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Notions of Domination for Some Classes of Graphs

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Abstract: If $G = (V, E)$ is a finite simple connected graph, a subset S of V is said to be a *dominating set* of the graph G if every vertex of G is either in S or is adjacent to at least one element of S. The minimum cardinality of a dominating set of G is called the *domination number* of G. In the present study, we consider variations of the concept of domination in a graph.

A subset S of V is said to be a *triple connected dominating set* of G if S is a dominating set and any set of three vertices in the subgraph $\langle S\rangle$ induced by S lie in a common path. In this case, we say that the graph is triple connected. The minimum cardinality of a triple connected dominating set is called the *triple connected domination number* of G and is denoted by $\gamma_{tc}(G)$. On the other hand, a subset S of V of a nontrivial connected graph G is said to be a *triple connected complementary tree dominating set*, if S is a triple connected dominating set and the induced subgraph $\langle V - S\rangle$ is a tree. The minimum cardinality of a triple connected complementary tree dominating set is called the *triple connected complementary tree domination number* of G and is denoted by $\gamma_{tct}(G)$.

In the papers entitled "Triple Connected Domination Number of a Graph" and "Triple Connected Complementary Tree Domination Number of a Graph" ,published in 2012 and 2013, respectively, by G. Mahadevan, S. Avadayappan, J. Paulraj Joseph, J. and T.Subramanian, exact values of both $\gamma_{tc}(G)$ and $\gamma_{tct}(G)$ for some classes and special graphs and bounds for these parameters for arbitrary finite simple connected graphs were determined. Their relationship with other graph theoretical parameters are also investigated. However, a number of errors in the results presented in these papers were noted. The corrected results are presented in this study.

Keywords: dominating set; domination number; triple connected graph; triple connected domination number; triple connected complementary tree domination number; graph parameter

1. INTRODUCTION

By a graph, we mean a pair $G = (V, E)$ where V is a non-empty set whose elements are called the *vertices* of G, while E is a possibly empty collection of unordered pairs of elements of V. The elements of E are called the *edges* of G. The cardinalities of V and E are called the *order* and the *size*, respectively, of the graph G. In this study, all graphs that will be considered are finite simple connected graphs.

A graph G is said to be *connected* if each pair of vertices in G are joined by a path. A connected graph which does not contain any cycle is a *tree*. If any set of three vertices in G are joined by a path, then G is said to be *triple connected*. A *vertex cut* of a connected graph is a subset H of V such that the graph $\langle V - H\rangle$ is disconnected. A maximal connected subgraph of a graph G is called a *component* of G.

If v is a vertex of a graph G, then v *dominates* itself and every vertex adjacent to v. If S is a non-empty subset of the vertex set V, then S is a *dominating set* of G if every vertex in G is dominated by at least one vertex in S. If the induced subgraph $\langle S\rangle$ is triple connected, then S is called a *triple connected dominating set* of G. In addition, if the induced subgraph $\langle V - S\rangle$ is a tree, then S is called a *triple connected complementary tree dominating set* of G. The cardinalities of the smallest triple connected dominating set and the the smallest triple connected complementary tree dominating set of a graph G are called the *triple connected domination number* and *triple connected complementary tree domination number*, respectively, of a graph G.

In this study, we determine conditions for the existence of triple connected or triple connected complementary tree dominating sets in a connected graph, bounds for the triple connected domination number and the triple connected complementary tree domination number of a graph, the relationship of either of these two domination parameters with other graph parameters, and the exact values of these domination parameters for some common classes of graphs and other special graphs.

The concept of the connected domination number of a graph was introduced in the paper by Sampathkumar and Walikar [3]. In this study, the value of the connected domination number of some common classes of graphs such as cycles, complete graphs, complete bipartite graphs, trees, and the join of a complete graph with a connected graph, were determined. In the paper by Muthammai and Bhanumathi [4], the concept of the complementary tree domination number of a graph G was discussed. This graph parameter, denoted by $\gamma_{ct}(G)$, was identified for the following classes of graphs: paths, cycles, complete graphs, star graphs, wheels, and the corona of two graphs. Bounds for this parameter for an arbitrary connected graph of order n were also determined. The conditions used to define the graph parameters presented in these two articles were extended to define the graph parameters that were included in the present study.

Several results included in the articles that were studied were presented without proof. In the course of providing proofs for these results, the researchers were able to identify counterexamples which showed that the stated results were incorrect. The corrected results are presented in the present paper.

2. THE TRIPLE CONNECTED DOMINATION NUMBER OF A GRAPH

Definition 1. Let G be a graph. Then

- A subset S of V of a nontrivial connected graph G is said to be a *triple connected dominating set* of G, if the following conditions hold:
  (a) The set S is a dominating set.
  (b) The induced subgraph $\langle S \rangle$ is triple connected.
- The *triple connected domination number* of G, denoted by $\gamma_{tc}(G)$, is the minimum cardinality of a triple connected dominating set of G.

Remark. If the triple connected dominating set of a connected graph exists, then the inequality $\gamma_{tc}(G) \geq 3$ should be satisfied.

Example 1. For the graph G shown in Figure 1, let $S = \{x_2, x_4, x_5\}$. It can be verified that S is a dominating set of G, and that $\langle S \rangle$ is triple connected. Thus, $S = \{x_2, x_4, x_5\}$ is a triple connected dominating set of minimum cardinality. Hence, $\gamma_{tc}(G) = |S| = 3$.

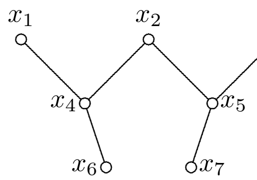

Fig. 1: The graph G with $\gamma_{tc}(G) = 3$.

The results that follow provide exact values for $\gamma_{tc}(G)$ for some common classes of graphs. These were stated without proof in Mahadevan et. al [].

Proposition 1. *For any path P_n of order n, where $n \geq 3$, we have*

$$\gamma_{tc}(P_n) = \begin{cases} 3, & \text{if } n < 6, \\ n-2, & \text{if } n \geq 6. \end{cases}$$

Proof. Let P_n be a path of order n, where $n \geq 3$ and let S be a triple connected dominating set of P_n.

Case 1: $n < 6$

Since the existence of a triple connected dominating set of a connected graph G implies that $\gamma_{tc}(G) \geq 3$, then we only need to show the existence of such a set S. For P_4, take any three consecutive vertices to be in S. For P_5, take all vertices to be in S except the pendant vertices. Therefore, S is a dominating set and $\langle S \rangle \cong P_3$ which is triple connected. Hence, $\gamma_{tc}(P_n) = |S| = 3$, if $n < 6$.

Case 2: $n \geq 6$

The only connected subgraphs of any path are paths, which are triple connected. If $|V - S| = 2$, there are two subcases to consider:

Subcase 1: If $V - S$ contains an end vertex and its neighbor, the end vertex is not dominated by any vertex in S, and hence S is not a dominating set.

Subcase 2: If $V - S$ contains the two end vertices, then S is a dominating set and $\langle S \rangle \cong P_{n-2}$. Hence S is a triple connected dominating set.

If $|V - S| > 2$, at least one vertex is not dominated by a vertex in S. Therefore, $\gamma_{tc}(P_n) = n - 2$, if $n \geq 6$. □

Proposition 2. *For any cycle C_n of order n, where $n \geq 3$, we have*

$$\gamma_{tc}(C_n) = \begin{cases} 3, & \text{if } n < 6, \\ n-2, & \text{if } n \geq 6. \end{cases}$$

Proof. Let C_n be a cycle of order n, where $n \geq 3$ and let S be a triple connected dominating set of C_n.

Case 1: $n < 6$

If $n < 6$, then $\langle S \rangle$ must be isomorphic to P_3, which is triple connected. Thus, $\gamma_{tc}(C_n) = |S| = 3$.

Case 2: $n \geq 6$

The only connected subgraphs of any cycle are paths, which are triple connected. If $|V - S| = 2$, then S is a dominating set and $\langle S \rangle \cong P_{n-2}$. Hence, S is a triple connected dominating set. If $|V - S| > 2$, then take any three consecutive vertices $a, b, c \in V - S$. The vertex b is not dominated by any vertex in S since the only neighbors of b are a and c. Therefore, exactly two vertices can be removed from C_n to obtain a triple connected dominating set of minimum cardinality. □

Proposition 3. *For any complete graph K_n of order n, where $n \geq 3$, we have $\gamma_{tc}(K_n) = 3$.*

Proof. Let K_n be a complete graph of order $n \geq 3$. If S contains only one vertex, then S is already a dominating set, since the vertices in a complete graph are pairwise adjacent. For a triple connected dominating set to exist, we only need to add two more vertices to S. Hence, $\langle S \rangle \cong C_3$ which is triple connected. Therefore, $\gamma_{tc}(K_n) = |S| = 3$. □

Proposition 4. *For any complete bipartite graph $K_{p,q}$ of order n, where $p+q = n$, $p, q \geq 2$, and $n \geq 4$, we have $\gamma_{tc}(K_{p,q}) = 3$.*

Proof. Let $K_{p,q}$ be a complete bipartite graph of order n, where $p + q = n$, $p, q \geq 2$, and $n \geq 4$. Let P and Q be the partite sets of $K_{p,q}$ with $|P| = p$ and $|Q| = q$. Let $x_1, x_2 \in P$ and let $y_1 \in Q$. Since any vertex in P dominates all the vertices in Q and any vertex in Q dominates all the vertices in P, the set $S = \{x_1, y_1\}$ is a dominating set. However, $\gamma_{tc}(G) \geq 3$ for any connected graph G. Hence, we add one more vertex, say x_i or y_j in S where $2 \leq i \leq p$ and $2 \leq j \leq q$. Therefore, $\langle S \rangle \cong P_3$ and S is a triple connected dominating set. Therefore, $\gamma_{tc}(K_{p,q}) = |S| = 3$. □

Proposition 5. *For any star S_n of order $n + 1$, where $n \geq 4$, we have $\gamma_{tc}(S_n) = 3$.*

Proof. Let F_n be a star of order $n+1$, where $n \geq 3$. Let x_0 be the central vertex of S_n, and let x_i, x_j be any two of the outer vertices of S_n. Then, the set $S = \{x_0, x_i, x_j\}$ is a triple connected dominating set of S_n of minimum cardinality. Hence, $\gamma_{tc}(S_n) = |S| = 3$. □

Proposition 6. *For any wheel W_n of order $n+1$, where $n \geq 3$, we have $\gamma_{tc}(W_n) = 3$.*

Proof. Let W_n be a wheel graph of order $n + 1$, where $n \geq 4$. The singleton set consisting of the central vertex is already a dominating set since the central vertex is adjacent to all other vertices in the wheel. Thus, we only need to add two more vertices to S for a triple connected dominating set to exist. Hence, $\langle S \rangle \cong P_3$ or C_3, both of which are triple connected. Therefore, $\gamma_{tc}(W_n) = |S| = 3$. □

Proposition 7. *For any helm graph H_n of order $2n+1$, where $n \geq 3$, we have $\gamma_{tc}(H_n) = n$.*

Proof. Let H_n be a helm graph of order $2n + 1$, where $n \geq 3$. The set consisting of all the support vertices in H_n is already a dominating set. In addition, all of these support vertices must be in S in order for $\langle S \rangle$ to be a triple connected. Since $\langle S \rangle \cong C_n$ for any such set S, S is a triple connected dominating set, and we have $\gamma_{tc}(H_n) = |S| = n$. □

Proposition 8. *For any bistar $B_{p,q}$ of order n, where $p+q+2 = n$, $p, q \geq 1$, and $n \geq 4$, we have $\gamma_{tc}(B_{p,q}) = 3$.*

Proof. Let $B_{p,q}$ be a bistar of order n, where $p+q+2 = n$, where $p, q \geq 1$, and $n \geq 4$. If the central vertices of the bistar are in S, then S is already a dominating set. Thus, to obtain a triple connected dominating set, we only need to add one more vertex to S. Hence, $\langle S \rangle \cong P_3$ and $\gamma_{tc}(B_{p,q}) = |S| = 3$. □

Proposition 9. *For any fan F_n of order $n + 1$, where $n \geq 4$, we have $\gamma_{tc}(F_n) = 3$.*

Proof. Let F_n be a fan of order $n+1$, where $n \geq 4$ and let x_0 be the central vertex of F_n. Let x_i, x_j be any two outer vertices of F_n. Then, the set $S = \{x_0, x_i, x_j\}$ is a triple connected dominating set of F_n of minimum cardinality, and therefore $\gamma_{tc}(F_n) = |S| = 3$. □

Proposition 10. *For any Friendship graph $\mathbb{F}_n$ of order $2n+1$, where $n \geq 2$, we have $\gamma_{tc}(\mathbb{F}_n) = 3$.*

Proof. Let $\mathbb{F}_n$ be a Friendship graph of order $2n+1$, where $n \geq 2$ and let x_0 be the central vertex of $\mathbb{F}_n$. The set consisting of the central vertex is already a dominating set since it is adjacent to all other vertices of $\mathbb{F}_n$. Let x_i, x_j be any two vertices of $\mathbb{F}_n$. Then, the set $S = \{x_0, x_i, x_j\}$ is a triple connected dominating set of F_n of minimum cardinality, and therefore $\gamma_{tc}(\mathbb{F}_n) = |S| = 3$. □

3. THE TRIPLE CONNECTED COMPLEMENTARY TREE DOMINATION NUMBER OF A GRAPH

In this section, results on the triple connected complementary tree domination number of a graph G will be discussed. Specifically, the corrected results from the paper by Mahadevan et.al. will be presented.

Definition 2. Let G be a graph. Then

- A subset S of V of a nontrivial connected graph G is said to be a triple connected complementary tree dominating set of G, if the following conditions hold:
  - (a) S is a triple connected dominating set.
  - (b) The induced subgraph $\langle V - S \rangle$ is a tree.
- The triple connected complementary tree domination number of G, denoted by $\gamma_{tct}(G)$, is the minimum cardinality of a triple connected complementary tree dominating set of G.

Remark. If the triple connected complementary tree dominating set of a connected graph exists, then the inequality $\gamma_{tct}(G) \geq 3$ should be satisfied.

Example 2. For the graph G shown in Figure 2, let S consist of the vertices x_1, x_2, and x_3. It can be verified that S is a triple connected dominating set of G, and $\langle V - S \rangle \cong P_4$, which is a tree. Thus, $S = \{x_1, x_2, x_3\}$ is a triple connected complementary tree dominating set of G of minimum cardinality. Hence, $\gamma_{tct}(G) = |S| = 3$.

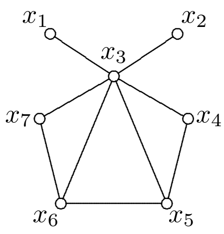

Fig. 2: The graph G with $\gamma_{tct}(G) = 3$.

The first result gives bounds for the triple connected complementary tree domination number of a graph.

Theorem 1. *For any graph G of order $n \geq 5$, we have $3 \leq \gamma_{tct}(G) \leq n-1$ and the bounds are sharp.*

Proof. Let S be a minimum triple connected complementary tree dominating set of G, so that $|S| = \gamma_{tct}(G)$. From the definition, we know that $\gamma_{tct}(G) \geq 3$. Suppose $|S| = n$. Then $V - S = \phi$ which implies that $\langle V - S \rangle$ does not exist. Hence S is not a triple connected complementary tree dominating set and $3 \leq \gamma_{tct}(G) \leq n-1$. To show that the bounds are sharp, we have $\gamma_{tct}(C_n) = 3$ and $\gamma_{tct}(P_n) = n-1$. □

The next set of results give the corrected exact value of $\gamma_{tct}(G)$ for some common classes of graphs:

Proposition 11. *For any complete bipartite graph $K_{p,q}$ of order n, where $n \geq 4$, $p \leq q$, and $p+q = n$, we have*

$$\gamma_{tct}(K_{p,q}) = \begin{cases} 3, & p = 2 \text{ or } 3 \\ p+1, & p > 3 \end{cases}$$

Proof. Let $K_{p,q}$ be a complete bipartite graph of order n, where $n \geq 4$, with $p \leq q$ and $p+q = n$. Let p and q be the number of vertices in the partite sets P and Q of $K_{p,q}$, respectively.

Case 1: $p = 2$ or 3

Subcase 1: $p = 3$

Let $p_1, p_2 \in P$ and $q_1 \in Q$. Let $S = \{p_1, p_2, q_1\}$. Then $\langle S \rangle \cong P_3$ and hence, S is a triple connected dominating set and $\langle V - S \rangle \cong S_{q-1}$

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which is a tree. This shows that S is a triple connected complementary tree dominating set of $K_{p,q}$. Since the minimum cardinality of a triple connected complementary tree dominating set is 3, we have $\gamma_{tct}(K_{p,q}) = |S| = 3$.

Subcase 2: $p = 2$

Let $p_1 \in P$ and $q_1, q_2 \in Q$. Let $S = \{p_1, q_1, q_2\}$. Then $\langle S \rangle \cong P_3$ and hence, S is a triple connected dominating set. Moreover, $\langle V - S \rangle \cong K_{1,q-2}$ which is a tree. This shows that S is a triple connected complementary tree dominating set of $K_{p,q}$. We have $\gamma_{tct}(K_{p,q}) = 3$.

Case 2: $p > 3$

The only induced subgraphs of $K_{p,q}$ which are trees are P_1, P_2, P_3, and a star. Since we want to maximize $|V - S|$, the induced subgraph that contains the most number of vertices is a star. Let the root vertex of the star be denoted by u. Since $p \leq q$, we choose $u \in P$. All the remaining vertices in $\langle V - S \rangle$ should come from Q, for otherwise $\langle V - S \rangle$ contains a cycle. Also, we only need to include exactly two vertices from Q in S, in order for $\langle S \rangle$ to be triple connected. Hence, S is a triple connected dominating set. Thus, we can maximize $|V - S|$ and minimize S by including all except vertices of Q in $V - S$, and we have $\langle V - S \rangle \cong S_{q-2}$ which is a tree and hence, S is a triple connected complementary tree dominating set. Therefore, $\gamma_{tct}(K_{p,q}) \leq (p - 1) + 2 = p + 1$.

Suppose $S \subseteq V(K_{p,q})$ such that $|S| = p$. Suppose $S \subseteq V(K_{p,q})$ such that $|S| = p$. The elements of S can come from either P or Q. If k represents the number of vertices in S that come from P, then $p - k$ is the number of vertices in S that come from Q. It can be verified that

- If $k = 1$ or $k = p - 1$, then $\langle S \rangle$ is not triple connected.
- If $k \in \{!2, 3, \ldots, (p - 2)\}$, then $\langle V - S \rangle$ is a cycle.

In each case, one can see that S is not a triple connected complementary tree dominating set. Thus, $\gamma_{tct}(K_{p,q}) = p + 1$. □

Proposition 12. *For any wheel W_n of order $n+1$, where $n \geq 3$, we have $\gamma_{tct}(W_n) = 3$.*

Proof. Let W_n be a wheel of order $n + 1$ where $n \geq 3$. We want to form a triple connected complementary tree dominating set S in W_n which is of minimum cardinality so the central vertex x_0 must be in S. If we include in S two adjacent outer vertices u and v, then $S = \{x_0, u, v\}$ is a triple connected dominating set. Moreover, $\langle V - S \rangle \cong P_{n-2}$ and hence a tree. This shows that S is a triple connected complementary tree dominating set of minimum cardinality, and we have $\gamma_{tct}(W_n) = 3$. □

Proposition 13. *For any fan F_n of order $n + 1$, where $n \geq 4$, we have $\gamma_{tct}(F_n) = 3$.*

Proof. Let F_n be a fan of order $n + 1$ where $n \geq 4$. Let S be a triple complementary tree dominating set of F_n so that $\langle V - S \rangle$ is a tree. Then, the set $S = \{x_0, x_i, x_j\}$, where x_0 is the central vertex of F_n, x_i, x_j are two vertices such that $\deg(x_i) = 2$ and $\deg(x_j) = 2$, is a triple connected complementary tree dominating set of minimum cardinality, so that $\gamma_{tct}(F_n) = |S| = 3$. □

Proposition 14. *For any friendship graph $\mathbb{F}_n$ of order $2n + 1$, where $n \geq 2$, we have $\gamma_{tct}(\mathbb{F}_n) = n - 2$.*

Proof. Let $\mathbb{F}_n$ be a friendship graph of order $2n+1$ where $n \geq 2$. The set $S = \{x_i, x_j | i, j \neq 0\}$, where x_i and x_j are adjacent, is a triple connected complementary tree dominating set such that $\gamma_{tct}(\mathbb{F}_n) \leq n-2$. If $|V - S| > 2$, Then there exist two vertices which belong to different triangles of $\mathbb{F}_n$. Hence, $\langle V - S \rangle$ is disconnected and therefore $\gamma_{tct}(\mathbb{F}_n) = n - 2$. □

The final two results describe the relationship of $\gamma_{tct}(G)$ with other graph parameters.

Theorem 2. *For any connected graph G of order $n \geq 5$ such that $\gamma_{tct}(G) \leq n - 2$, we have the following:*

(a) $\gamma_{tct}(G) + \kappa(G) \leq 2n - 3$.

(b) The bound is sharp if and only if $G \cong K_n$.

Proof. Let G be a connected graph of order $n \geq 5$ such that $\gamma_{tct}(G) \leq n - 2$. Recall that $\kappa(G)$ is the vertex connectivity of G.

(a) We know that $\kappa(G) \leq n - 1$ and that $\gamma_{tc} \leq n - 2$. Hence, $\gamma_{tc}(G) + \kappa(G) \leq 2n - 3$.

(b) Suppose that the bound is sharp. Then, $\gamma_{tct}(G) + \kappa(G) = 2n-3$. This is only possible if $\gamma_{tct}(G) = n-2$ and $\kappa(G) = n - 1$. We know that $\gamma_{tct}(K_n) = n - 2$, where $n \geq 5$. We also know that $\kappa(K_n) = n - 1$. Suppose that $G \ncong K_n$. Then there exists $x \in V(G)$

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such that $\deg(x) = n - 1$. Hence, $N(x)$ is a vertex cut such that $|N(x)| < n - 1$. Since $\kappa(G) < n - 1$, we have a contradiction. Therefore, the bound is sharp only if $G \cong K_n$. Conversely, if $G \cong K_n$, then $\gamma_{tct}(K_n) + \kappa(K_n) = (n - 2) + (n - 1) = 2n - 3$. □

Theorem 3. *For any connected graph G of order $n \geq 5$ such that $\gamma_{tct}(G) \leq n - 2$, we have the following:*

(a) $\gamma_{tct}(G) + \chi(G) \leq 2n - 2$.

(b) The bound is sharp if and only if $G \cong K_n$.

Proof. Let G be a connected graph of order $n \geq 5$ such that $\gamma_{tct}(G) \leq n - 2$. Recall that $\chi(G)$ is the chromatic number of G.

(a) We know that $\chi(G) \leq n$ and that $\gamma_{tc} \leq n-2$. Hence, $\gamma_{tc}(G) + \chi(G) \leq 2n - 2$.

(b) Suppose that the bound is sharp. Then, $\gamma_{tct}(G) + \chi(G) = 2n - 2$. This is only possible if $\gamma_{tct}(G) = n - 2$ and $\chi(G) = n$. It is known that $\gamma_{tct}(K_n) = n - 2$, where $n \geq 5$. We also know that $\chi(K_n) = n$ since the vertices in a complete graph are pairwise adjacent. Suppose that $G \ncong K_n$. Then there exist at least two vertices $x, y \in V(G)$ that are not adjacent. Thus, vertices x and y can have the same color. Since $\chi(G) < n$, we have a contradiction. Hence, the bound is sharp only if $G \cong K_n$. Conversely, if $G \cong K_n$, then $\gamma_{tct}(K_n) + \chi(K_n) = (n - 2) + n = 2n - 2$. □

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