A note on open tournament theory

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Abstract: Tournament theory hypothesizes that effort is positively related with an increase in the prize spread (Lazear and Rosen, 1981). However, this does not hold true in all cases. Sheremeta and Wu (2012) identified an empirical puzzle, which shows that effort should not change when the prize spread does not. They find that decreasing the prize spread by increasing the loser's prize did not result in the agent's effort to decrease but rather the agent's effort appeared to increase. Given this gap in the literature, we introduce a third player in the tournament using the standard model of tournament theory; the third player represents external agents given some constant prize. Using comparative statics, effects of the prize spread on effort are determined. We find that effort from risk-neutral and risk-averse agents increases when the winner's prize increases and the internal prize spread increases; and effort also increases when the loser's prize increases and the internal prize spread decreases. Effort also increases when the internal prize spread is held constant but the internal-external prize spread increases. Hence, we are able to show that the existence of external agents plays a role in situations where tournament theory may be applied (e.g., wages and hiring), implying that the internal-external prize spread is as important to analyze as the internal prize spread.

Keywords: tournament theory; prize spreads; effort probabilities; labor economics; personnel economics

1. INTRODUCTION

The original tournament theory model was introduced by Lazear and Rosen (1981) [LR, hereafter]. Some propositions in the LR tournament model were not met by the results of a study by Sheremeta and Wu (2012) [SW, hereafter]. In the LR tournament model, it is assumed that as the prize spread of the winner and loser changes, the effort of the agents will also directly change despite whether the change in spread is caused by a change in the winner's prize or the loser's prize. SW relaxed the assumption on separable agent utility and induced these changes in their empirical test. They showed that increasing the loser's prize, i.e., decreasing the prize spread, does not appear to reduce agents' effort, and increasing the loser's prize, holding the prize spread constant, results in an increase in the effort of the agents.

From an efficiency wage perspective, results based on tournament theory imply that a player's effort is critical for the firm as effort has a direct relationship on their output. When workers or agents receive a higher wage, it will result in an increase in their productivity due to the possibility of being replaced or losing their job.

Sparks (1986) discusses that there exists an efficiency wage effect because agents face a threat of dismissal. To further encourage agents to exert more effort, the firm may increase their wages. The higher wage acts as an incentive for agents to exert more effort. Subsequently, increasing wages also increases the "opportunity cost of dismissal." This pertains to the cost that an agent loses upon exiting the firm. With a higher wage, the agent loses the opportunity of earning a higher salary when she leaves the firm. Thus, agents tend to exert more effort to avoid the cost of dismissal. An increase in effort then leads to an increase in productivity.

The empirical puzzle presented by SW indicates that even as the prize spread remains constant for both winner and loser, the effort of both parties still increases. This may be due to career concerns of the "losing agent." Gibbons and Murphy's (1991) two-period model refers to a situation where the contract is already made, and the prospective laborers are free to choose whether they want to enter into the contract's compensation scheme, which will reveal their intention because of their being underpaid in the first period, and overpaid in the second. It is optimal in the sense that there is minimal required surveillance because the effort of the worker during the first period would be greater than the actual return, thus it would discourage them from shirking, and to avoid the risk of getting fired.

A study by Szymanski and Valletti (2004) discusses the benefits of having a strong second prize in provoking every contestant to produce maximum effort. They focus on the case where many weak contestants are against a strong participant, shifting the focus from the first prize to the second prize; this induces more effort from an individual contestant and the total effort as well.

In this paper, we aim to assess a standard tournament model with the possibility of an external agent and by using comparative statics. Agents facing different wages, i.e., prizes, may react differently when there is a thread of being replaced or of being dismissed, following Sparks (1986). Since a player's effort is critical for the firm as effort has a direct relationship with their output, this study aims to theoretically test if the existence of an external agent's constant prize possesses a threat to the internal agents and if this may create behavioral changes in effort. We intend to show that with the existence of an external player, effort is not dependent on the internal prize difference but rather on the external prize difference. Findings from this exercise can be useful in understanding how wages and prize spreads can elicit the desired effort by firms.

In Section 2, we discuss the framework used and the methodology. In Section 3, we present our key results, and we conclude the paper in Section 4.

2. FRAMEWORK & METHODOLOGY

2.1 The standard tournament model

The standard model of tournament theory considers two risk-neutral agents competing in a tournament. These agents are hired by a risk-neutral principal. Agent$i$'s performance can be denoted as y_i which is stochastically related to effort in the functional form

$$y_i = e_i + \epsilon_i \quad \text{(Eq.1)}$$

where

e_i = effort exerted by individual i
ϵ_i = random variable with mean zero and density function $f(\epsilon_i)$

We also assume that ϵ_i is independent and identically distributed across agents with mean zero. The agent's cost of exerting effort is defined by $c(e_i)$ where $c'(e_i) > 0$ and $c''(e_i) > 0$, i.e., the cost function is convex. The principal evaluates each agent's performance, y_i, and not their effort, e_i. This behavior allows for the existence of a moral hazard.

In the two-agent tournament model, agents compete for two prizes, W_1 and W_2, where W_1 is the winner's prize and W_2 is the loser's prize. The probability of agent i outperforming agent j is defined as P_i. Hence, $P_i = \Pr(y_i > y_j)$. We also assume that

$\frac{\partial P_i}{\partial e_i} > 0$ since as effort increases, the probability that she outperforms the other agent increases.

The two risk-neutral agents then decide on their effort simultaneously based on their expected profit defined as

$$E\left(\pi_i^A\right) = P_i W_2 + (1 - P_i)W_2 - c(e_i). \qquad \text{(Eq.2)}$$

Since agents maximize their expected profit to determine their optimal effort, the following condition is derived:

$$\frac{\partial E(\pi_i^A)}{\partial e_i} = 0 \rightarrow \frac{\partial P_i}{\partial e_i}(W_1 - W_2) = \frac{\partial c(e_i)}{\partial e_i} \qquad \text{(Eq. 3)}$$

which is true for individuals i and j. With

$$\frac{\partial P_i}{\partial e_i}(W_1 - W_2) = \frac{\partial c(0)}{\partial e_i},$$

the LR tournament model suggests that if the objective function is exhibiting some degree of concavity (possibly strict concavity), then

$$\frac{\partial c(e_j)}{\partial e_j} = \frac{\partial P_i}{\partial e_i}(W_1 - W_2) = \frac{\partial c(e_i)}{\partial e_i}$$

implies that there is a symmetric Nash equilibrium such that

$$e_i = e_j = e(W_1, W_2).$$

With identical effort, there is a fifty percent chance that each agent will win the tournament if the players are risk-neutral.[1]

The agents' reactions to change in the tournament prize structure can then be measured from the reaction functions, using comparative statics. The LR tournament model's propositions shows that a risk-neutral agent optimally responds to a change in the tournament prize structure as follows:

a) effort increases in W_1 holding W_2 constant (i.e. the prize spread increases);
b) effort decreases in W_2 holding W_1 constant (i.e. the prize spread decreases);
c) effort does not change with a change in W_2 holding the prize spread constant; and,
d) an increase in either the prize spread or in W_2 (holding W_1 constant) relaxes the agents' participation constraint and induces more participation.

Now, suppose that the expected benefit of the agents is expressed in expected utility such that

$$E[U(\pi_i^A)] = P_i U(W_2) + (1 - P_i)U(W_2) - c(e_i). \qquad \text{(Eq. 4)}$$

We note that the agent's effort will behave in the same manner, excluding $c(e_i)$. This is because the effect on effort does not directly come from the prize spread but from the utility spread. Note that $\Delta[U(W_1) - U(W_2)]$ is different from $\Delta[W_1 - W_2]$. This is true by concavity of the utility function.

2.2 The open tournament model

For this extended model, we consider the following hypotheses:

1. Two risk-neutral internal players are in the tournament with the existence of a risk-neutral external player.
2. The higher performing agent receives the winner's prize, W_1, and the lower performing agent receives the loser's prize, W_2. The external agent receives a constant external prize, W_3, where $W_1 > W_2 > W_3$. Hence, $(W_1 - W_3) > (W_2 - W_3)$.
3. The cost of effort of any agent i is convex, defined as $c(e_i)$ wherein $c'(e_i) > 0$ and $c''(e_i) > 0$.
4. There are three mutually exclusive and exhaustive probabilities: the probability of winning P_1, the probability of losing P_2, and the probability of being an external player P_3 or P_E (called the external probability). The sum of P_1 and P_2 is the overall probability of entering the tournament P_I, called the internal probability.
5. $\frac{\partial P_I}{\partial e} > 0$; i.e., the probability of entering the tournament increases as the effort of an agent increases. This results in $\frac{\partial P_1}{\partial e} + \frac{\partial P_2}{\partial e} > 0$.
6. Agents simultaneously maximize expected profit.

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[1] Note that such hypothesis may be unrealistic, or possibly simplistic. As such, extensions will be done by relaxing this hypothesis and to further investigate how agents behave with non-identical cost functions.

7. No collusion exists between agents.

Hence, this open tournament model differs from the LR model by assumptions 1, 2, 4, and 5. We also assume homogenous agents, and there is only one time period (i.e., static).

3. RESULTS AND DISCUSSION

3.1 The model with risk-neutral agents

The three agents (two internal and one external) maximize their expected profit by deciding on their effort levels. The expected profit is given by

$$E(\pi_i) = P_1W_1 + P_2W_2 + P_3W_3 - c(e). \qquad \text{(Eq.5)}$$

Since $P_3 = 1 - P_1 - P_2$, substituting into Eq. 5 yields

$$E(\pi_i) = P_1(W_1 - W_3) + P_2(W_2 - W_3) + W_3 - c(e) \qquad \text{(Eq.6)}$$

Because this is a simultaneous move game, the necessary condition generates the best response function for each agent and is expressed by

$$\frac{\partial E(\pi_i)}{\partial e} = \frac{\partial P_1}{\partial e}(W_1 - W_3) + \frac{\partial P_2}{\partial e}(W_2 - W_3) - c'(e) = 0 \qquad \text{(Eq.7)}$$

We note that the best response function is now a function of two prize spreads: that between the winner's prize and the external prize, and that between the loser's prize and the external prize. This is referred by the literature as the relative prize spreads.

The sufficiency condition must also be satisfied, i.e.,

$$\frac{\partial^2 P_1}{\partial e^2}(W_1 - W_3) + \frac{\partial^2 P_2}{\partial e^2}(W_2 - W_3) - c''(e) < 0 \qquad \text{(Eq.8)}$$

We obtain the total differential of the best response function with respect to e, W_1, and W_2, and rearrange the terms to obtain the following derivatives:

$$\left.\frac{de}{dW_1}\right|_{dW_2=0} = \frac{-\frac{\partial P_1}{\partial e}}{\frac{\partial^2 P_1}{\partial e^2}(W_1 - W_3) + \frac{\partial^2 P_2}{\partial e^2}(W_2 - W_3) - c''(e)} > 0 \qquad \text{(Eq. 9)}$$

$$\left.\frac{de}{dW_2}\right|_{dW_1=0} = \frac{-\frac{\partial P_2}{\partial e}}{\frac{\partial^2 P_1}{\partial e^2}(W_1 - W_3) + \frac{\partial^2 P_2}{\partial e^2}(W_2 - W_3) - c''(e)} > 0 \qquad \text{(Eq.10)}$$

$$\left.\frac{de}{dW_2}\right|_{\frac{dW_2}{dW_1}=1} = \frac{-\frac{\partial P_1}{\partial e} - \frac{\partial P_2}{\partial e}}{\frac{\partial^2 P_1}{\partial e^2}(W_1 - W_3) + \frac{\partial^2 P_2}{\partial e^2}(W_2 - W_3) - c''(e)} > 0 \qquad \text{(Eq. 11)}$$

We note that the denominator of Eq. 9 to 11 are negative based on the sufficiency condition in Eq. 8.

3.2 The model with risk-averse agents

The players in the open tournament model may also behave as risk-averse agents, where they maximize expected utility, which is dependent on profit.

We redo the necessary and sufficiency conditions as in the case of risk averse agents (replacing the prize spreads with the concave utility function dependent on the relative prize spreads). These generate the following derivatives:

$$\left.\frac{de}{dW_1}\right|_{dW_2=0} = \frac{-\frac{\partial P_1}{\partial e}U'(W_1)}{\frac{\partial^2 P_1}{\partial e^2}[(U(W_1) - U(W_3)] + \frac{\partial^2 P_2}{\partial e^2}[U(W_2) - U(W_3)] - c''(e)} > 0 \qquad \text{(Eq.12)}$$

$$\left.\frac{de}{dW_2}\right|_{dW_1=0} = \frac{-\frac{\partial P_2}{\partial e}U'(W_2)}{\frac{\partial^2 P_1}{\partial e^2}[(U(W_1) - U(W_3)] + \frac{\partial^2 P_2}{\partial e^2}[U(W_2) - U(W_3)] - c''(e)} > 0 \qquad \text{(Eq.13)}$$

$$\left.\frac{de}{dW_2}\right|_{\frac{dW_2}{dW_1}=1} = \frac{-\left[\frac{\partial P_1}{\partial e}U'(W_1) + \frac{\partial P_2}{\partial e}U'(W_2)\right]}{\frac{\partial^2 P_1}{\partial e^2}[(U(W_1) - U(W_3)] + \frac{\partial^2 P_2}{\partial e^2}[U(W_2) - U(W_3)] - c''(e)} > 0 \qquad \text{(Eq.14)}$$

Again, by the concavity of the utility function[2],

[2] Note that to do comparative statics given in equations 9 to 14, we assume that the hypotheses of the implicit function theorem to hold on the function $e(W_1, W_2)$.

$$U'(W_1) < U'(W_2).$$

We arrive at the following propositions, noting that collusion between the agents is not possible and the tournament is static. We can observe that the agents' reactions are only based on the internal-external prize spread and are not affected by other agents' effort, as in the standard tournament theory. Since agents are homogenous, all have similar reaction functions since there does not exist any player that poses a threat of being dismissed. Thus, we are able to measure changes in effort due to the internal-external prize spread. We infer the following results, based on Eq. 9 to 11 (for risk-neutral agents) and Eq. 12 to 14 (for risk-averse agents).

1. Effort increases as the prize of the winner increases, holding the prize of the loser constant, i.e., the internal prize spread increases and the internal-external prize spread also increases. This is from Eq. 9 and 12.
2. Effort increases as the prize of the loser increases, holding the prize of the winner constant (i.e., the internal prize spread decreases, but the internal-external prize spread increases). This is from Eq. 10 and 13.
3. Effort increases as the prize of the loser increases, holding the internal prize spread constant (i.e., the internal-external prize spread increases). This is from Eq. 11 and 14.

The general hypothesis of the LR tournament model is that the agent's effort reacts directly to the prize spread. Therefore, it should not increase when the prize spread remains constant. Our results address the empirical puzzle raised by SW with the presence of an external agent receiving a constant prize. Our model proposes that agents also react to an internal-external prize spread.

Due to the extension in our model, we are able to derive that the best response functions of our agents are also in response to the internal-external prize spread. In consideration of the external agents, the comparative statics show that an increase in the internal prize increases effort. The agents' risk behavior makes it more interesting as we show that the internal prize is directly related to effort whether an agent is risk-neutral or risk-averse.

This is further supported in the literature. Szymanski and Valletti (2004) discuss the benefits of having strong second prizes in inducing every agent to exert maximum effort. Hence, the three agents' efforts are positively affected by an increase in the loser's prize.

Eriksson (1996) shows that a larger prize spread directly affects the performance of firms. In our model, we can generate a general hypothesis that with increasing firm wages, the general labor market performance is affected since external agents are increasing effort to increase the probability of entering the tournament, i.e., of being hired.

The extended model fits labor situations where there is a high possibility for employees to be dismissed or to be demoted. If this is the case, the firm has the capability to replace non-performing agents from a pool of people who want to join the tournament. In our model, the pool of people who can replace the agents inside the tournament are represented by the external players. This usually happens when there is a high level of unemployment, when employers have bargaining power, and when the abilities of the agents inside the firm are homogenous with people who want to join the tournament. To induce positive effort from employees, the principal should set a higher external prize spread, and setting the loser's prize higher than the reservation wage. This is supported by Berkhout, Hartog, and Ophem (2011) on reservation wages and starting wages. They have shown that on the average, in the Netherlands, recent higher education graduates accept wages that are eight percent above their reservation wage. Thus, setting a higher external prize spread would induce positive effort from the external player for her to be included in the tournament.

Considering the Philippine labor market for business processing outsourcing (BPO), the starting wage is high compared to other industries. In the BPO, more laborers are attracted to work for the said industry. Therefore, the increase in the level of competition increases the effort of the general public who want to participate; while the agents who already had temporarily secured the job would be threatened by the increase in participation and the associated high cost of dismissal. Hence, the general reaction of these agents may tend of perform better to at least secure their spot in the industry.

Moreover, the standard economic theory predicts that firms will tend to lower their wages until the supply of workers decreases (Sparks, 1986). Assuming that wages are nonsticky, after an increase in wages, to induce effort and attract a supply of workers, firms will react by lowering wages until the supply of workers decreases. Hence, employees should not impulsively react to an increase in wages because it may lead to crowding in a certain industry, since an increase in wages does not necessarily mean that it would stay that level for a long period of time. Therefore, in the long run, it may not be beneficial for employees to crowd into one firm since wages will diverge back to its normal level.

Hence, the role of an external agent brings important results to the literature and the highlights that a prize spread between an existing player and a potential entrant has some bearing on the resulting effort of all players.

4. CONCLUSIONS

The LR tournament model is standard in the literature but an empirical puzzle was raised by SW. We find that by introducing an external agent, the threat of being replaced changes the expected behavior of the internal agents.

Our results do not necessarily contradict the LR tournament model but, in a way, supports it. Our general result shows that risk-neutral and risk-averse agents' efforts increase with an increase in the internal prize structure. Agents take into consideration the presence of an external player in the market. The reaction comes from the prize spread of the internal-external rather than from the internal prizes. This is due to the higher cost of dismissal faced by internal agents.

Hence, the presence of an external agent does affect the decision of internal agents in terms of their effort and, ultimately, this will affect the performance of firms. This result is quite important in pay decisions taken by a firm.

This conclusion from the paper opens recommendations for academic research. Together with Spark's (1986) theory, this shows that there is a need to do an empirical study on the "cycling" effect of wages and effort for a definite period of time. The backbone of such research should focus on the cycle's length of time following that the assumptions hold true.

Another aspect that could be focused on is the puzzle of risk-behavior. Risk-neutral and risk-averse agents, both having direct response to the prize structure changes, remain unexplained. The possible reason for this could be the following: (a) risk preference may not seem to matter since the internal agents might feel secure in their job after the realization that an increase in effort would secure them a place inside the tournament – this seems to be valid based on the literature regarding career concerns (Gibbons and Murphy, 1992); (b) risk-aversion could also amplify the effect on effort – this is evident as the marginal utility causes a multiplier effect in the comparative statics analysis. However, this needs to be assessed further with data as several terms of the denominator follow a linear trend. We note that utility, as a function of prizes, is concave in nature and causes a different kind of trend. This then causes a problem in the comparison between the comparative statics of risk-neutral and risk-averse agents.

Further research could also test the possible impact of a collusion of agents on effort. From the results, we can hypothesize that collusion among agents increases effort to a certain limit to prohibit one agent from getting a certain rank. In such a situation, the principal does not benefit well from the tournament.

Another concern is whether signals on gender matter in hiring external agents. Agents with differing gender characteristics may have a different response to changes in the prize spread, whether external or internal. Literature on labor economics and gender has demonstrated that gender discrimination may exist significantly among workers, relative to the expected discrimination that may happen between the employer and the employee. Also, internal agents may experience some phenomenon of "glass-ceiling" especially when particular jobs are considered in implementing the open tournament model (Eswaran, 2014).

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