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A Sequencing Heuristic to Minimize Weighted Flowtime in the Open Shop

Eric A. Siy
Department of Industrial Engineering
email : eric.siy@dlsu.edu.ph

Abstract: The open shop is a job shop with no precedence constraints on the job-machine operations: as long as the job passes through the prescribed set of machines, the jobs can processed in any order of machining, with deterministic processing times, and no preemption. Minimizing weighted flowtime can represent the time that the job spends inside the production system. Flowtime is the difference between completion time and the time when the job was released. For simplicity, it is assumed in this paper that all jobs were available for scheduling at time=0, and hence, flowtime is the completion time C_j of each job. The longer a job is in the system, a certain delay cost over the duration is ascribed by the job's weight. Pinedo (2008) showed that the minimizing weighted completion time sequencing problem is NP-hard, which justifies the development of heuristic procedures to curtail the schedule search process. This paper presents a search heuristic that uses the concept of duespan, developed by the author (Siy, unpublished MS thesis, 1999), to create an initial sequence on the bottleneck machine. Improvement of sequences on the non-bottleneck machines is subsequently performed to arrive at an approximate-optimal sequence. The paper concludes by showing how the heuristic performs against evolutionary genetic algorithm search process in Excel Solver, and offers promising results with less computational effort.

Key Words: Open Shop Scheduling; Weighted Flowtime; Production Control

1 INTRODUCTION

1.1 Open Shop Sequencing with Total Weighted flowtime as scheduling criteria

The open shop is job sequencing problem where n jobs (jobs i=1,2..n) require processing in a subset of m machines (machine j=1,2..m) with deterministic processing time Pij. Each job has a weight Wj corresponding to its importance (i.e. costs) as the job spends time in the open shop system. It is assumed that once a job has begun in a machine, it cannot be pre-empted by other jobs and is committed to finish according to its processing time Pij. The open shop is similar to the flow shop and the job shop but is distinct due to having the jobs not requiring any particular sequence of machining. As long as the job undergoes processing under the machines with non-zero processing times, the open shop does not constrain job sequencing, unlike the flowshop (unidirectional sequence) and job shop (job-specific machine precedence constraints).

The weighted flowtime is the scheduling criterion to be minimized. Assumed is that all jobs are available initially at time t=0, and the completion time of each job j in any machine i is defined as flowtime. When each job's weight Wj is multiplied with the same job's completion time Cj, and these products are summed for all jobs, we have the scheduling criterion weighted flowtime (ΣWjCj). This criterion can be thought of as the total cost of having the n jobs spend their time in the system of machines to which they must be processed under. The longer jobs spend in the system due to sub-optimal scheduling, the higher weighted flowtime results, and a significant cost higher than necessary may be incurred.

Common open shop applications of this scheduling criterion is testing and maintenance and teacher-class time-tabling. Testing in a production system typically requires inspectors (machines) to test different products (jobs) for quality. Different inspectors may specialize in testing different product quality characteristics, but the workpiece to be tested cannot be tested simultaneously by different inspectors due to physical colocality. It does not matter in which order the tests are made as long as each workpiece undergoes the battery of tests completely before complete testing status can be declared on each workpiece.

Maintanance requires machine specialists (machines) to check a number of machines (jobs) for failures and repairs. In any number of machines needed to be maintained, prolonging the maintenance of each machine means the longer the downtime of the respective individual machines: possibly affecting productivity due to the necessary delay of maintenance. Some machines have a higher value in downtime, and hence a weight may be assigned to these machines to represent the disruptive downtimes that may result.

Teacher-class time-tabling can also be seen as an open shop problem instance: teachers have to be assigned lecture classes for a day in school but there are no order restrictions on which class should come first, as long as the class is scheduled in. Some teachers are paid higher rates, and a higher weight for their time may be assigned. Teachers of lower rates may be assigned longer in-between classes breaks since their time may not be as expensive.

Minimizing total weighted completion times $\Sigma W_j C_j$ in an open shop is NP hard (Pinedo, 2008). For open shop setup of at least two machines, NP-hardness can be described as not easy to solve in polynomial time, specifically for search algorithms that use complete enumeration. This is the motivation for this paper's proposed heuristic. This paper is also an alternative solution procedure to the Lower Bound Heuristic presented by this author previously (Siy, 2015 DLSU Research Congress).

2. HEURISTIC DEVELOPMENT

To demonstrate how the heuristic proceeds, consider the open shop problem in Table 1 from a previous paper (Siy, 2015).

Table 1. Open Shop processing times for illustration

Machine\Job	J1	J2	J3	Sum
M1	4 hrs	9	3	16
M2	3	0	8	11
M3	6	5	10	21
Sum	13	14	21	
Weight W_j	1	2	3	

As the third machine M3 has the highest total processing time of 21 hours, a relationship between this theoretical minimum processing time and the completion times of the other jobs may be made. This relationship may be used for prioritizing jobs specifically on machine M3. This machine M3 may be called the "Bottleneck" machine, from which a sequence may be developed first due to its salient effect on the other machine's sequences. (Siy, 1999)

Define a term called duespan (Dj) as the difference between the theoretical minimum makespan (Cmax) and each job's minimum total completion time $\sum P_{ij}$ (Siy. 1999)

$$D_j = (C_{max}) - \sum P_{ij} \qquad (Eq.\ 1)$$

Where: Dj = duespan

Cmax = maximum theoretical completion time of any job in the set of jobs

$\sum P_{ij}$ = sum of a job j processing times on all m machines (summation index i)

The higher a job's due span Dj, the earlier is its possible total completion time. Later total processing times would have a relatively lower duespan.

The proposed heuristic follows the procedure below:

1. Determine the theoretical minimum makespan.
2. Determine the duespan for each job.
3. On the machine with the longest total processing time (bottleneck machine M'), schedule the jobs via descending order of weighted duespans.
4. For the other machines, create a sequence following the series given on the bottleneck machine, but with special priority to jobs with higher weights.
5. Proceed to a neighborhood search process for available time windows as long as completion times of later jobs are not affected.

This heuristic would henceforth be referred to as the weighted duespan heuristic.

3. HEURISTIC RESULTS

Table 2 demonstrates how the heuristic procedure provides a sequence of Job2-Job1-Job3 on machine M3.

Table 2: Steps 1-3 of Proposed Heuristic on Three jobs on Three machines

Machine\Job	J1	J2	J3	Sum
M1	4	9	3	16
M2	3	0	8	11
M3	6	5	10	21
Sum	13	14	21	
Duespan	21-13 =8	21-14 =7	21-21 =0	
x Weight	1	2	3	
=Weighted Duespan (Sequence on M3)	8 (2nd)	14 (1st)	0 (Last)	

Step 4 of the heuristic can now be done. From the developed sequence Job2-Job1-Job3, we can repeat this same sequence on all three machines just as in a flowshop, as shown in Figure 1. One can note that since there is no processing time of Job 2 on machine 2, so no blue bar is shown on M2.

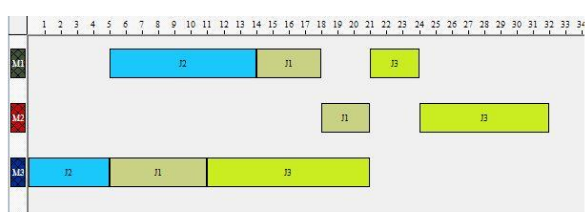

Fig. 1: Initial permutation schedule for open shop example in Table 1

Step 5 Neighborhood search can proceed. This process is simply looking for time windows to have jobs sequenced late in the initial schedule to be moved earlier into. This step exploits the characteristic of open shops where jobs can be processed in any machining order. Visual inspection of the Gantt Chart on Figure 1 shows that

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there exists front-end idle time on machine 1 and machine 2. We could exploit this time window by placing jobs to the front (i.e. Job 3 in Machine M1, and move jobs 1 and 3 forward in Machine M2.) This completes a neighborhood search, and a schedule can be presented in Figure 2, with the computed flowtime in Table 3.

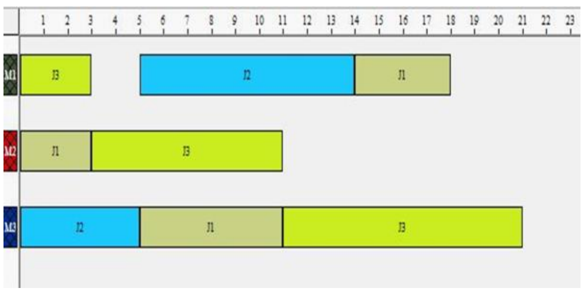

Fig. 2. Proposed schedule by sequencing heuristic

Table 3: Computed Flowtime for generated schedule

Job	J1	J2	J3
Completion time	18	14	21
x Weight	1	2	3
= Weighted Flowtime	18	28	63
Sum Wtd flow			109

The weighted flowtime for this schedule is 109. A test for optimality can now be done. Since there is only three jobs and a finite number of possible permutation schedules based on the possible sequences on the bottleneck machine, the evolutionary (genetic algorithm) search solution method of Excel Solver may be used to find the best solution across all $(3!)^3=216$ possible permutation schedules.

Table 4 shows results from the permutation schedules. The minimum possible weighted flowtimes given initial seed schedules on machine M3: (Siy, 2015) are shown for the six (3!=6) possible permutations for three jobs. It can be seen that the optimal weighted flowtime is indeed 109, as found by the heuristic.

A second problem is similarly presented (Siy, 2015), and the same optimal schedule was found through the present heuristic. Table 5 is the problem information for three jobs on two-machine open shop. Table 6 shows how the weighted duespan heuristic determines the sequence on the bottleneck machine M2.

Table 4. Lower bounds for all M3 sequences for illustrative example (*from Siy, 2015*)

Completion times				
M3 Sequence	J1	J2	J3	Wtd Flowtime
123	14	20	21	117
132	16	21	24	130
213	18	14	21	109
231	24	14	21	115
312	16	21	21	121
321	21	15	21	114

Table 5. Second illustrative example of Open Shop

Machine\Job	J1	J2	J3
M1	6 hrs	3	7
M2	2	5	10
Weight W_j	1	2	3

Table 6: Steps 1 to 3 of the heuristic applied to second illustrative example

Machine\Job	J1	J2	J3	Sum
M1	6	3	7	16
M2	2	5	10	17
sum	8	8	17	
Duespan x Weight	9 x1	9 x2	0 x3	
=Wtd Duespan	9	18	0	
Sequence on M2	2nd	1st	last	

Figure 3 is the open shop schedule that results in the least weighted flowtime for the sequence J2-J1-J3. The neighborhood search step moved Job 3 on machine one to the front of the sequence.

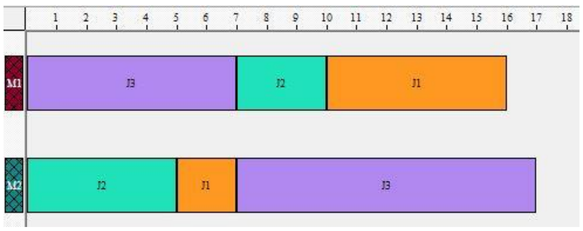

Fig. 3 Proposed schedule for illustrative problem 2.

The weighted duespan heuristic gave a schedule whose total weighted flowtime is 87 (Table 7).

Table 7: Determination of weighted flowtime for the schedule generated in fig. 3.

	J1	J2	J3	
Completion	16	10	17	
Weight	1	2	3	TotalWtdFlow
WtdFlow	*16*	*20*	*51*	**87**

By complete enumeration of all 36 possible sequences (as shown Table 8), there exists two schedules with the minimum weighted flowtime of 87. The proposed heuristic was able to determine one of them. (M1: J3-J2-J1; M2: J2-J1-J3)

Table 8: Complete enumeration of sequences for the two-machine three jobs open shop problem in Table 5

M2\M1	123	132	213	231	312	321
123	114	109	100	92	96	87
132	148	133	100	116	108	111
213	104	109	105	92	96	87
231	101	126	111	98	102	90
312	97	103	97	108	118	118
321	95	108	98	104	119	119

One can compare this heuristic with this another previous one (Siy, 2015) that uses a lower bound calculation for bottleneck machine sequence (recommending a Shortest Processing time (SPT) based criteria for initial sequence choice). Both came up with the same optimal sequences for the two same presented problems. This present heuristic, however, is rather definite (one could say, stubborn) in its recommended sequence, compared to the search -based branch-and-bound process favored by the previous work. It remains to be seen in future validations if the same detection of optimal sequences may be possible for bigger problems (more than 3 machines and more than 4 jobs).

4. CONCLUSIONS

The proposed weighted duespan heuristic first presented for minimizing makespan and total weighted tardiness in the open shop has applications in minimizing weighted flowtime in the same open shop setup. This result seems to replace the same flowtime open shop heuristic that uses lower bound calculations for generating schedules in the open shop problem.

In the same track as the previous study, future efforts could be given to program a higher number of jobs and machines to see how the weighted duespan heuristic performs in identifying the optimal (or near-optimal) sequences.

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