A Comparison of ARMA-GARCH and Bayesian SV Models in Forecasting Philippine Stock Market Volatility

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Abstract: Forecasting volatility is vital in the financial field since it represents one of the risk indicators available. Several models are incorporated by several analysts, investors, and traders but no single superior model was found. In this paper, we present Bayesian and Non-Bayesian methods in forecasting the Philippine Stock Market volatility. The Autoregressive Moving Average – Generalized Autoregressive Conditional Heteroskedasticity (ARMA-GARCH) and the Stochastic Volatility (SV) models were introduced in this paper. Since the best representation for the Philippine market is the Philippine Stock Exchange Index (PSEI), it is then used as the primary data for the paper. The assumptions for the ARMA-GARCH models are tested first to ensure convergence, then the selection for the autoregressive and moving average terms are chosen based on the automatic selection in the *forecast* package in R studio. After which, the parameters for the GARCH model is estimated using the R package *rugarch*. For the Bayesian SV model, the Markov Chain Monte Carlo (MCMC) sampler, embedded in the R package *stochvol,* was used to generate forecasts for the volatility. Results show that ARMA(5,0,4)-GARCH(1,1) model performed significantly better than the SV model in forecasting the stock market volatility with forecasts yielding the lowest mean absolute percentage error (MAPE) and root mean squared error (RMSE).

Key Words: stock market volatility; Bayesian SV; ARMA; GARCH; MAPE

1. INTRODUCTION

In the financial and economic field, volatility refers to the spread of all likely outcomes of an uncertain variable. In this study, the standard deviation of the daily returns of closing stock prices was used. Volatility has been the subject of trading, financial regulation, monetary policy, and macroeconomy. Thus, it is established that volatility is vital to the financial world (Poon, 2015).

Although asset returns are of equal importance, risks should be monitored as well. To create sound investment decisions, risks measure potential losses and volatility plays its role as the purest form of risk in financial markets. Volatility also affects the public confidence which in return, provide an impact on the global economy (Marra, 2015).

A vast collection of literature regarding volatility can be found in journals. Consequently, several models are also used in forecasting volatility. However, no single superior model was found. In this paper, the researchers compared Bayesian and Non-Bayesian approaches in forecasting volatility, namely the Bayesian Stochastic Volatility (SV) and the Autoregressive Moving Average – Generalized Autoregressive Conditional Heteroskedasticity (ARMA-GARCH) models.

2. METHODOLOGY

2.1 Data

Closing stock prices of the Philippine Stock Exchange index (PSEi) from January 3, 2012 to August 25, 2017 was used. Formerly called the PHISIX, the PSEi was carefully selected because it is the representation of the general movement of the stock market. It is a fixed basket composed of 30 common stocks listed companies. Moreover, it serves as a benchmark in measuring the performance of the Philippine stock market (PSE Academy, n.d.).

Since intra-daily data is difficult to obtain, the volatility of each month is instead computed and the data set is split into two parts: the training set and the validation set with a ratio of 80-20%, respectively. Thus, the training set includes the monthly volatility computations for January 3, 2012 to July 15, 2016 while the rest were included in the validation set.

The number of observations utilized in the study constitute to $n = 1{,}377$. The daily returns are computed using the difference of the natural logarithm of the closing price for the two consecutive trading days.

R studio was used in constructing the ARMA-GARCH and the Bayesian SV models. For the SV model, the R package *stochvol* was used. For the ARMA-GARCH model, the R packages *rugarch, aTSA, forecast, lmtest, zoo* were used.

2.2 Model Fitting and Forecasting

In the following section, we present two different models in forecasting stock market volatility namely (i) Bayesian SV model and (ii) ARMA-GARCH model.

For the Bayesian SV model, the log-returns were demeaned first prior to running the MCMC sampler to avoid zero returns. After setting up the data, the prior hyperparameters for the parameter vector $\theta=(\mu,\phi,\sigma\eta)^T$ were selected. The hyperparameters used will follow that of Table 1. Other arguments in the *stochvol* package were set to default, as they were found to suffice in most situations. After running the sampler, the draws of the predicted volatility are extracted for easier comparison.

Table 1. Validated parameters for Bayesian SV.

Prior	Parameter	Estimates	Validation	Source
Level μ	b_μ	0	Not influential to empirical data containing large samples ($n \leq 1{,}000$)	Kastner, 2016
	B_μ	100		
Persistence φ	α_0	20	To avoid zeroing out the μ terms in the SV model	Kim, et al, 1998
	b_0	1.5		
Volatility of log-variance σ_η	B_σ	0.5	Based on usage experience	Kastner, 2016

For the ARMA-GARCH model, the primary concern is to construct a model that satisfies the assumptions of the ARMA and GARCH model and to select the most accurate one based on forecast errors. The optimal parameters p and q are selected using the auto-selection of the *forecast* package, based in the algorithm in yielding the lowest Akaike Information Criterion (AIC) and Bayes Information Criterion (BIC) estimates. After which, the presence for ARCH effects are tested using the Lagrange-Multiplier (LM) test. If heteroskedasticity is confirmed to be present, the GARCH model will be fitted to describe the behavior of the variance equation. The parameters up to GARCH (2,2) were fitted and the lowest AIC and SBC parameter estimate was chosen to be the final model.

3. RESULTS AND DISCUSSION

3.1 Summary statistics

Since the log-returns of the PSEi were used for model building, it is necessary to show the formal tests in order to verify the behavior of the plots. The corresponding tests for normality, distributional properties, stationarity and autocorrelations are the Shapiro-Wilk, Augmented Dickey-Fuller (ADF) test, and Ljung-Box (LB) test, respectively. Table 2 shows the summary statistics and the p-values for the aforementioned tests. All the p-values for the tests are less than 0.001, implying that the data is significantly different from a normal distribution, is stationary, and that there is serial correlation in the PSEi returns data. The existence of serial correlation means that the PSEi returns data should be accounted for in the mean equation for the model building process. The significance of the LB of the squared returns indicate the existence of inter-temporal dependence on the variance. Overall, the preliminary tests suggest that the data is non-normal with thick tails together with strong dependence and exhibit volatility clustering.

Table 2. Descriptive statistics for the PSEi returns.

Descriptive Statistics	PSEi Returns
N	1377
Mean	0.000436
STD	0.010441
Variance	0.000109
Kurtosis	5.294743
Skewness	-0.72832
Shapiro Wilk test	0.94736 *$p < 0.0001$*
Augmented Dickey-Fuller Test (12)	49.15 *$p = 0.0010$*
Ljung-Box Test (12) return	43.17 *$p < 0.0001$*
Ljung-Box 2 (12) squared returns	427.01 *$p < 0.0001$*

3.2 ARMA-GARCH Model Fitting

The results from the Ljung-Box tests indicate that the mean and variance of the series were needed to be accounted for, hence satisfying the preliminary conditions of ARIMA-GARCH modelling. The ARIMA model was used for the returns while the GARCH model was used for the variance of the returns or the volatility. Results show from the auto-selection algorithm in the *forecast* package that the best model is ARIMA(5,0,4). Table 3 shows the values of the parameters for the ARIMA(5,0,4). Consequently, Table 4 shows the AIC and BIC estimates for the aforementioned model.

Table 3. *ARIMA (5,0,4) Parameter Estimates*

Parameter	Estimate	Std. Error
AR1	0.45397	0.19991
AR2	-0.1185	0.0706
AR3	-0.8003	0.05705
AR4	0.53067	0.18309
AR5	-0.0783	0.03539
MA1	-0.3802	0.19837
MA2	0.03751	0.05487
MA3	0.79032	0.0585
MA4	-0.5581	0.17076
INTERCEPT	0.00054	0.00028

Table 3 (continuation)

Parameter	z value	Pr(>|z|)	
AR1	2.2709	0.02316	*
AR2	-1.6789	0.09317	
AR3	-14.027	< 2.2e-16	*
AR4	2.8984	0.00375	*
AR5	-2.2127	0.02692	*
MA1	-1.9167	0.05528	
MA2	0.6836	0.49423	
MA3	13.5109	< 2.2e-16	*
MA4	-3.268	0.00108	*
INTERCEPT	1.9279	0.05387	

*significant at 0.05 significance level

Table 4. AIC and BIC estimates.

Test Statistic	Estimate
AIC	-6882.48
BIC	-6827.42

Table 5 summarizes the results of the ARCH test and it clearly indicates the presence of ARCH effect based from the relatively small p-value, which are significantly less than the significance level 0.05.

This indicates that the homoscedasticity assumption is invalid, which is a desired assumption in ARCH models.

Table 5. LM Test for Heteroskedasticity

Lag	Order	LM Statistics	*p*-value	
1	4	1010	<0.0001	*
2	8	474	<0.0001	*
3	12	273	<0.0001	*
4	16	200	<0.0001	*
5	20	159	<0.0001	*
6	24	131	<0.0001	*

** significant at 0.05 significance level*

Table 6 shows the different orders of the GARCH process with its corresponding AIC and BIC estimates. Based on the results, GARCH (1,1) yielded the lowest AIC and BIC estimates and as such it was utilized in the study.

Table 6. AIC and BIC estimates.

Model	AIC	BIC
ARMA(5,0,4)-GARCH(1,1)	-6.5087	-6.4452
ARMA(5,0,4)-GARCH(2,1)	-6.5069	-6.4388
ARMA(5,0,4)-GARCH(1,2)	-6.5049	-6.4368
ARMA(5,0,4)-GARCH(2,2)	-6.5053	-6.4326

Table 7 shows the parameter estimates for the ARIMA(5,0,4)-GARCH(1,1) model. The volatility estimates were obtained from the ARIMA(5,0,4)-GARCH(1,1) model utilizing the rolling forecast with re-estimation. However, since the actual data only provides the monthly volatility, the forecasted daily volatility are aggregated into a monthly volatility by averaging the daily volatility forecasts to match the actual forecasts.

Table 7. *ARIMA(5,0,4)-GARCH(1,1) Parameters*

Parameter	Estimate	Std. Error	p-value	
MU	0.000879	0.000218	0.000056	*
AR1	1.25248	0.012075	<0.0001	*
AR2	-1.49512	0.010489	<0.0001	*
AR3	1.305712	0.009264	<0.0001	*
AR4	-0.77919	0.012625	<0.0001	*
AR5	0.008929	0.009726	0.358589	
MA1	-1.20751	0.000116	<0.0001	*
MA2	1.410097	0.00602	<0.0001	*
MA3	-1.25818	0.000107	<0.0001	*
MA4	0.682284	0.008671	<0.0001	*
OMEGA	0.000005	0.000003	0.079403	
ALPHA1	0.114901	0.005332	<0.0001	*
BETA1	0.838492	0.019415	<0.0001	*
SHAPE	6.293456	1.030934	<0.0001	*

**significant at 0.05 significance level*

The equation for the mean and variance model is as follows:

$$(1 - 1.25B + 1.50B^2 - 1.31B^3 + 0.78B^4 - 0.01B^5)r_t = (1 + 1.21B - 1.41B^2 + 1.26B^3 - 0.68B^4)\varepsilon_t, \quad (1)$$

where,

$$\varepsilon_t \sim t(0, h_t) \quad (2)$$

$$h_t = 0.000005 + 0.114901\varepsilon_{t-1}{}^2 + 0.838492h_{t-1}{}^2 \quad (3)$$

3.3 Bayesian SV Model Fitting

The data is prepared first by demeaning it beforehand. After which, the prior estimates were specified and the main stochastic volatility sampler is run. Hence, Bayesian inference via the Markov Chain Monte Carlo method is employed. The main sampler is run with the prior estimates for a total of 10,000 iterations.

It should be noted that in the SV model, the time-varying volatility is assumed to follow a stochastic evolution, instead of a deterministic one. Following Kim et al.(1998) specifies the SV model as:

$$y_t = e^{h_t/2}\epsilon_t \quad (4)$$

and (5)

$$h_t = \mu + \phi(h_{t-1} - \mu) + \sigma_{\eta_t}$$

where ϵ_t and η_s are independent and identically distributed standard normal innovations and are independent for $t, s \in \{1, \dots, T\}$.

The Markov Chain Monte Carlo algorithm was utilized to obtain draws from the posterior distribution of the desired random variables, specifically the latent log-variances h and the parameter vector θ. The MCMC specifications is further elaborated by Kastner & Frühwirth-Schnatter (2014). The strategy employed in the algorithm implemented in the R stochvol package is the use of "ancillary-sufficiency interweaving strategy (ASIS)" which has been introduced by Yu and Meng (2011) for state-space models. The ASIS exploits that for certain parameter constellations, sampling efficiency improves drastically when considering a non-centered version of the state-space model. This move can be achieved by transferring the level μ and volatility σ_η to the observation process shown in Equation (4) through reparameterization of h. In the case of the SV model however, there is no single superior parameterization. In some cases, the standard parameterizations perform better than that of the non-centered versions, and vice-versa. To solve the problem, the parameter vector θ is sampled twice, through the centered and the non-centered parameterizations. This method of combining best of different worlds allows efficient inference regardless of the underlying process of one algorithm (Kastner, 2016).

Table 8 shows the posterior parameter estimates for the SV model.

Table 8. *Summary of 10000 MCMC draws after a burn-in of 1000*

Estimates	**Mean**	**SD**
MU	-9.4505	0.16161
PHI	0.944	0.01821
SIGMA	0.2601	0.04029
$EXP^{(MU/2)}$	0.0089	0.00073
$SIGMA^2$	0.0693	0.02141

After obtaining the parameter estimates of the stochastic volatility model, forecast for volatility is obtained using the posterior estimates earlier. The number of forecasts for each period is equivalent to the number of iterations in the main sampler used. In order to obtain a single forecast for a period, the mean of the 10,000 iterations were obtained for each forecast.

The forecasting equation is as follows:

$$r_t = e^{ht/2}\epsilon_t, \quad (6)$$

where,

$$h_t = -9.45 + 0.94(h_{t-1} + 9.45) + 0.0693 \quad (7)$$

3.4 Forecasting

In this stage, the volatility forecasts for the two models are obtained and forecasting accuracy is then measured using the Mean Absolute Percentage Error (MAPE) and Root Mean Squared Error (RMSE). Table 9 summarizes the forecasting accuracy results for the ARIMA(5,0,4)-GARCH(1,1) and Stochastic Volatility model. The results indicate that the ARIMA(5,0,4)-GARCH(1,1) has significantly outperformed the SV model in forecasting the monthly volatility based from the relatively lower MAPE and RMSE.

Table 9. *Forecasting Accuracy of GARCH and SV*

Model	MAPE	RMSE
ARMA(5,0,4)-GARCH(1,1)	0.163986564	0.0013292
Stochastic Volatility	0.304762864	0.002794453

4. CONCLUSIONS

Forecasting volatility is a significant aspect in portfolio management in terms of risk and pricing strategies. This research has utilized the Bayesian Stochastic Volatility model that is rarely used in many practical applications due to the difficulty in estimation. Specifically, the researchers have found a way to efficiently estimate the stochastic volatility model through the use of Markov Chain Monte Carlo method. Additionally, the researchers have utilized a forecasting technique namely the rolling window with re-estimation in the ARIMA(5,0,4)-GARCH(1,1) that produces more efficient volatility forecasts. From the two models, the ARIMA(5,0,4)-GARCH(1,1) have outperformed the Stochastic Volatility model in forecasting the monthly volatility of the PSEI. It signified that the Non-Bayesian method utilized in this paper have outperformed the Bayesian method in forecasting the PSEi volatility.

5. Recommendation

In this paper, PSEi was utilized and future researches may also opt to apply the forecasting models to specific stock prices in the Philippines as the PSEi is the weighted capitalization of the thirty different firms' market value. Future researchers may also consider to use daily volatility by obtaining an intraday data in order to have more accurate volatility forecasts. The GARCH(1,1) model can be further improved by exploring the different varieties of the GARCH. For the SV model, researchers are advised to test different prior estimates for the parameter vector $\theta = (\mu, \phi, \sigma_\eta)^T$ with supporting literature.

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