thus, the conditional variance equation is specified as:

$$\ln h_t = \omega' z_t + \sum_{i=1}^{q} \left[\alpha_i (|\varepsilon_{t-1}| + \gamma_i \varepsilon_{t-i}) + \sum_{j=1}^{p} \beta_j \ln h_{t-j} \right] (6)$$

with $\varepsilon_t = u_t / \sqrt{h_t} \sim N(0,1)$ or suitable distribution.

The Conditional Error Distribution

All of the above GARCH models are estimated using Maximum Likelihood (or Pseudo ML) Procedure, bringing to fore the choice of the most appropriate distribution of ε_t . In this study, five alternative error distributions are considered depending on the shape of the empirical distribution of the residuals. These are the following:

1. Standard Normal
2. Student's t
3. Generalized Error Distribution (GED)
4. Skewed Student's t
5. Skewed GED

Thus, in analyzing an empirical model for Bitcoin daily return, three specifications should be formulated:

1. The Mean equation
2. The Conditional Variance equation, and
3. The Error distribution

2.4 Data

Daily historical data on the closing price of Bitcoin in US$ per coin over the uninterrupted period of April 26, 2013 to March 15, 2019, involving 2,148 observations constitutes the data base of the study. Source of data is www.CoinMarketCap.com .

3. RESULTS AND DISCUSSION

3.1 Descriptive Analysis

As applied to the daily Bitcoin prices for the sample period involving 2,148 observations, the continuously confounded daily rate of return is computed and subjected to a battery of descriptive and inferential procedures. Initial assessment of the time graph of the return series reveals a great deal of special stylized facts. Shown in Figure 1 below, daily returns somewhat cluster around a constant value which can be considered as its long run equilibrium average return. Taking a hint on this observed stationary behavior, the mean equation of the return series may be specified as a constant plus a time varying noise element. The time graph also reveals a phenomenon of volatility clustering, as evidenced by episodes of wild swings and tranquil periods. As seen here, wild swings exceed calm episodes. Also, intense volatility clustering is noted during the 4th quarter of 2013, the entire year of 2017 and the 4th quarter of 2018. Table 1 is constructed to provide additional information on important statistical properties of the daily Bitcoin return series.

Figure 1. Daily Returns on Bitcoin, April 26, 2013 to March 15, 2019

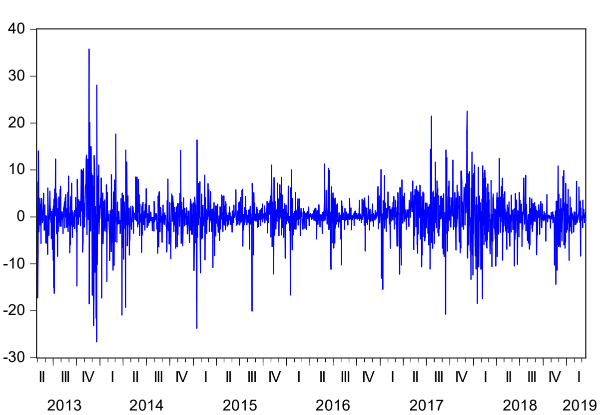

Table 1. Descriptive Statistics of Daily Bitcoin Returns

Statistics	Sample Values
Mean	0.15765
Median	0.18354
Minimum	-26.620
Maximum	35.745
Standard Deviation	4.3347
CV	27.495
Skewness	-0.18724
Excess Kurtosis	7.9427

It may be gleaned from Table 1 that the daily returns on Bitcoin is relatively symmetric but highly spread out, with a mildly negative skewness and a great deal of leptokurtosis.

3.2 Statistical Diagnostics

Table 2. Testing for Normality, Unit Roots, Presence of ARCH Effects and Testing for Adequacy of the Mean Equation of the Bitcoin Daily Returns

Normality Test (Null: Normal)	Test Statistic	p-value
Jarque-Bera	5656.2	0.00000
Shapiro-Wilks	0.883823	2.10e-037
Lilliefors	0.124639	0.00000
Doornik-Hansen	1611.32	0.00000

Unit Root Test (Unit root null)	Test Statistic	p-value
ADF Test	-8.4002***	3.242e-014
PP Test	-46.1953***	0.0001
KPSS (Stationary null)	0.136816ns	>0.10

Lagrange Multiplier (LM) Test for ARCH Effects (Null: No ARCH Effect)	Test Statistic	p-value
ARCH(1)	LM = 203.42***	3.75585e-046
ARCH(7)	LM = 261.94***	7.94634e-053
ARCH(14)	LM = 306.36***	5.57106e-057

RAMSEY Reset of the Mean Equation
(Null: Mean equation is adequately specified)

	Test Statistic	p-value
Reset Test	F(2, 2146) = 0	0.9999

The different panels of Table 2 provide a wealth of statistical evidence on the various stylized facts of the daily Bitcoin returns series during the sample period.

1. The four powerful normality tests confirm, beyond reasonable doubt the absence of normality in the Bitcoin return series.
2. Stationarity is more than adequately validated through the conventional unit root tests.
3. The presence of the ARCH effects, or volatility clustering is formally proved through three

Lagrange Multiplier tests, thus justifying the modeling of the conditional variance equation simultaneous with the mean equation.

4. Furthermore, the specification of the mean equation that it is equal to a constant plus a "white noise" is deemed adequate through the Ramsey Reset. This result preclude the use of the GARCH variant called GARCH-M (or GARCH in Mean) which augment the mean equation by a GARCH term, thus incorporating the phenomenon of risk-return "trade-off".

3.3Modeling Conditional Variance through Variants of GARCH Model

Using the stylized facts uncovered in the descriptive analysis, together with the results of the various statistical tests, modeling of the conditional variance in tandem with the mean equation model has become imperative. The seven (7) different GARCH variants discussed in section 2.3 as the alternative conditional variance formulations for the mean equation are implemented In addition; the three (3) different assumptions on the error distribution give rise to a total of twenty-one (21) alternative models for the daily returns of the Bitcoin. To remain parsimonious, these models consider only p = 1 and q = 1 for good reasons. For one, GARCH(1,1) has been considered as the "gold standard" in the literature since adding more ARCH and GARCH terms (i.e., p, q > 1) rarely add more information and more significant coefficients to infer on the data generating process (DGP) of the series. Furthermore, this parsimonious model has been known to be robust in modeling countless applied phenomena (Engle 2001). Tables 3, 4, and 5 show the results of implementing the different GARCH variants using the Normal, Student's t, and GED as error distribution respectively. The skewed versions of the Student's t and GED are not considered because of the observed symmetry of the returns.

Examining the results presented in the three tables, it has become clear that volatility clustering is valid for Bitcoin returns by virtue of the significant estimates of the α and β parameters in all variants. However, volatility persistence in all models, captured by the estimate of $\alpha+\beta$ is significantly greater than 1. This is an indication of the unpredictable future volatility, since according to the formula of unconditional future volatility:

$$h^2 = \frac{\omega}{1-\alpha-\beta} \qquad (7)$$

(Engle 2001), the unconditional variance may have negative value. Hence, when $\alpha+\beta>1$, the steady state standard deviation h may become unbounded, , in other words, it may reach explosive levels. All gamma parameters of the asymmetric models (GJR, TARCH, APARCH and EGARCH) in Tables 3, 4, and 5 reveal the absence of asymmetry and leverage effects. This implies that any negative shock (bad news) concerning Bitcoin does not increase volatility asymmetrically more than any positive shock (good news) of equal intensity. Incidentally EGARCH can only be estimated under GED error.

4. CONCLUSIONS

The advent of the "high tech" era and the increased use of the internet provide the impetus for the proliferation of so-called "virtual communities" which innovate to come up with "virtual currencies" for use in settling their transactions. The pioneer and the biggest among these currencies is the Bitcoin.

This study is not about discussing the merits and flaws of Bitcoin as money, but about examining its statistical properties, particularly its risk-return profile that may offer hints on its being a viable investment instrument. To this end, the study employed the different variants of the GARCH model for the analysis and the following are the conclusions reached:

1. Long-run daily return on Bitcoin is highly significantly positive.
2. Returns are generally symmetric.
3. Bitcoin has very high unconditional volatility, and is subject to sudden, massive, price swings.
4. Symmetric but non-normal error distribution gives better results.
5. Parsimony should be maintained in the conditional variance equation.
6. There exists no "leverage effect".
7. There is a potential for volatility of returns to become highly explosive thus fueling the occurrence of "bubbles".

Table 3. Estimates of the Alternative GARCH Models for the Daily Returns for Bitcoin using Normal Error Distribution

Coefficients/ Models	GARCH (Bollerslev)	GARCH (Taylor/ Schwert)	APARCH (Ding, et. al.)	NARCH (Higgins and Bera)	GJR (Glosten, et. al.)	TARCH (Zakoian)	EGARCH (Nelson)
Mean Equation							
Constant	0.0986735 (0.1122)	0.157974 (0.0040)**	0.131873 (4e-075)***	0.153143 (0.0506)*	0.103520 (0.0911)	0.131873 (1. e-101)***	Matrix not + Definite
Variance Equation							
Omega (ω)	0.124743 (0.0913)	0.737038 (8.40e-039)***	0.727027 (0.0063)**	0.703274 (0.0096)**	0.404526 (0.1021)	0.732918 (0.0025)**	"same"
Alpha (α)	0.124743 (0.0004)***	0.148200 (1.41e-066)***	0.149389 (1.75e-08)***	0.148921 (5e-08)***	0.124387 (0.0005)***	0.149217 (5e-09)***	"same"
Beta (β)	0.862956 (1e-099)***	0.856385 (0.0000)***	0.856097 (2.5e-220)***	0.856926 (2e-199)***	0.839883 (6.43e-287)***	0.856016 (8e-219)***	"same"
Gamma (γ)			0.0679161 (0.4915)		-0.0136600 (0.8221)	0.0697097 (0.4031)	"same"
Delta (δ)			1.01637 (0.0028)**	1.08984 (0.0016)**			

Table 4. Estimates of the Alternative GARCH Models for the Daily Returns for Bitcoin using Generalized Error Distribution

Coefficients/ Models	GARCH (Bollerslev)	GARCH (Taylor/ Schwert)	APARCH (Ding, et. al.)	NARCH (Higgins and Bera)	GJR (Glosten, et. al.)	TARCH (Zakoian)	EGARCH (Nelson)
Mean Equation							
Constant	0.1297157 (0.0000)***	0.128214 (0.0000)***	0.128214 (0.0000)***	0.125324 (0.0000)***	0.138968 (0.0000)***	0.128214 (0.0000)***	0.128214 (0.0000)***
Variance Equation							
Omega (ω)	0.195973 (0.0322)*	0.386997 (0.0023)**	0.373683 (0.0054)**	0.378057 (0.0041)**	0.186067 (0.0438)*	0.385125 (0.0026)*	-0.143707 (2.75e-09)***
Alpha (α)	0.174370 (1.63e-08)***	0.181313 (2.67e-015)***	0.182027 (3.19e-013)***	0.182373 (1.47e-013)***	0.171680 (6.80e-08)***	0.180804 (8.42e-015)***	0.318852 (5.27e-018)***
Beta (β)	0.844985 (9.91e-258)***	0.856653 (0.00000)***	0.856840 (0.00000)***	0.856128 (0.00000)***	0.848002 (5.17e-238)***	0.857099 (0.00000)***	0.971263 (0.00000)***
Gamma (γ)			-0.0178959 (0.7596)		-0.0522907 (0.2268)	-0.0166235 (0.7812)	0.0117735 (0.4722)
Delta (δ)			1.042253 (5.49e-07)***	1.03376 (3.66e-07)***			

Table 5. Estimates of the Alternative GARCH Models for the Daily Returns for Bitcoin using Student's t Distribution

Coefficients/ Models	GARCH (Bollerslev)	GARCH (Taylor/ Schwert)	APARCH (Ding, et. al.)	NARCH (Higgins and Bera)	GJR (Glosten, et. al.)	TARCH (Zakoian)	EGARCH (Nelson)
Mean Equation							
Constant	0.137607 (0.0002)***	0.123146 (0.0013)**	0.128613 (0.0006)***	0.122910 (2e-019)***	0.146363 (9e-05)***	0.130138 (2e-015)***	Estimate did not converge
Variance Equation							
Omega (ω)	0.208490 (0.0861)*	0.309767 (0.0196)*	0.301388 (0.0234)*	0.312592 (0.0187)*	0.188502 (0.1093)	0.300037 (0.0235)*	Estimate did not converge
Alpha (α)	0.360343 (0.0003)***	0.261937 (4e-010)***	0.257922 (5.98e-07)***	0.259693 (2e-07)***	0.352207 (0.0004)***	0.258977 (1.11e-09)***	Estimate did not converge
Beta (β)	0.835817 (0.0000)***	0.855885 (0.0000)***	0.857994 (0.00000)***	0.856128 (0.00000)***	0.839883 (6e-287)***	0.857899 (0.00000)***	Estimate did not converge
Gamma (γ)			-0.0575051 (0.3313)		-0.0675936 (0.1132)	-0.0578015 (0.3178)	Estimate did not converge
Delta (δ)			0.991313 (2.78e-07)***	0.982477 (5e-08)***			

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6. REFERENCES

Briere, M., Oosterlinck, K. and A. Szafarz. (2015) Virtual Currency, Tangible Return: Portfolio Diversification with Bitcoin. Journal of Asset Management 16(6), pp. 365-373

Bollerslev, T. P. (1986). Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 31: 307-327.

Ding, Z. X., R. Engle and C. W. F. Granger (1993). A long memory property of stock markets returns and a new model. Journal of Empirical Finance 1: 83-106.

Engle, R. (1982). Autoregressive conditional heteroskedasticity with estimates of the u.k. Inflation. Econometrica 50: 987-1008.

Engle, R. (2001). GARCH 101: The use of ARCH/ GARCH Models in Applied Econometrics. Journal of Economic Perspectives 15(4): 157-168

European Central Bank (ECB) (2012) *Virtual Currency Schemes*. http://www.ecb.int/pub/pdf/ other/virtualcurrencyschemes201210en.pdf

Glosten, L., R. Jagannathan and D. Runkle (1993). Relation between expected value and the nominal excess return on stocks. Journal of Finance 48: 127-138.

Higgins, M. and Bera, A. (1992). A class of nonlinear arch models. International Economic Review 33: 137-158.

Nelson, D. B. (1991). Conditional heteroskedasticity in assets returns: a new approach. Econometrica 59: 347-370.

Schwert, W. (1990). Stock volatility and the crash of '87. Review of Financial Studies 3: 77-102.

Taylor, S. (1986) Modelling Financial Time Series, Wiley.

Zakoian, J. M. (1994). Thresold heteroskedastic models. Journal of Economic Dynamic and Control 18: 931-955.