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Fitting a Leslie matrix model for the population of the Philippines and other Asian countries

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Abstract:
One of the problems of many countries is the rapid increase in population. This huge population growth rate, creates risks not only to the people but also to the environment. For this reason, several countries such as the Philippines attempted to resolve this issue by putting reproductive health policies into action. In this study, we constructed a Leslie matrix model by using fertility, survival rates, and population data to forecast whether the population of the Philippines and other selected Asian countries, is increasing or decreasing. For further insights, we also determined the value of dominant eigenvalue and mean absolute percentage error (MAPE) of the other selected Asian countries for comparison in the same year. In 2005, Philippines had a dominant eigenvalue λ, less than 1 which indicates that the population was actually decreasing with a MAPE of $9-13\%$ which explains why the forecasting is not acceptable. As a result, we propose sensitivity analysis to countries with MAPE of greater than 10% to give a valid forecast.

Keywords: Leslie matrix model, dominant eigenvalue, mean absolute percentage error

1. INTRODUCTION

As of 2017, the Philippines has a population close to a 100 million and the rate of increase in population is extremely high. In 2011, the Philippines is among the most populous country in the world. However, the growth rate of the Philippines is slowly declining but this growth rate is still higher than the average global population. It is expected that the Philippines will even have a faster population growth rate than India and China. In Southeast Asia, the Philippines is only second to Indonesia in terms of population size. However, in the next 20 years, the greatest relative increase in population for Southeast Asia will be in the Philippines. The continued positive population growth rate and its slow decline in the Philippines are due to the continued relatively high total fertility rate. High fertility is due to unwanted child-bearing, and a desired family size of more than two children. Population growth and population structures are two of the main factors for sustainable development. Thus, there is a need of precise calculation for future population. Several countries have already expressed their concern about high population growth and started formulating reproductive health policies. Unpredictable population growth puts pressures on natural resources, human well-being, and global warming. With this, many countries including the first world countries have raised concerns regarding increasing population growth rates. As one of the countries that signed the 1967 United Nation Declaration of Population and the 1994 Statement of Population Stabilization of World Leaders, the Philippines recognized the population problem as a principal element in long-term planning if governments are to achieve their economic goals and full the aspirations of their people. Due to this concern, there is a great need to forecast population.

There are various methods used for forecasting population, each assuming various factors and assumptions. In [1], [6], both made use of the Leslie matrix to forecast the population. In [1], a Leslie model for Ghana is fitted using accessible fertility and survival rates from the World Health Organization and the Ghana Statistical Service. In [6] another study on the use of the Leslie matrix model was conducted. A Leslie matrix is made that predicts female population in the United States with 5-year intervals from the years 2000 to 2020. Testing the accuracy of this framework, the forecasted population of the year 2010 is compared with the actual information of the female population in the United States received from the 2010 U.S.census. In view of the results, forecast is not dependable for the age class 0-10 also for the age class of 85 years and beyond. This result is upheld by our percent mistake computations, which demonstrate greatest error in forecast to be 8%, though at extremes it surpasses well over half. An attempt is made in this paper for the population of the Philippines using the base year 2005 and 2014, we forecast for the population of years 2010 and 2019, respectively. Furthermore, a forecast for the year 2015 was determined using the 2005 as the base year and the forecasted 2010 population. Whenever the actual population is available, the MAPE is a method to check between the actual and forecasted population is computed. As a rule of thumb, if the MAPE is less than 10% [46], the forecast is acceptable. Moreover, this procedure was applied to other selected Asian countries such as Azerbaijan, Iran, Israel, Japan, Kyrgyztan, Maldives, Singapore and Thailand . A country is selected if there is an available population and fertility data and survival rates. Two sets which are 5 years apart are needed for each country.

2. LESLIE MATRIX MODEL

The simplest age-structured model for population forecast is the Leslie Model. The model was named after Patrick H. Leslie (1900-1974) who created and developed it in 1945. Leslie matrix is a discrete, age-structured model of population growth that is well-known and mostly used in population ecology. The matrix form makes the model flexible and mathematically easy to study. The Leslie matrix (also called the Leslie Model) is also one of the best-known ways to project the present state of a population into the future; either as an attempt to forecast the age distribution, or as a way to evaluate life history hypotheses by considering different sets of survival and fertility parameters and where only one sex, usually the female, is considered. This is also used to model the changes in a population of organisms over a certain period. Leslie matrix is generally applied to populations with annual breeding cycle. In many species, reproduction is highly age-dependent. For instance, periodical cicadas spend 13-17 years in the nymphal stage; they only reproduce once in their lifetime. Many animals, such as humans, elephants, etc., do not reproduce during their first years and their reproductive success is age-dependent. To model such situations, age-dependent population models are appropriate. Patrick Leslie introduced matrix models that have discrete age classes with synchronous reproduction [16]. The models are parameterized by age-specific survival probabilities and average number of female offsprings. An age-structured population with k age classes is described by a population vector of length k where F_j denotes the number of females in age class j. Individuals within an age class are assumed to have equal birth and death probabilities. Survival from age class j to $j+1$ is given by probability S_j; each of the F_j in age class j give birth to $n_j(t)$ at time t on the average.

In Matrix notation, the Leslie matrix model is as follows:

$$\begin{bmatrix} n_1(t+1) \\ n_2(t+1) \\ \vdots \\ n_k(t+1) \end{bmatrix} = \begin{bmatrix} F_1 & F_2 & \dots & \dots & F_k \\ S_1 & 0 & \dots & \dots & 0 \\ 0 & S_2 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & S_{k-1} & 0 \end{bmatrix} \begin{bmatrix} n_1(t) \\ n_2(t) \\ \vdots \\ n_k(t) \end{bmatrix}$$

$$\mathbf{n(t+1)} = \mathbf{Ln(t)}$$

The matrix contains both the survival probabilities (just below the diagonal) and the fertility rates (in the first row). The matrix $\mathbf{L}$ is called the Leslie matrix.

An extension of the fundamental Leslie matrix is the two-sex model that enables one to get the quantity of both males and females in the population from age zero to the most extreme age [3]. Let the vector

$$n_j(t) = \begin{bmatrix} n_{0F}(t) & \dots & n_{kF}(t) & n_{0M}(t) & \dots & n_{kM}(t) \end{bmatrix}^T,$$

represent the number of females n_{jF} and males n_{jM}, of age $j = 1, 2, \dots, k$ in the population at time t.

3. EIGENVALUE AND SENSITIVITY ANALYSIS

The Leslie matrix model is one of the most popular tool used in population dynamics. The behavior of a complex system is described with eigenvalues and eigenvectors. One of the eigenvalues of the Leslie matrix is called the dominant eigenvalue . The dominant eigenvalue λ, is equal to the eigenvalue with the highest magnitude. If $\lambda > 1$ then the population is said to be increasing; if $\lambda < 1$, then the population is said to be decreasing; and if $\lambda = 1$, then the population is said to be stable. The annual rate of increase or decrease r, of the population is given by the logarithm of the dominant eigenvalue, that is, $r = \log(\lambda)$. The sensitivity analysis is done by changing an entry in the Leslie matrix, this entry must either be a survival or fertility rate. Then, the new dominant eigenvalue is computed, to see if there is a increase or decrease in its value. The fundamental equation for the sensitivity of the dominant eigenvalue to a little change in the component $a_{i,j}$ is presented as : $S = \frac{\partial \lambda}{\partial a_{i,j}}$. This shows that S represents the ratio of the change in λ to the change in the $(i, j)th$ element of the Leslie matrix.

4. DATA DESCRIPTION

The data used were obtained from two main sources. These are the World Health Organization [30] and United Nations Statistical Yearbook for years 2003 to 2015. The population data were obtained from theUnited Nation Statistical Yearbook, while the fertility rates were computed from the data given in the United Nations Statistical Yearbook. The fertility rates for women F_{jW} were computed from the data given in United Nations Statistical Yearbook. Fertility rates for women were computed by getting the quotient of the number of female live births to the number of women in the same age group. Similarly, the fertility rates for men F_{jM} is the quotient of the number of male live births to the number of women in the same age group. Survival rates were also obtained from World Health Organization [30]. The data used for the selected Asian countries were the latest available data such that there is a population forecast for the year 2017 or beyond.

5. METHODOLOGY

For each country it was checked if two sets of data are available. These two sets of data must be 5 years apart. For instance, the population data for the year Y and year $Y + 5$ are available. Considering Y as the base year, a Leslie matrix for year Y was constructed. This matrix was used to obtain a population forecast for the year $Y + 5$ and compared to the actual data of the year $Y + 5$. This was done by computing for the *mean absolute percentage error* or MAPE. Computed by $MAPE = \frac{\sum |e_t/A_t|}{n} \times 100$, where e_t is the forecast error for period t, n is the number of periods of evaluation and A_t is the actual population for period t. If the MAPE obtained is less than 10% then the resulting forecast population is acceptable. Then, a forecast for the population $Y + 10$ is determined. Using R statistical software the value of the dominant eigenvalue for year Y is computed and if also for year $Y + 5$, if data is complete. Furthermore, for the Philippine data, sensitivity analysis was undertaken for the year 2005.

6. SUMMARY AND CONCLUSION

6.1. The Philippines

Using the data available for 2005, a two sex Leslie matrix was constructed. Using this Leslie matrix a forecast for the population of 2010 was obtained. However, this forecast yielded a MAPE of 9.38% for females, 12.14% for males and 10.29% for the total population. Using same the Leslie matrix, a forecast for the year 2015 was also obtained and the results of the MAPE were around 16%, as expected it gave poorer results that the 2010 forecast. The forecasted, actual, and data are presented in Tables 1, 2 and 3. To compare these results, a graph is given in Figure 1.

The results of by the Philippine forecast using the Leslie matrix model are considered poor. One possible reason for this is due tot he fact that the Leslie matrix for the 2005 data gave a dominant value λ of 0.80. This value, since it is less than unity, suggests that the Philippine population of 2005 was decreasing. However, it is a known fact that Philippine population is increasing. This conflicting result between theoretical and actual, could possibly explain the poor result given by the Leslie matrix model. Hence, a sensitivity analysis was performed. The results are presented in Table 4. An arbitrary number is chosen to add at a time for each male fertility entry The arbitrary number chosen is 0.9. This increased the forecasted male population, at the same time the dominating eigenvalue increased for each changed of entry and the MAPE gave results less than 10%.

Another Leslie matrix was constructed using the 2014 Philippine data, the latest available. However, this 2014 matrix gave a dominant eigenvalue of 0.7. Again, since λ is less than unity, this goes against the increasing behaviour of Philippine population. This conflicting result between the actual growth in population and the value of λ, suggests that the a 2019 forecast may not be acceptable.

Age Group	F_F	S_F	F_M	S_M
0-4	0	0.968192	0	0.960279
5-9	0	0.997	0	0.997
10-14	9.11E-05	0.997	8.91E-05	0.997
15-19	0.016266	0.996	0.017581	0.994
20-24	0.057889	0.995	0.062742	0.99
25-29	0.063754	0.993	0.069509	0.986
30-34	0.049347	0.991	0.053525	0.983
35-39	0.034296	0.987	0.03688	0.976
40-44	0.013762	0.983	0.014517	0.969
45-49	0.002053	0.975	0.002072	0.954
50-54	0.000159	0.965	0.000182	0.935
55-59	0	0.943	0	0.891
60-64	0	0.917	0	0.854
65-69	0	0.868	0	0.779
70-74	0	0.802	0	0.718
75-79	0	0.675	0	0.577
80-84	0	0.498	0	0.446
λ= 0.7962077				
log(λ)= -0.2278952				

Table 1: 2005 Philippine data on fertility, survival and dominant eigenvalue

	Base Population	Actual Population		Forecasted Population	
Age Group	2005	2010	2015	2010	2015
0-4	4721115	4940573	5505300	809264	888734
5-9	4643067	4989256	5217700	4570946	783523
10-14	4500519	4942604	4973400	4629138	4557233
15-19	4229087	4773848	4924800	4487017	4615250
20-24	3905441	4151657	4739400	4212171	4469069
25-29	3541009	3677412	4109200	3885914	4191110
30-34	3160534	3329347	3639700	3516222	3858712
35-39	2776133	2956630	3294000	3132089	3484576
40-44	2374323	2692927	2919400	2740043	3091372
45-49	2006520	2312840	2648300	2333960	2693463
50-54	1631337	1940898	2258600	1956357	2275611
55-59	1319097	1511287	1872200	1574240	1887885
60-64	1013026	1164283	1429700	1243908	1484509
65-69	767324	817330	1066400	928945	1140664
70-74	546329	650410	710700	666037	806324
75-79	374459	421036	520800	438156	534162
80-84	330630	248251	498500	252760	295755
Total Population	41839950	45520589	50328100	41377167	41057951

Table 2: Philippine female population in 2005, 2010, 2015 and forecast for 2010 and 2015

	Base Population	Actual Population		Forecasted Population	
Age Group	2005	2010	2015	2010	2015
0-4	4937632	5293211	5822000	876396	962374
5-9	4832467	5332287	5453300	4741504	841585
10-14	4792979	5237006	5310500	4817970	4727280
15-19	4418572	4931506	5212100	4778600	4803516
20-24	3983027	4256999	4904000	4392061	4749928
25-29	3557779	3746311	4223300	3943197	4348140
30-34	3141953	3443582	3702300	3507970	3887992
35-39	2756653	3057323	3391300	3088540	3448335
40-44	2374463	2778661	2997000	2690493	3014415
45-49	2006056	2367809	2702900	2300855	2607088
50-54	1629315	1953952	2271400	1913777	2195015
55-59	1296672	1475861	1830900	1523410	1789382
60-64	963875	1064116	1335800	1155335	1357358
65-69	704079	680227	912000	823149	986656
70-74	475228	492152	538500	548478	641233
75-79	298154	286079	349400	341214	393807
80-84	232487	145937	277500	172035	196880
Total Population	42401391	46543019	51234200	41614982	40950983

Table 3: Philippine male population in 2005, 2010, 2015 and forecast for 2010 and 2015

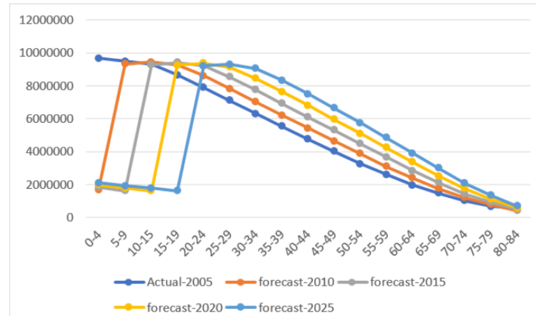

Figure 1: Philippine total population by age

a_{ij}	λ	Change	Sensitivity	MAPE (male)	MAPE (total)	Total Population (male)
$a_{18,3}$	1.0250	0.2288	0.4575	7.640	7.964	45665449
$a_{18,4}$	1.0199	0.2237	0.4473	7.912	8.105	45421161
$a_{18,5}$	1.0162	0.2200	0.4400	8.235	8.272	45129879
$a_{18,6}$	1.0134	0.2172	0.4344	8.600	8.461	44801890
$a_{18,7}$	1.0110	0.2148	0.4296	8.980	8.657	44459463
$a_{18,8}$	1.0089	0.2127	0.4255	9.365	8.856	44113502
$a_{18,9}$	1.0069	0.2107	0.4213	9.767	9.064	43751873
$a_{18,10}$	1.0048	0.2086	0.4172	10.135	9.254	43420850
$a_{18,11}$	1.0025	0.2063	0.4125	10.510	9.449	43083186

Table 4: Sensitivity ratios

6.2. Selected Asian countries

The MAPE results for the other Asian countries are given below. A graphical representation of the actual and forecasted population are also presented. The actual and forecasted for the male and female population of the other Asian Countries are in the Appendix. Any countries which gave a MAPE of less than 10% are presented. Countries like India, Malaysia, Qatar, and Oman which gave MAPE results of greater than 10% were not further studied.

Azerbaijan and Iran

For Azerbaijan data, the base year was 2007. A forecast for years 2012 and 2017 was obtained. The MAPE comparing the actual and forecast for 2012 data were all between $8 - 10\%$ (see Tables 5). For Iran data, the base year was 2006. A forecast for years 2011 and 2016 was obtained. The MAPE comparing the actual and forecast for 2011 data were all between $7 - 9\%$ (see Table 5).

Azerbaijan			Iran		
MAPE2012 (Female)	MAPE2012 (Male)	MAPE2012 (Total)	MAPE2011 (Female)	MAPE2011 (Male)	MAPE2011 (Total)
9.518327	8.58709	8.955652	8.714229	7.401127	7.929842
λ= 0.7402159			λ= 0.7318664		
log(λ)= -0.3008134			log(λ)= -0.3121573		

Table 5: MAPE and λ for Azerbaijan and Iran

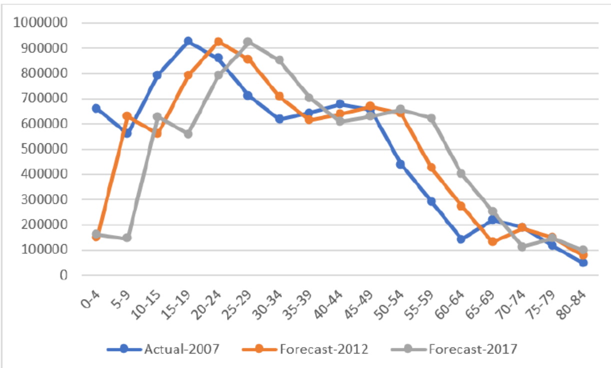

Figure 2: Total actual and forecast population for Azerbaijan

Israel and Japan

For Israel data, the base year was 2009. A forecast for years 2014 and 2019 was obtained. The MAPE comparing the actual and forecast for 2014 data were all between 5 − 7% (see Table 6). For Japan data, the base year was 2006. A forecast for years 2011 and 2016 was obtained. The MAPE comparing the actual and forecast for 2011 data were all between 5 − 7% (see Table 6).

Israel			Japan		
MAPE2014 (Female)	MAPE2014 (Male)	MAPE2014 (Total)	MAPE2014 (Female)	MAPE2014 (Male)	MAPE2014 (Total)
6.172699	5.821402	5.995517	5.594347	6.115957	5.799441555
λ= 0.8292283			λ= 0.7377181		
log(λ)= -0.1872598			log(λ)= -0.3041935		

Table 6: MAPE and λ for Israel and Japan

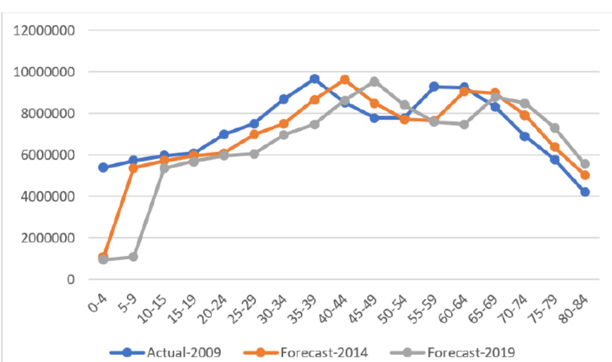

Figure 5: Total actual and forecast population for Japan

Singapore and Thailand

For Singapore data, the base year was 2009. A forecast for years 2014 and 2019 was obtained. The MAPE comparing the actual and forecast for 2012 data were all between 6−7% (see Table 8). For Thailand data, the base year was 2007. A forecast for years 2012 and 2017 was obtained. The MAPE comparing the actual and forecast for 2012 data were all between 7 − 8% (see Table 8).

Singapore			Thailand		
MAPE2014 (Female)	MAPE2014 (Male)	MAPE2014 (Total)	MAPE2012 (Female)	MAPE2012 (Male)	MAPE2012 (Total)
6.866022869	6.998102	6.866450091	7.115986382	7.100806	7.115551447
λ= 0.7397779			λ= 0.7275956		
log(λ)= -0.3014053			log(λ)= -0.3180099		

Table 8: MAPE and λ for Singapore and Thailand

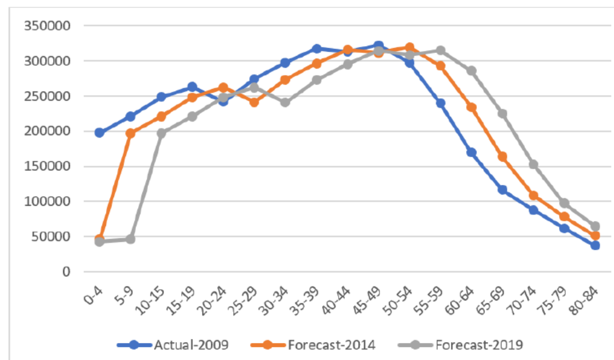

Figure 8: Total actual and forecast population for Singapore

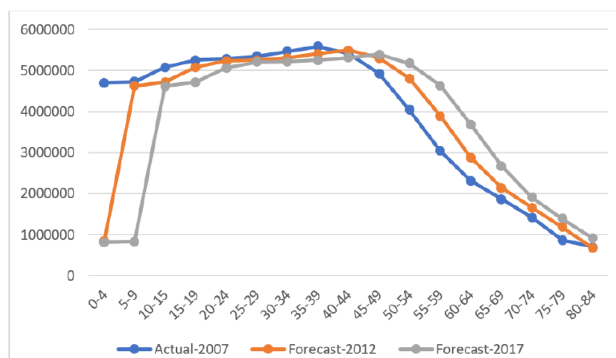

Figure 9: Total actual and forecast population for Thailand

7. RECOMMENDATIONS, SUMMARY AND CONCLUSIONS

The Leslie matrix was used to forecast the population of a country. This forecast was used to compare with actual population for the same year. The MAPE value resulted to values less than 10% which implies that the forecast is acceptable. In fact, Japan and Israel got the lowest MAPE ranging from $6-7\%$. This suggests that the forecast is close to the actual population. However, the Leslie matrix was not applicable to some countries such as Oman and Qatar which has a MAPE of between $18-28\%$. The Philippine population also did not return acceptable results. For selected countries with MAPE of at least 10%, we recommend other models or method to be applied. We also recommend sensitivity analysis in countries with MAPE of at least 10% to improve forecasted values. Another important observation is that the Leslie Matrix forecasts are generally not acceptable for the age 0-4. This is shown in the high APE for this age range in almost all countries.

It is also clear that the forecast will be better, if the dominant eigenvalue λ agrees with how the population is known to behave. In other words, countries known to have increasing population, should have Leslie matrices formed from their populations having λ values of greater than unity; and those with known decreasing population should yield Leslie matrices with λ values less than unity. Hence, we can suggests that a preliminary investigation of the λ value obtained from population data of a country we first obtained. If this value agrees with the behaviour of the population, then proceed with the forecast. If the value disagrees, then use a different method to forecast the population.

The Leslie matrix model are dependent on fertility rates, thus so are the projected values. It is possible that the projected population is poor because of the inaccurate fertility rates available. Thus, we suggests performing sensitivity analysis, with the aim of improving the fertility rates and thus give a better forecast. Furthermore, we also recommend that the Leslie matrix model be used to other countries aside from Asian countries.

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