



Relative Prices and Real GDP

Jesus C. Dumagan, Ph.D.

Adjunct Professor, School of Economics

*Consultant, Angelo King Institute for Economic and Business Studies
De La Salle University, Manila, Philippines*

Email: jesus.dumagan@dlsu.edu.ph or jcdcu91@yahoo.com.

Abstract: Current official procedures for real GDP in chained prices or in constant prices ignore *relative prices*—ratios of industry GDP deflators to the economy's GDP deflator—and consequently yield economically misleading results by *understating* (overstating) *level* contributions of industries with *above* (below) average relative prices and also *understating* (overstating) *growth* contributions of industries with *rising* (falling) relative prices. These results are illustrated by US GDP in chained prices and Philippine GDP in constant prices. However, the above misleading results could be mitigated by this paper's *general* formulas applicable for determining level and growth contributions to GDP either in chained or in constant prices. While allowing for differences and changes in relative prices, these general formulas encompass existing formulas as *special* cases of constant relative prices.

Key Words: Real GDP; GDP residuals; relative prices; index numbers.

1. INTRODUCTION

This paper points out that differences and changes in relative prices have *real* effects—separate from quantity changes—but relative prices are ignored in current official procedures for determining industry contributions to *level* and *growth* of real GDP in chained prices or in constant prices. Consequently, ignoring relative prices results in “non-additivity” residuals in the above industry contributions to GDP in chained prices and also makes similar industry contributions to GDP in constant prices—although there are no residuals—questionable as presently calculated.

Analytically, relative price is a ratio of one price to another and the price in the denominator may be chosen arbitrarily. However, since this paper is concerned with industry contributions to level and growth of real GDP, it is appropriate to define relative price as the ratio of an industry's GDP deflator to the overall GDP deflator. That is, overall real GDP is the numeraire. By this definition, relative price is the *real price* measured in “GDP basket” per unit of an industry's real GDP. Relative prices play a pivotal role in this paper's analytic framework.

The rest of this paper is organized as follows. Section 2 presents general formulas for

level and growth of GDP in chained or in constant prices to show (i) the effects of *differences* in relative prices between industries on their *level* contributions and (ii) the effects of *changes* in relative prices of industries on their *growth* contributions. It is shown that current formulas for level and growth contributions are *special* cases of this paper's general formulas when relative prices are constant. Section 3 applies this paper's framework to US GDP in chained prices to show that existing *residuals* from “non-additivity” are procedural in nature and, therefore, avoidable. The same framework is also applied to Philippine GDP in constant prices to show that—although there are no residuals—contributions to GDP level and growth are, nevertheless, questionable. Section 4 concludes this paper.

2. A GENERAL (GEN) FRAMEWORK FOR GDP

In period t , let there be nominal prices, p_{it}^j , and quantities, q_{it}^j of $i = 1, 2, \dots, N$ commodities, where $j = 1, 2, \dots, M$ are mutually exclusive subsets of “similar” commodities and $M < N$ since each j contains at least one i and some j contains more

than one. By definition, nominal GDP or GDP in current prices is,

$$Y_t \equiv \sum_j Y_t^j \quad ; \quad Y_t^j \equiv \sum_i p_{it}^j q_{it}^j. \quad (\text{Eq. 1})$$

The relative change of nominal GDP from $t-1$ to t or from 0 to t combines changes in prices and quantities. These changes can, however, be decomposed into a product of a *price index*, which measures relative change in prices, holding quantities constant and a *quantity index*, which measures relative change in quantities, holding prices constant. To see the decomposition, let the price index be $P_{t-1,t}$ and the quantity index be $Q_{t-1,t}$ that link $t-1$ to t . Similarly, let the corresponding indexes be $P_{0,t}$ and $Q_{0,t}$ that link 0 to t . Hence,

$$\frac{Y_t}{Y_{t-1}} = \frac{\sum_i p_{it}^j q_{it}^j}{\sum_i p_{i,t-1}^j q_{i,t-1}^j} = P_{t-1,t} \times Q_{t-1,t} \quad (\text{Eq. 2})$$

$$\frac{Y_t}{Y_0} = \frac{\sum_i p_{it}^j q_{it}^j}{\sum_i p_{i0}^j q_{i0}^j} = P_{0,t} \times Q_{0,t}. \quad (\text{Eq. 3})$$

Two pairs of price and quantity indexes that satisfy (Eq. 2) and (Eq. 3) are employed in current practice. One is the pair of Paasche price and Laspeyres quantity indexes and the other is the pair of Fisher price and Fisher quantity indexes (Balk, 2010).

For GDP in constant prices, $P_{0,t}$ and $Q_{0,t}$ are *direct* Paasche price and Laspeyres quantity indexes with a *fixed* base period 0. For GDP in chained prices, period 0 is treated as a reference period that need not be fixed. Two alternative pairs of chained price and quantity indexes are now employed. These are the chained Fisher price and Fisher quantity indexes in Canada and US and the chained Paasche price and Laspeyres quantity indexes in the EU and other countries.¹

GDP either in chained or in constant prices of the economy, X_{t-1} and X_t , and of an industry, X_{t-1}^j and X_t^j , are obtained by deflating (i.e., dividing) GDP in current prices by price indexes or by inflating (i.e., multiplying) base-year GDP by quantity indexes. That is, from (Eq. 2) and (Eq. 3),

$$X_{t-1} \equiv \frac{Y_{t-1}}{P_{0,t-1}} = Y_0 \times Q_{0,t-1} \quad (\text{Eq. 4})$$

$$X_t \equiv \frac{Y_t}{P_{0,t}} = Y_0 \times Q_{0,t-1} \times Q_{t-1,t} \quad (\text{Eq. 5})$$

$$X_{t-1}^j \equiv \frac{Y_{t-1}^j}{P_{0,t-1}^j} = Y_0^j \times Q_{0,t-1}^j \quad (\text{Eq. 6})$$

$$X_t^j \equiv \frac{Y_t^j}{P_{0,t}^j} = Y_0^j \times Q_{0,t-1}^j \times Q_{t-1,t}^j. \quad (\text{Eq. 7})$$

The procedures in (Eq. 1) to (Eq. 7) have generality because they are valid for GDP in chained or in constant prices, regardless of the underlying price and quantity indexes. For this reason, the above procedures will henceforth be referred to in this paper collectively as the GEN framework for expository purposes.

2.1 Contributions to Level of GDP

(Eq. 1) to (Eq. 7) imply that the economy's GDP, X_t , may be expressed as the weighted sum of each industry's GDP, X_t^j , where the weight is r_t^j , ratio of the industry GDP deflator, $P_{0,t}^j$, to the economy's GDP deflator, $P_{0,t}$. That is,

$$X_t = \sum_j r_t^j X_t^j \quad ; \quad r_t^j \equiv \frac{P_{0,t}^j}{P_{0,t}}. \quad (\text{Eq. 8})$$

In (Eq. 8), the economy's GDP deflator is the denominator, i.e., real GDP is the *numeraire*. Thus, r_t^j is the "real price" in terms of the economy's "GDP basket" of an industry's GDP, X_t^j , so that the real value $r_t^j X_t^j$ is an industry's contribution to the economy's GDP level, X_t . Since $r_t^j X_t^j$ is measured in the *same* units as X_t , it is additive to yield $X_t = \sum_j r_t^j X_t^j$ in (Eq. 8). For this reason, this equation is this paper's GEN formula for *additive* industry contributions to level of GDP in chained or in constant prices.²

If *all* prices change in the same proportion at the same time, i.e., constant relative prices, *all* price indexes are equal so that $r_t^j = r_{t-1}^j = 1$ in (Eq. 8). Unless this condition holds, it follows that,

¹ For references in country practices of GDP in chained prices, see Dumagan (2014a & 2014b).

² Tang and Wang (2004) applied relative prices to industry GDP in (Eq. 8) to obtain aggregate labor productivity when GDP is in chained prices. However, Dumagan (2013) *generalized* Tang and Wang's framework to GDP in chained or in constant prices.

$$r_t^j \equiv \frac{P_{0,t}^j}{P_{0,t}} \neq 1 \quad ; \quad X_t \neq \sum_j X_t^j. \quad (\text{Eq. 9})$$

It appears from (Eq. 8) that *ignoring* r_t^j amounts to assuming $r_t^j = 1$ for all j that if not true results in “non-additivity” of GDP in chained prices in (Eq. 9).³ This implies that non-additivity is procedural in nature and, therefore, avoidable. However, this finding is contrary to the prevailing view in theory and practice (Balk, 2010).

In concept, $P_{0,t}$ is like the average of $P_{0,t}^j$ for all industries. Therefore, $P_{0,t}$ lies between the extreme values of $P_{0,t}^j$ so that $\infty > r_t^j \equiv P_{0,t}^j/P_{0,t} > 0$ but in practice should not be too far away above or below 1. Hence, recalling that an industry’s contribution to the level of X_t is $r_t^j X_t^j$, it follows that ignoring r_t^j *understates* level contributions of industries with *above* average prices or $r_t^j > 1$ and, conversely, *overstates* level contributions of industries with *below* average prices or $0 < r_t^j < 1$.

2.2 Contributions to Growth of GDP

From (Eq. 4) to (Eq. 8), the relative change in GDP is,

$$\frac{X_t}{X_{t-1}} = \sum_j r_t^j w_{t-1}^j \frac{X_t^j}{X_{t-1}^j} \quad ; \quad w_{t-1}^j \equiv \frac{X_{t-1}^j}{X_{t-1}}. \quad (\text{Eq. 10})$$

Moreover, (Eq. 1), (Eq. 8), and (Eq. 10) imply,

$$r_{t-1}^j w_{t-1}^j = \frac{Y_{t-1}^j}{Y_{t-1}} \quad ; \quad \sum_j \frac{Y_{t-1}^j}{Y_{t-1}} = 1. \quad (\text{Eq. 11})$$

Together, the above results yield the GEN formula for the growth rate of the economy’s GDP in chained or in constant prices given by,

$$\frac{X_t}{X_{t-1}} - 1 = \sum_j \left[\frac{Y_{t-1}^j}{Y_{t-1}} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right) + \frac{X_{t-1}^j}{X_{t-1}} (r_t^j - r_{t-1}^j) \right]. \quad (\text{Eq. 12})$$

In (Eq. 12), each industry’s growth contribution has two parts. One part is,

$$\text{PGE (pure growth effect)} \equiv \frac{Y_{t-1}^j}{Y_{t-1}} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right). \quad (\text{Eq. 13})$$

PGE is the industry’s GDP growth weighted by its share in nominal GDP. The other part is,

$$\text{PCE (price change effect)} \equiv \frac{X_t^j}{X_{t-1}^j} (r_t^j - r_{t-1}^j). \quad (\text{Eq. 14})$$

PCE comes from a change in relative prices.

The official US Bureau of Economic Analysis (BEA) formula (Moulton and Seskin, 1999) for an industry’s growth contribution is the formula for a component’s contribution to growth of the Fisher quantity index that underpins US real GDP. BEA’s formula is “exact” in that the sum of growth contributions equals growth of the Fisher index. This formula is mathematically equivalent to a different-looking formula derived by Dumagan (2002). Using the latter for comparison, it can be shown that BEA’s formula is *approximately* equal to PGE in (Eq. 13). The minor difference is that while the weights in BEA’s formula also sum to 1, each weight is approximately equal to the industry’s share in nominal GDP, Y_{t-1}^j/Y_{t-1} . Although “exact,” BEA’s formula has residuals because it measures contributions to growth of the Fisher quantity index that by construction holds prices constant.⁴ Hence, BEA’s formula does not capture growth effects of relative price changes and, thus, PCE in (Eq. 14) constitutes BEA’s residual. Therefore, BEA *understates* growth contributions of industries with *rising* relative prices, i.e., $r_t^j - r_{t-1}^j > 0$, and *overstates* growth contributions of industries with *falling* relative prices, i.e., $r_t^j - r_{t-1}^j < 0$.

Finally, if relative prices are constant, i.e., $r_{t-1}^j = r_t^j = 1$ for all j and t , the GEN GDP level formula in (Eq. 8) becomes $X_t = \sum_j X_t^j$, which is the formula in current practice. Also, the GEN GDP growth formula in (Eq. 12) becomes $(X_t/X_{t-1}) - 1 = \sum_j \{w_{t-1}^j [(X_t^j/X_{t-1}^j) - 1]\}$, which is the growth formula in current practice. Thus, the GEN framework encompasses current practices as special cases of constant relative prices.

⁴ The US GDP quantity index is the Fisher index (Fisher, 1922), the geometric mean of Laspeyres and Paasche indexes. The Laspeyres quantity index holds prices constant by keeping the prices in $t-1$ while the Paasche quantity index holds prices constant by keeping those in t . Hence, the Fisher quantity index holds prices constant at the “average” of the prices in the two periods.

³ Non-additivity is universal in countries that have adopted GDP in chained prices. For references in country practices, see Dumagan (2014a & 2014b).

3. APPLICATIONS OF GEN

This paper's GEN framework for industry contributions to level and growth of GDP is applicable to US GDP in chained prices and to Philippine GDP in constant prices.

3.1 Application to GDP in Chained Prices

Due to space limitations, the US GDP data for this application are omitted in this paper but their source is identified in Table 1 that presents the application results. It can be verified from the data that $r_{t-1}^j \neq 1$ and $r_t^j \neq 1$. Therefore, $X_{t-1} = \sum_j r_{t-1}^j X_{t-1}^j$ and $X_t = \sum_j r_t^j X_t^j$ imply that if relative prices are ignored or dropped then for industries with $r_{t-1}^j, r_t^j > 1$, i.e., above average prices, level contributions are understated while for those with $0 < r_{t-1}^j, r_t^j < 1$, i.e., below average prices, level contributions are overstated. These results imply that "non-additivity," i.e., $X_t \neq \sum_j X_t^j$, is due to *ignoring* r_t^j in (Eq. 8). Thus, it is merely procedural and, hence, avoidable.

Table 1 shows the results of applying PGE in (Eq. 13) and PCE in (Eq. 14) to US GDP and also shows BEA's own results. It is interesting to note that for the same industry BEA's growth contribution equals PGE when PGE is rounded to two decimal places. This confirms that BEA's growth contribution captures almost solely PGE and almost totally excludes PCE. Thus, for all industries, BEA yields 2.50 percent while this paper's GEN framework yields PGE + PCE = 2.78 percent, the actual 2012 GDP growth.

Table 1 shows positive PCE for industries with rising relative prices and negative PCE for industries with falling relative prices. Therefore, by excluding PCE, BEA understates growth contributions of the former industries and overstates growth contributions of the latter.

BEA's exclusions of PCE could result in sign reversals of growth contributions. Table 1 shows three sign reversals in the case of **utilities**, **nondurable goods**, and **state and local government**. For example, the growth contribution of **nondurable goods** switches from positive (0.23) according to this paper's GEN to negative (-0.03) according to BEA. Hence, excluding PCE could make BEA's contributions misleading.

3.2 Application to GDP in Constant Prices

GDP of industries and of the economy are in constant prices if $P_{0,t-1}^j, P_{0,t}^j, P_{0,t-1}$, and $P_{0,t}$ are *direct* Paasche price indexes and $Q_{0,t-1}^j, Q_{0,t}^j, Q_{0,t-1}$, and $Q_{0,t}$ are *direct* Laspeyres quantity indexes in (Eq. 4) to (Eq. 7). In this case, $X_{t-1} = \sum_j X_{t-1}^j$ and $X_t = \sum_j X_t^j$. Therefore, since (Eq. 8) also applies when the deflators are direct Paasche price indexes, it follows that for GDP in constant prices,

$$X_{t-1} = \sum_j r_{t-1}^j X_{t-1}^j ; X_{t-1} = \sum_j X_{t-1}^j \quad (\text{Eq. 15})$$

$$X_t = \sum_j r_t^j X_t^j ; X_t = \sum_j X_t^j \quad (\text{Eq. 16})$$

It is important to note that the first equalities in (Eq. 15) and (Eq. 16) are true when GDP is either in chained or in constant prices. However, the second equalities are true only when GDP is in constant prices. These second equalities illustrate the "traditional" (TRAD) aggregation of GDP in constant prices.

(Eq. 15) and (Eq. 16) imply two ways of computing industry contributions to growth of GDP in constant prices given by,

$$\begin{aligned} \frac{X_t}{X_{t-1}} - 1 &= \sum_j \left[\frac{Y_{t-1}^j}{Y_{t-1}} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right) + \frac{X_t^j}{X_{t-1}^j} (r_t^j - r_{t-1}^j) \right] \\ &= \sum_j \frac{X_{t-1}^j}{X_{t-1}^j} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right). \end{aligned} \quad (\text{Eq. 17})$$

Note that the first line in (Eq. 17) is the GEN growth formula in (Eq. 12) while the second line is the TRAD formula for growth contributions to GDP in constant prices in current official procedures, e.g., by NEDA (2011). Using the fact that $X_{t-1}^j/X_{t-1} = (Y_{t-1}^j/Y_{t-1})/r_{t-1}^j$ from (Eq. 10) and (Eq. 11), the above TRAD formula becomes,

$$\frac{X_t}{X_{t-1}} - 1 = \sum_j \frac{Y_{t-1}^j/Y_{t-1}}{r_{t-1}^j} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right). \quad (\text{Eq. 18})$$

It follows from (Eq. 13) and (Eq. 18) that TRAD and PGE are related by,

$$\begin{aligned} \text{TRAD} &\equiv \frac{X_{t-1}^j}{X_{t-1}^j} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right) \\ &= \frac{Y_{t-1}^j/Y_{t-1}}{r_{t-1}^j} \left(\frac{X_t^j}{X_{t-1}^j} - 1 \right) = \frac{\text{PGE}}{r_{t-1}^j}. \end{aligned} \quad (\text{Eq. 19})$$

The results of applying TRAD, PGE, and PCE to Philippine GDP are presented in Table 2. Also due to space limitations, the data inputs for this application are omitted in this paper but their source is identified in Table 2.

It is important to note analytically in (Eq. 17) and (Eq. 19) and empirically in Table 2 that the sum of TRAD necessarily equals the sum of (PGE + PCE) for *all* industries, which was 7.18 percent in 2013. However, TRAD may differ from (PGE + PCE) for each industry and the implications of this difference for the questionability of TRAD are explained in the following discussion.

It turns out in (Eq. 19) that $TRAD = PGE/r_{t-1}^j$ where $r_{t-1}^j \equiv P_{t-1}^j/P_{t-1}$ and P_{t-1} is the average of P_{t-1}^j for all industries. Hence, for industries with $r_{t-1}^j > 1$, relative prices are above average and $TRAD < PGE$. In contrast, for those

with $0 < r_{t-1}^j < 1$, relative prices are below average and $TRAD > PGE$. That is, TRAD understates (overstates) growth contributions of industries with above (below) average relative prices.

PCE captures growth effects of price changes from r_{t-1}^j to r_t^j that TRAD ignores. Hence, TRAD could yield a positive growth contribution when (PGE + PCE) is negative. This is shown, for example, in Table 2 by **mining and quarrying**. This industry had a negative PCE that more than offset the positive PGE to end up with a negative (-0.078) overall growth contribution but TRAD showed a positive (0.013) growth contribution.

The above examples indicate that TRAD could yield misleading and, thus, questionable results.

Table 1. Industry contributions to US GDP growth

	BEA		GEN		Actual
	GDP growth 2012		PGE 2012	PCE 2012	GDP growth 2012
		(1)	(2)	(1)+(2)	
Contribution to GDP growth (percentage point)					
Agriculture, forestry, fishing, and hunting	0.00	0.004	-0.004	0.000	
Mining	0.35	0.370	-0.286	0.084	
Utilities	0.03	0.032	-0.094	-0.062	
Construction	0.14	0.141	0.020	0.161	
Durable goods	0.26	0.264	-0.005	0.259	
Nondurable goods	-0.03	-0.031	0.264	0.233	
Wholesale trade	0.15	0.150	0.087	0.237	
Retail trade	0.08	0.076	0.035	0.111	
Transportation and warehousing	0.03	0.031	0.070	0.101	
Information	0.21	0.208	-0.066	0.142	
Finance and insurance	0.15	0.151	0.158	0.309	
Real estate and rental and leasing	0.28	0.281	0.095	0.375	
Professional, scientific, and technical services	0.29	0.289	-0.022	0.267	
Management of companies and enterprises	0.15	0.148	-0.022	0.126	
Administrative and waste management services	0.11	0.109	0.008	0.117	
Educational services	0.01	0.011	0.022	0.033	
Health care and social assistance	0.19	0.192	-0.009	0.183	
Arts, entertainment, and recreation	0.02	0.022	0.006	0.028	
Accommodation and food services	0.07	0.074	0.049	0.123	
Other services, except government	0.04	0.040	0.014	0.054	
Federal government	-0.05	-0.048	-0.053	-0.101	
State and local government	0.02	0.023	-0.027	-0.004	
Sum	2.50	2.54	0.24	2.78	
US GDP percent growth	2.78			2.78	
Residuals: "Not allocated by industry"	0.28			0.00	

Source: Data inputs from BEA may be found in Table 1 (Dumagan, 2014a). BEA results are copied from BEA while GEN results are the author's calculations broken out into PGE and PCE.

Table 2. Industry contributions to Philippine GDP growth

	TRAD	GEN		Actual
	GDP growth 2013	PGE 2013	PCE 2013	GDP growth 2013
		(1)	(2)	(1)+(2)
Contributions to GDP growth (percentage point)				
Agriculture and forestry	0.106	0.117	0.066	0.184
Fishing	0.015	0.014	0.004	0.017
Mining and quarrying	0.013	0.014	-0.091	-0.078
Manufacturing	2.269	2.108	-0.790	1.317
Construction	0.529	0.574	0.142	0.717
Electricity gas and water supply	0.167	0.174	-0.003	0.170
Transport communication and storage	0.428	0.363	-0.072	0.291
Trade & repair of vehicles, personal, & household goods	0.948	1.003	0.344	1.347
Financial intermediation	0.854	0.913	0.076	0.988
Real estate renting and business activity	0.935	1.005	0.183	1.187
Public administration, defense, and social security	0.166	0.166	0.063	0.229
Other services	0.751	0.689	0.121	0.810
Sum = Philippine GDP percent growth	7.18	7.14	0.04	7.18

Source: All calculations are by the author using the data in Table 1 (Dumagan, 2014b) and the formulas for TRAD, PGE and PCE.

4. CONCLUSIONS

Current official GDP procedures ignore relative prices and consequently yield misleading results where level contributions of industries with above (below) average relative prices are understated (overstated) while growth contributions of industries with rising (falling) relative prices are understated (overstated) as shown by US GDP in chained prices and Philippine GDP in constant prices. However, these misleading results could be mitigated by this paper's general formulas for level and growth contributions to GDP in chained or in constant prices. While allowing for differences and changes in relative prices, the general formulas encompass those in current practice as special cases of constant relative prices.

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