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Application of Parametric Bootstrap in Analyzing the Siler Model on Female Mortality of the Philippines

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Abstract: Finding appropriate models capable of fully explaining Philippine mortality has always been a challenge. The lack of readily available and up to date complete life tables has also added to the difficulty of finding models that fit well. In response to this, the five-parameter Siler mortality model has been applied to abridged female life tables of the Philippines and its regions. The Siler model is given by

$$\mu_t = a_1 e^{-b_1 t} + a_2 + a_3 e^{b_3 t}$$

where a_1 is the magnitude of mortality at the moment of death, b_1 represents the rate at which early mortality declines with age, a_2 gives the rate of death regardless of age, a_3 is the hazard rate at age 0 and b_3 is the rate at which the hazard increases with age.

This model was applied to the female mortality of the Philippines taken from Cabigon, J (2009). *2000 Life Table Estimates for the Philippines, Its Regions and Provinces By Sex.*Quezon City: Commission on Population. Furthermore, to assess the validity of the parameter estimates, the parametric bootstrap technique was used. The results suggest that the Siler model is an appropriate model for the female mortality of the Philippines and its regions due to the relatively low losses observed. The parametric bootstrap also proved useful in analyzing the parameter estimates and their implications.

Key Words: Siler mortality model; abridged life tables, parametric bootstrap

1. INTRODUCTION

Mortality models have been used in the field of actuarial science for a long time to explain the behavior of mortality of a group of lives. Two of the oldest models that are still in use today are the models proposed by Gompertz and Makeham in 1825 and 1860, respectively (Gage and Mode, 1993). The Makeham model, also called the Gompertz-Makeham model, was a revision proposed by William Makeham to the existing model by Benjamin Gompertz. Makeham added a constant hazard to the Gompertz model to account for age-independent mortality. Since then, more models have been developed by adding competing hazards such as the models proposed by Theile in 1871, Siler in 1979, and Heligman and Pollard in 1980.

The Carriere mixture model, which consists of the Gompertz, Inverse Gompertz, Weibull and Inverse Weibull models, was used to fit the 1973-1978 PICM by Rodriguez and Wee (2007). They found that the Carriere model was able to explain the behavior of Philippine mortality better than its individual components. It was concluded that the Carriere model is able to model Philippine mortality well.

Jos (2014) used the eight-parameter Heligman-Pollard Model to model the mortality of the Philippines, Malaysia, Singapore and Thailand. It was shown that the model fits the Singapore mortality and Thailand mortality considerably better than the model's fit to the Philippine mortality and Malaysian mortality. However, the significance of the estimates of the model's parameters were not established for lack of statistical tests used.

These studies done on Philippine mortality all fitted their respective models to mortality gathered from complete life tables which list survivorship in single years. This poses a problem to modelling the mortality of places where only abridged life tables are constructed due to the under-registration of deaths which happens frequently in the Philippines. One of the models which can utilize abridged life tables in explaining the mortality trends of various areas is the Siler model.

The Siler competing hazards model is a simple five-parameter model of mortality which was first used for modelling animal mortality and is now used for modelling human mortality. Siler added a third component that represents the mortality at early age to the Gompertz-Makeham model. The risk of death at this stage often starts out high then declines rapidly. The force of mortality is given by

$$\mu_t = a_1 e^{-b_1 t} + a_2 + a_3 e^{b_3 t}.$$

The first term is the one added to the Gompertz-Makeham model. It is defined by a_1, the magnitude of mortality at the moment of birth, and b_1 the rate at which early mortality declines with age. The second term represents the deaths that occur regardless of age, and is defined by the single parameter a_2, Makeham's constant. The third term is the traditional Gompertz model, where a_3 is the component's hazard at age 0 and b_3 is the rate at which this hazard increases with age (Gage and Dyke, 1986). These three components of the Siler model are referred to as immature, age-independent, and senescent respectively; these are assumed to be competing but non-interacting causes of death or clusters of causes of death (Hoppa, 2002).

Gage and Dyke (1986) fitted the Siler model to 40 abridged life tables from 20 countries across the world and compared the fit to the four-parameter Logit model of Ewbank et al (1983). The results suggest that the Siler model fits the life tables 25% better than the Logit model. In obtaining estimates for the parameters, they used a non-linear curve fitting package called MODFIT that employs a Marquardt-Levenberg algorithm which uses iterations of least squares to fit functions. They then used a Monte Carlo simulation procedure to check for the success of the model.

Rodriguez and Wee (2007) also used a statistical procedure called parametric bootstrap which suggested the statistical significance of the fit of the Carriere model to Philippine mortality. This procedure is similar to the Monte Carlo simulation done by Gage and Dyke (1986) since bootstrap is considered a specific case of a Monte Carlo method for it also relies on random resampling (www.astrostatistics.psu.edu). Ostaszewski and Rempala (2000) suggested the use of bootstrap to provide estimates for the standard error and bias of a statistic. It was also shown that estimates computed using the bootstrap method are better approximations to the model parameters compared to traditional normal approximation.

This study will follow the procedures of Rodriguez and Wee (2007) but instead of the Carriere mixture model, the proposed model is the Siler model. It will be fitted to abridged life tables for female mortality of the Philippines and its regions. This is done to allow for more conservative results since the Siler model assumes homogeneity among individuals (Wood, Holman, O'Connor and Ferrell, 2002) and there is lower variability in females, as compared to males (Geher, 2014).

2. METHODOLOGY

2.1 Data

The life tables used in this study may be found in a monograph by Cabigon (2009). The mortality tables, being abridged, were divided into age intervals which start with ages less than 1, then ages 1-4, followed by 5 year age intervals up until 95-99.

2.2 Analysis

2.2.1 Parameter Estimation

The Solver Add-in function of Microsoft Excel was used to obtain estimates of the parameters of the Siler model. Initial estimates used by Gage and Dyke (1986) were also utilized in this study considering how the Siler model was shown to have been a good fit to the mortality experience of both developed and third-world countries. These estimates were derived from fitting the Siler model to Level 15 of the Coale and Demeny West life tables. Gage and Dyke (1986) chose this level considering how its values represent the middle range of human mortality experience. The initial values of a_1, b_1, a_2, a_3 and b_3 are 0.215, 1.44, 0.0032, 0.000192, and 0.0818 respectively.

Apart from initial estimates, another requirement of the Solver Add-in would be a loss function which it shall minimize in order to arrive at better estimates for the particular mortality experience being studied. The loss function used is

$$L = \sum_{x=0}^{95} \left(1 - \frac{{}_n\hat{q}_x}{{}_nq_x}\right)^2,$$

$$where\ x = 0,1,5,10,15,\ldots,90,95$$

$$\text{and } n = \begin{cases} 1 \text{ when } x = 0, \\ 4 \text{ when } x = 1, \\ 5 \text{ when } x = 5,10,\ldots,95. \end{cases}$$

2.2.2 Parametric Bootstrapping

Once the estimates are found to produce a good fit to the data being examined, it is appropriate to evaluate their precision and accuracy with the help of parametric bootstrapping. The first step was to generate 20,000 random ages-at-death which followed the Siler distribution by applying the probability integral transform. However, due to the complexity of solving for the inverse distribution function of the Siler model, the bisection method was used to solve for the ages-at-death instead. A pseudo life table was then constructed by simply using the 20,000 ages-at-death to calculate ${}_nd_x$ and l_x for each age interval. The procedure of parameter estimation stated in Section 3.2.1 was afterwards performed to the pseudo life table in order to obtain the pseudo parameter estimates for this particular life table. Note that the initial estimates used this time would be the estimates which were used to generate the 20,000 ages-at-death.

The entire method of generating pseudo parameter estimates was then repeated 1000 times. Afterwards, the bootstrap approximation of standard error and bias may be obtained. In order to facilitate the bootstrapping process, the researchers constructed a tool comprised of formulas and macros in Microsoft Excel which would automatically calculate the ages-at-death and produce the bootstrap estimates.

3. RESULTS AND DISCUSSION

The parameters of the Siler model were estimated to give the best fit to the data by minimizing the loss function using the Solver Add-In of Microsoft Excel. Table 1 shows the estimates for the parameters of the Siler model for every life table it was fitted to with their respective minimum loss.

Table 1. Parameter Estimates of the Siler Model

Life Table	a_1	b_1	a_2	a_3	b_3	Loss
Philippines	0.079171	1.354543	0.000683	0.000048	0.090893	0.092666
NCR	0.063709	1.543408	0.000379	0.000041	0.093723	0.076203
CAR	0.072009	1.382116	0.000338	0.000127	0.079452	0.485414
Region I	0.075469	1.402625	0.000589	0.000062	0.085400	0.169356
Region II	0.073581	1.443287	0.000516	0.000110	0.081226	0.541795
Region III	0.071404	1.478657	0.000597	0.000039	0.092674	0.333896
Region IV A	0.071339	1.447899	0.000540	0.000041	0.091720	0.241271
Region IV-B	0.088257	1.293495	0.000886	0.000084	0.086655	0.241640
Region V	0.080005	1.371643	0.000739	0.000049	0.090368	0.333221
Region VI	0.081936	1.329703	0.000758	0.000044	0.091621	0.346977
Region VII	0.075451	1.402456	0.000600	0.000053	0.088733	0.298904
Region VIII	0.089856	1.277604	0.000933	0.000082	0.085767	0.288394
Region IX	0.085637	1.276833	0.000785	0.000063	0.087856	0.581294
Region X	0.077523	1.201199	0.000392	0.000081	0.085944	0.685544
Region XI	0.082453	1.294478	0.000685	0.000064	0.088560	0.240494
Region XII	0.083118	1.336802	0.000732	0.000096	0.084135	0.302889
Region XIII	0.090329	1.221368	0.000878	0.000070	0.089470	0.297814
ARMM	0.109475	1.132261	0.000631	0.000707	0.063086	0.361780

The model for Philippine female mortality resulted in a loss of 0.092666, the second lowest minimum loss among the models. The model with the lowest minimized loss is the model for the female mortality in the National Capital Region (NCR) with a minimum loss value of 0.076203. While Northern Mindanao (Region X) has the highest minimum loss among the models with a value of 0.685544. Rodriguez and Wee (2007) considered the models they produced, with minimum losses of 1.454 and 0.773, to be appropriate models for Philippine mortality. In comparison, all 18 life tables resulted in minimum losses less than 0.773. Comparing to the loss of the Heligman-Pollard model fitted to Philippine mortality (1.39881719) by Jos (2014), again, all life tables were superior in terms of minimum loss. Also, the losses of the Philippines and NCR are less than the loss for the models of Singapore (0.23722477 for male and 0.22898870 for female) and Thailand (0.20491924 for male and 0.15031391 for female) which were considered the best fit Heligman-Pollard models.

To test the unbiasedness of the bootstrap parameter estimates, the bias and relative bias for the estimates of each parameter were computed. The relative bias is the quotient of the parameter estimate value given in Table 1, divided by the computed bias presented in Table 2. Table 2 also reports the relative bias for each estimate.

Hidiroglou, Morry, Dagum, Rao, Sarndal (1984) stated that whenever the relative bias is less than five percent (5%), the bias is negligible. Since the resulting bias for the parameter estimates have negative values, the bias will be considered negligible if the relative bias is between -5% and +5%. Following this criterion, all parameter estimates are found to have negligible biases. Thus, the bootstrap parameter estimates are taken to have unbiased estimators for their respective parameters.

Parametric bootstrapping was used to approximate the standard errors and expectations of the parameter estimates obtained in the study. With these values, coefficients of variation were then computed in order to further assess the precision of the estimates. These values are shown in Table 3.

All the parameter estimates of the first component and the rate of increase of the hazard of senescence were found to have good precision. Some estimates of the constant hazard and initial hazard of senescence on the other hand were found to have coefficients of variation greater than 0.10, which may suggest a lack in precision. The estimate of the constant hazard of the ARMM is the least precise, followed by that of the Cordillera Administrative Region, Region X, Cagayan Valley Region (Region II), and NCR. Coefficients of variations of Ilocos Region (Region I), IV-A, and Western Visayas (Region VI) on the other hand were found to have slightly exceeded 0.10. Lastly, a majority of the initial hazard of senescence estimates had coefficients of variations close to 0.10 which may suggest that prudence still be exercised in using these estimates.

The immaturity component of mortality represents a child's improving ability to adapt to his environment across time. A child whose immune and physiological system are able to adjust to environmental challenges is referred to as someone who has overcome immaturity. In this study, it is of interest to determine the proportion of individuals that are able to "overcome immaturity" or in simpler terms, survive the immaturity hazard. Figure 1 below presents the proportion of individuals that are expected to survive the immaturity component in their respective areas.

It may be observed that on the average, the ARMM has the least proportion surviving the immaturity component (0.928711) whereas the NCR has the highest (0.959562). Central Visayas (Region VII), along with the regions in Luzon except for MIMAROPA (Region IV-B), are shown to expect a larger number of individuals overcoming immaturity as opposed to the rest of the regions in the country. Lastly, the Bicol Region (Region V) is expected to roughly have the same proportion of individuals surviving the immaturity hazard with that of the Philippines as a whole.

The survivorship curves of the age-independent and senescent component of the Siler model, denoted by $l_2(t)$ and $l_3(t)$ respectively, are illustrated in Figure 2 below. The estimates used in generating the curves were that of the Philippines.

The Philippine data shows that on the average, 94.3% of newborns in the Philippines are able to adapt to their environment and survive the immaturity component of mortality with an initial hazard of 0.079171 that declines exponentially at a rate of 1.353543. This decline is shown in Figure 2 by the great decrease of the hazard between ages 0 and 1. The hazard decreases less abruptly between ages 1 and 5, and after age 5, the hazard remains unchanged for the rest of the individuals' lives. This suggests that after the first year of life, the risk of dying due to the first component greatly decreases, and it may be said that the immaturity hazard is most felt during the first year of life. The survivors of the immaturity component then undergo a constant hazard of 0.000683 which is quite small. This may imply that a majority of them are expected to survive external causes of death that are independent of age, as seen in Figure 2. Lastly, the initial hazard of senescence, 0.000048, is shown to increase exponentially by a factor of 0.090893 as the individuals grow older. This may be observed in Figure 2, where the expected number of survivors of the hazard caused by aging begins to rapidly decline when the individuals are around the age of 40.

Table 2 . Bias and relative bias for bootstrap parameter estimates

Life Table		a_1	b_1	a_2	a_3	b_3
Philippines	Bias	-0.0000341	-0.0012092	-0.0000173	0.0000001	0.0000497
	Relative Bias	-0.0431%	-0.0893%	-2.5378%	0.1873%	0.0547%
NCR	Bias	-0.0000148	-0.0035175	-0.0000658	-0.0000001	0.0001173
	Relative Bias	-3.9051%	-0.2279%	-0.1033%	-0.2637%	0.1251%
CAR	Bias	-0.0000552	-0.0025134	-0.0000142	-0.0000003	0.0000885
	Relative Bias	-0.0766%	-0.1819%	-4.2088%	-0.2459%	0.1114%
Region I	Bias	-0.0000379	-0.0025117	-0.0000173	0.0000002	0.0000277
	Relative Bias	-0.0502%	-0.1791%	-2.9314%	0.3008%	0.0324%
Region II	Bias	-0.0000329	-0.0024434	-0.0000155	0.0000000	0.0000604
	Relative Bias	-0.0447%	-0.1693%	-2.9974%	-0.0383%	0.0744%
Region III	Bias	-0.0000458	-0.0029463	-0.0000165	0.0000000	0.0000581
	Relative Bias	-0.0641%	-0.1993%	-2.7583%	0.1241%	0.0627%
Region IV-A	Bias	-0.0000785	-0.0036690	-0.0000165	0.0000000	0.0000572
	Relative Bias	-0.1101%	-0.2534%	-3.0536%	0.1189%	0.0623%
Region IV-B	Bias	-0.0000124	-0.0007072	-0.0000134	-0.0000004	0.0001062
	Relative Bias	-0.0140%	-0.0547%	-1.5104%	-0.4390%	0.1226%
Region V	Bias	0.0000382	-0.0010388	-0.0000171	0.0000001	0.0000391
	Relative Bias	0.0477%	-0.0757%	-2.3198%	0.2414%	0.0433%
Region VI	Bias	-0.0010064	0.0101940	-0.0000298	-0.0000004	0.0001528
	Relative Bias	-1.2283%	0.7666%	-3.9322%	-0.8636%	0.1667%
Region VII	Bias	-0.0000401	-0.0025918	-0.0000176	0.0000002	0.0000197
	Relative Bias	-0.0531%	-0.1848%	-2.9248%	0.3786%	0.0222%
Region VIII	Bias	-0.0000049	-0.0005370	-0.0000133	-0.0000004	0.0001137
	Relative Bias	-0.0054%	-0.0420%	-1.4210%	-0.4772%	0.1326%
Region IX	Bias	-0.0000104	-0.0013131	-0.0000160	0.0000000	0.0000597
	Relative Bias	-0.0121%	-0.1028%	-2.0327%	-0.0368%	0.0679%
Region X	Bias	-0.0002824	-0.0042161	-0.0000150	0.0000001	0.0000491
	Relative Bias	-0.3643%	-0.3510%	-3.8129%	0.1241%	0.0572%
Region XI	Bias	-0.0000152	-0.0009647	-0.0000149	-0.0000001	0.0000730
	Relative Bias	-0.0184%	-0.0745%	-2.1730%	-0.0948%	0.0825%
Region XII	Bias	0.0000361	-0.0002759	-0.0000143	-0.0000002	0.0000722
	Relative Bias	0.0434%	-0.0206%	-1.9535%	-0.1610%	0.0858%
Region XIII	Bias	-0.0000796	-0.0018026	-0.0000140	-0.0000003	0.0001164
	Relative Bias	-0.0881%	-0.1476%	-1.5901%	-0.4677%	0.1301%
ARMM	Bias	-0.0001478	-0.0026445	-0.0000129	-0.0000020	0.0000647
	Relative Bias	-0.1350%	-0.2336%	-2.0381%	-0.2840%	0.1026%

Table 3. Coefficient of variation for bootstrap parameter estimates.

Life Table	a_1	b_1	a_2	a_3	b_3
Philippines	0.0510162	0.0422850	0.0881862	0.1133182	0.0165657
NCR	0.0580103	0.0492121	0.1297489	0.1155938	0.0167331
CAR	0.0528338	0.0427267	0.2064436	0.0966621	0.0163253
Region I	0.0530179	0.0432853	0.1047329	0.1095272	0.0168303
Region II	0.0528738	0.0437247	0.1361597	0.0970169	0.0159650
Region III	0.0537532	0.0451145	0.0919689	0.1124633	0.0160946
Region IV-A	0.0536353	0.0445073	0.1000027	0.1120642	0.0161892
Region IV-B	0.0473500	0.0422897	0.0792032	0.0914795	0.0142319
Region V	0.0512188	0.0429155	0.0824723	0.1110483	0.0162919
Region VI	0.0612190	0.0537078	0.1007222	0.1126654	0.0160380
Region VII	0.0529759	0.0432435	0.0999155	0.1101730	0.0164587
Region VIII	0.0468017	0.0422814	0.0766132	0.0937830	0.0145651
Region IX	0.0489045	0.0425163	0.0807390	0.0983222	0.0149802
Region X	0.0543701	0.0463681	0.1634788	0.1073043	0.0170255
Region XI	0.0501522	0.0430702	0.0914583	0.1022145	0.0155713
Region XII	0.0501175	0.0431432	0.0963258	0.0953784	0.0153349
Region XIII	0.0462774	0.0412694	0.0780356	0.0955774	0.0144695
ARMM	0.0395650	0.0352880	0.2538983	0.0691018	0.0152250

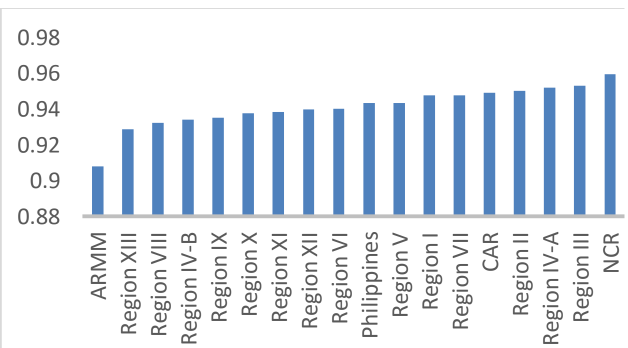

Figure 1. Proportion of individuals expected to survive the immaturity component

The hazard function of the Philippines is expressed as

$$\mu_x = 0.09171e^{-1.354543x} + 0.000683 + 0.000048e^{0.090893x}$$

Its curve is illustrated in Figure 3.

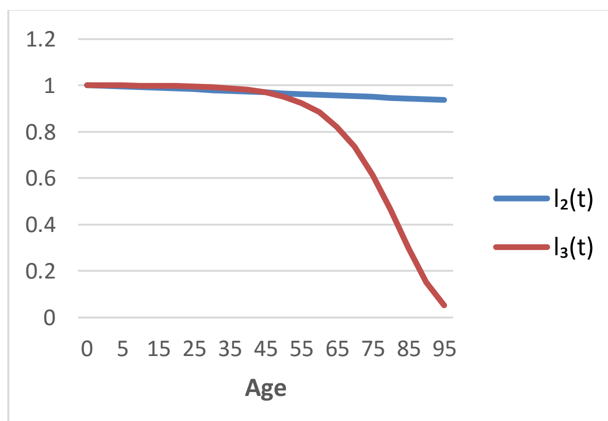

Figure 2: Survival curves of the age-independent and senescence component.

It may be observed that the immaturity hazard rapidly declines until age 5 which is the expected age an individual from the Philippines shall be able to fully adapt to his environment physiologically. The child's immunity is also expected to have been developed by this time. Once

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an individual reaches age 40 however, it is expected that the hazard due to aging shall begin to be felt. The same pattern may be observed in the survival curves in Figure 4 where the decline of the curve begins to be slower at age 5, until the individual reaches age 40. It is then at this age that the individual's chances of survival begins to rapidly decline across time.

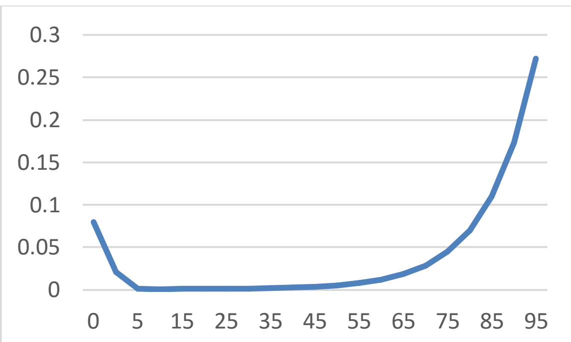

Figure 3. The hazard for females in the Philippines across the ages

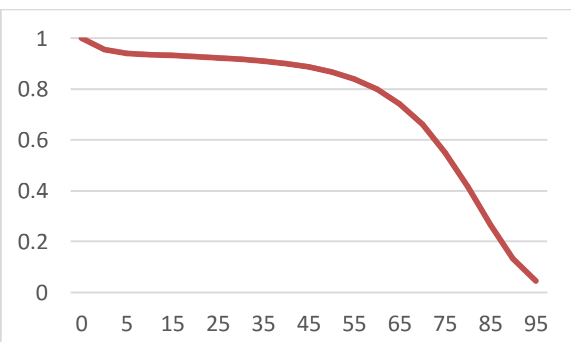

Figure 4. The proportion of survival of females in the Philippines across ages

4. CONCLUSIONS

The Siler model of mortality is a good fit to the abridged female life tables of the Philippines and its regions regardless of the varying levels of mortality the different areas may experience. Parametric bootstrapping has played a significant role in assessing the validity of the parameter estimates and understanding their implications. It has also been useful in identifying significant differences in the mortality experiences of both the Philippines and its regions.

5. ACKNOWLEDGMENT

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