Analysis of Model for Long-term Rabies Incidence in a Dog Population

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Abstract: Rabies is a fatal zoonotic disease and it remains to be a priority health concern in the Philippines. Dogs are the principal carrier of rabies. Despite the fatality of rabies, it is a vaccine-preventable disease. Today, national health officials have been conducting mass dog vaccination campaigns with the goal of having a rabies-free Philippines by the year 2020. However, statistical information about the disease poses a challenge and gives rise to doubts on whether the goal is achievable. In line with the goal, we aimed to see whether eliminating dog rabies in the country is possible by analyzing the long-term rabies incidence from an SEIR-based mathematical model for rabies transmission in the presence of mass vaccination. Some model parameters were calibrated from actual data sets. An analytic expression for the long-term rabies incidence in terms of key epidemiological parameters were derived. Numerical simulations were performed to see the relative contribution of varied parameters on controlling the long-term rabies incidence. Our preliminary findings pointed to annual dog birth rate as potentially an important driver of rabies incidence in the country.

Key Words: Basic Reproduction Number; Rabies Vaccination; Numerical Simulation; SEIR

1. INTRODUCTION

Rabies is a zoonotic disease caused by RNA viruses of the Lyssavirus genus of *Rhabdoviridae* family that is commonly present in the saliva of an infected animal, such as dogs (WHO, 2017), and is normally transmitted through a bite (Drew, 2004; Rupprecht, 2017). It is estimated to have caused 55,000 deaths every year, with 56% occurring in Asia and 43.6% in Africa. Rabies is present all over the world except in Antarctica, and in countries like Hawaii, Japan, and Singapore (WHO, 2017).

According to the Center for Disease Control and Prevention (2017), dogs exposed to rabies develop clinical symptoms after being exposed to the virus (incubation). This period for exposed dogs depends on the virus entry location and its load (Drew, 2004). Once symptoms occur, rabies is almost always fatal. Fortunately, it is a vaccine-preventable disease. Some control methods for rabies are mass dog vaccination, dog population management, and dog movement control (WHO, 2017). Among these control methods, mass dog vaccination is the most effective given that no less than 70% of the total dog population must be vaccinated (Beran, 1982; Coleman and Dye, 1996; WHO, 2004). Today, Philippines have been demonstrating mass dog vaccination campaigns in line with the goal of having a rabies-free Philippines by the year 2020. Mass dog vaccination campaigns done in Cebu City and Ilocos Norte reduced the total cost of vaccination and increase the vaccination coverage in the area, reducing the reported number of canine rabies cases (Miranda, 2017; Valenzuela et al., 2017). However, the continuing increase of the dog population in the country from almost 8.9 million in 2007 to 10 million in 2011 (DOH, 2012) might be a reason why rabies remains to be a public health problem in the Philippines, killing 200-300 Filipinos annually (DOH, 2017).

In line with the goal of having a rabies-free Philippines by 2020 and inspired by the work of Zhang et al. (2001), this study aimed to determine whether eliminating dog rabies in the country is achievable using the Susceptible-Exposed-Infected-Recovered (SEIR) epidemiological modelling approach. This approach has been widely used in terms of analyzing and understanding the transmission dynamics and control methods for infectious diseases like in the rabies transmission in China. In this study, the SEIR modelling framework of Zhang et al. (2011) for the dog population was adopted with minor modification on parameter definitions and basic assumptions.

This study addressed the importance of understanding the drivers of rabies incidence in the long-run analytically and via numerical simulations. The study mainly focused on rabies transmission in dog population only because dogs are the principal carrier of rabies and it is essential to eliminate the disease in the dog population (Davlin & VonVille, 2012).

2. METHODOLOGY

2.1 Model Description and Parameters

An SEIR epidemiological model was proposed in this study to depict the transmission of rabies among dogs and vaccination as a control strategy. the dog population was divided into four classes vary with time t (in years): $S(t)$ healthy dogs but are likely to acquire the disease; $E(t)$ dogs that have been infected with the disease but are not yet infectious; $I(t)$ dogs that are infectious; and $R(t)$ dogs that have been administered with anti-rabies vaccine and cease to be infectious.

Dog population was assumed to be mixed and homogenous. All members of the population interact with each other with all susceptible dogs facing the same risk of exposure to the virus by those already infected. A proportion of R can become susceptible due to loss of vaccine immunity (λ). Also, a proportion of S and E can become recovered when receiving anti-rabies vaccine (k). Moreover, a proportion of E develop clinical rabies during the incubation period, σ. All infectious dogs die due to the fatality of the disease hence there is no chance for them to recover at rate μ. Furthermore, the population under consideration is in an open environment. There is a constant flux of susceptible dogs due to birth rate (A) and natural death occurs for dogs in compartments S, E, and R. Dogs in S, E, and R have the same natural death rate, m. In addition, no migration in the populations is assumed. Disease transmission occurs when there is an interaction between I and S at a transmission rate β.

The rate of change for the susceptible, exposed, infective, and recovered class is given by Eq. (1), (2), (3), and (4), respectively.

$$dS/dt=A+\lambda R-S(k+m)-\beta SI \qquad \text{(Eq. 1)}$$
$$dE/dt=\beta SI-E(m+\sigma+k) \qquad \text{(Eq. 2)}$$
$$dI/dt=\sigma E-\mu I \qquad \text{(Eq. 3)}$$
$$dR/dt=k(E+S)-R(\lambda+m) \qquad \text{(Eq. 4)}$$

where:

A = annual crop of dogs
m = natural mortality rate of dogs
μ = rabid dog mortality rate
β = direct transmission rate of rabies
k = vaccination rate
$1/\sigma$ = incubation period
λ = dog loss rate of vaccine immunity

The model schematic diagram is shown in Fig. 2. Boxes represent the compartments S, E, I, and R. Solid arrows indicate the movement between compartments.

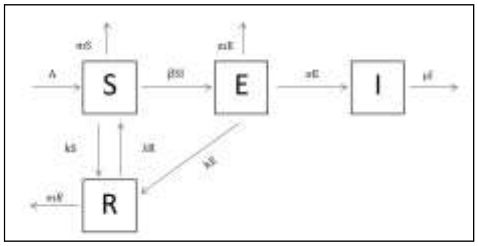

Fig. 1. Schematic diagram of the SEIR model for rabies transmission.

2.2 Long-term Rabies Incidence

The long-term rabies incidence, which are associated to the equilibria of the system, were obtained by setting the left-hand sides of Eq. (1) to (4) to zero. The equations were manipulated algebraically to determine the equilibrium values of S, E, I, and R in terms of the model parameters. The theoretical long-term incidence is described by the equilibrium value of infectious dog population expressed in terms of the varying parameters A, k, λ, and β and, denoted as $I(A, k, \lambda, \beta)$.

The analyses were done by fixing the rate of loss of vaccine immunity (λ) and direct transmission rate (β) while varying the annual crop of dogs (A) and vaccination rate (k). The function $I(A, k, \lambda, \beta)$ is represented in terms of color maps of two parameters, e.g. A and k, to assess how the long-term rabies incidence vary over the parameter space for a fixed λ and β. Furthermore, a Jacobian matrix was obtained to aid in computing the stability of the equilibrium, i.e., if the maximum eigenvalue is negative then long-term incidence is described by the endemic steady-state, otherwise, the long-term incidence is the disease-free steady-state.

Furthermore, numerical simulation of the system was performed to determine how the rabies incidence varies over time t (in years) for various parameter combinations. The following initial conditions were used for the numerical simulations: $1.03\text{x}10^7$ dogs in the susceptible class $S(0)$, $2\text{x}10^3$ dogs for the exposed class $E(0)$, $2\text{x}10^2$ dogs for the infected class $I(0)$, and $2\text{x}10^6$ dogs for the recovered class $R(0)$. $S(0)$ was based on the dog-to-human ratio of 1:10 from the 2015 human population in the Philippines while $E(0)$, $I(0)$, and $R(0)$ were assumed. Other parameter values used are $m=0.08$ *per year,* $\sigma=6$ *per year,* and $\mu=1$ per year. All the simulations for the mathematical models were implemented using Octave 4.2.1.

3. RESULTS AND DISCUSSION

3.1 Endemic Steady-States

Setting Eq. (1) to (4) to zero and upon manipulating them algebraically, the steady-states S^*, E^*, I^*, and R^* classes were derived as shown below.

$$S^*=(\mu(m+k+\sigma))/\sigma\beta \quad \text{(Eq. 5)}$$

$$E^*=(S^*[(k+m)(\lambda+m)-\lambda k]-A(\lambda+m))/(\lambda k-[(m+k+\sigma)(\lambda+m)]) \quad \text{(Eq. 6)}$$

$$I^* =(\sigma E^*)/\mu \quad \text{(Eq. 7)}$$

$$R^*=(k(S^*+E^*))/(\lambda+m) \quad \text{(Eq. 8)}$$

The stability of the steady-state Eq. (5) to (8) is determined numerically.

3.2 Long-term Rabies Incidence

Using the derived steady-states from Eq. (5) to (8) and by computing the corresponding maximum eigenvalue of the associated Jacobian matrix, the long-term rabies incidence was expressed as a function of the parameters A, k, λ, and β, computed using parameter values in Section 2.2, and color maps were generated for the analysis of the long-term rabies incidence as a function of A and k with fixed λ, and β, as displayed in Fig. 2.

As seen in Fig. 2, decreasing β increases the chance in the (A,k) parameter space to have a zero long-term incidence. When β is increased a thousand times, i.e. from $\beta=10^{-6}$ to $\beta=10^{-3}$, the model predicts a varied long-term incidence when $A<550{,}000$ and in this scenario, the vaccination rate (k) has a major role in decreasing the disease incidence. When A is below the threshold 550,000, we can ensure that zero disease incidence is achieved when the vaccination rate is at least 80%.

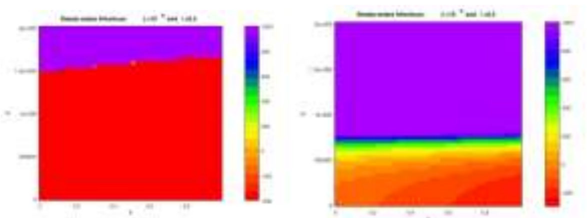

Fig. 2. Color maps showing long-term rabies incidence as a function of *A* and *k* with $\beta=10^{-6}$ (left) and $\beta=10^{-3}$ (right) for λ=0.5 (2 years).

Further analysis was done by seeing the effect of rate of loss of vaccine immunity (λ) on the long-term rabies incidence by varying λ to a fixed transmission rate (β). From the color maps displayed in Fig. 3, it is clear that the long-term rabies incidence is not that sensitive to λ.

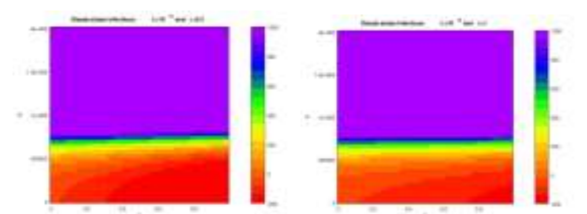

Fig. 3. Long-term incidence as a function of *A* and *k* for low value of λ (left) and high value of λ (right) with other parameters held fixed.

4. CONCLUSIONS

Our preliminary findings pointed to mathematical modelling as an important tool to make reasonable projections about rabies disease. Using Philippine parameter setting, our model suggests that the Philippines can be rabies-free if the annual dog birth rate is small enough (<500,00) and the vaccination rate is large enough (>80%). Moreover, our simulation shows that annual dog birth rate is an important driver of disease incidence in the country. The model used in this study can be studied thoroughly if dog population data is present. Furthermore, the results of this study would aid the government in making policies regarding rabies elimination, e.g. dog population control. We also recommend that a monitoring of dog population should be done to have actual measures of annual dog birth rate.

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