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A Calibrated Fuzzy AHP Approach to Derive Priorities in a Decision Model for Low Carbon Technologies

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Abstract: Analytic Hierarchy Process (AHP) is one of the most widely used multi-criteria decision analysis (MCDA). It has been applied to a wide spectrum of problems in domains such as business, conflict resolution, environment and energy planning, among others. AHP breaks down the decision problem in a hierarchic structure and derives priorities to rank the alternatives. In this study, a variant of AHP is proposed to model the vagueness of judgment involved in the decision making process. Instead of a 9-point scale typically used in AHP for pairwise comparison, a fuzzy scale was calibrated and used to derive the weights via fuzzy preference programming. An illustrative case study is presented which uses the proposed method to prioritize low-carbon technologies for electricity generation in the Philippines.

Key Words: AHP; fuzzy scale; fuzzy preference programming; low carbon technologies

1. INTRODUCTION

Analytic Hierarchy Process (AHP) is a relative measurement theory which was initially introduced by Saaty (1980) in the late 1970's to help decision makers in quantifying priority weights among alternatives. AHP decomposes complex decision problems into hierarchical structure with the goal (objectives) found on top of the hierarchy, criteria and sub-criteria located at the mid-level of the hierarchy, and the decision alternatives are found at the bottom of the hierarchy. The local priorities are then derived from the pairwise comparative judgment matrices which were populated by the intensity of dominance of one element over the other. The dominance can be interpreted in terms of importance, preference, or likelihood. It then provides a computational framework by which such local priorities can be unified into a coherent decision framework. However, the classical AHP seems insufficient to capture the vagueness of the judgments of decision-makers when pairwise comparisons were being done. Thus, fuzzy numbers have been used in order to account for the uncertainties due to vagueness that the classical AHP models are unable to consider.

2. METHODOLOGY

The method used by Ishizaka and Nguyen (2013) was adopted to calibrate the fuzzy scale used in the study. The areas of the geometric figures used were retained in the calibration but a unique set of shapes were introduced by the researchers. The shapes created are shown in Figure 1 along with their areas and dimensions.

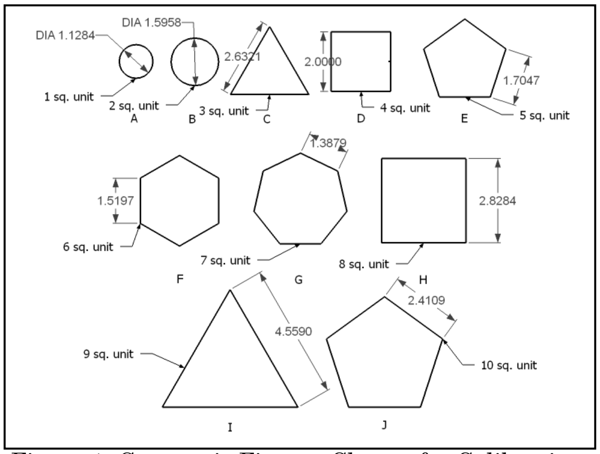

Figure 1. Geometric Figures Chosen for Calibration

These shapes were then included in a survey instrument which was then answered by different respondents coming from differing age groups. The survey is used to measure in pairwise comparison how one area is perceived to be larger than the other area using linguistic scale. Table 1 presents an example of the obtained survey results in the form of a pairwise comparison matrix that uses the verbal scale. Shown in Table 2 is the pairwise comparison matrix from the actual area values of the geometric figures.

Table 1. Respondent Pairwise Comparison Matrix Using the Verbal Scale

	A	B	C	D	E	F	G	H	I	J
A										
B	Mod/Str									
C	Str	Mod								
D	Str	Mod	Equ							
E	Str/verStr	Mod/Str	Equ/Mod	Equ/Mod						
F	Str/verStr	Mod/Str	Equ/Mod	Equ/Mod	Equ					
G	Str/verStr	Mod/Str	Equ/Mod	Equ/Mod	Equ	Equ				
H	Str/verStr	Mod/Str	Equ/Mod	Equ/Mod	Equ	Equ	Equ			
I	verStr/Extr	Str	Mod	Mod	Equ/Mod	Equ/Mod	Equ/Mod	Equ/Mod		
J	verStr	Str	Mod	Mod	Equ/Mod	Equ/Mod	Equ/Mod	Equ/Mod	Equ	

Table 2. Pairwise Comparison Matrix for the Actual Area Values

	A	B	C	D	E	F	G	H	I	J
A										
B	2									
C	3	1.50								
D	4	2.00	1.33							
E	5	2.50	1.67	1.25						
F	6	3.00	2.00	1.50	1.20					
G	7	3.50	2.33	1.75	1.40	1.17				
H	8	4.00	2.67	2.00	1.60	1.33	1.14			
I	9	4.50	3.00	2.25	1.80	1.50	1.29	1.13		
J	10	5.00	3.33	2.50	2.00	1.67	1.43	1.25	1.11	

The verbal judgements were matched to the actual pairwise comparison values. The corresponding actual values assigned to each of the verbal judgements were grouped together as seen in Table 3. The lower and upper limits of each of the verbal judgement are represented by the minimum and maximum values present under the respective verbal judgement. The modal values are the arithmetic averages of the values present under each verbal judgement.

Table 3. Matching Table

Scale	Equ	Equ/Mod	Mod	Mod/Str	Str	Str/verStr	verStr	verStr/Extr	Extr
	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00
	1.33	1.67	1.50	2.00	3.00	5.00	10.00	9.00	
	1.20	2.00	2.00	2.50	4.00	6.00			
	1.40	2.33	3.00	3.00	4.50	7.00			
	1.60	2.67	3.33	3.50	5.00	8.00			
	1.17	1.25	2.25	4.00					
	1.33	1.50	2.50						
	1.14	1.75							
	1.11	2.00							
		1.80							
		2.00							
		1.50							
		1.67							
		1.29							
		1.43							
		1.13							
		1.25							
Lower Limit	1.00	1.13	1.50	2.00	3.00	5.00	7.00	8.00	9.00
Modal Value	1.25	1.72	2.51	3.17	4.30	6.40	8.50	8.50	9.00
Upper Limit	1.60	2.67	3.33	4.00	5.00	8.00	10.00	9.00	9.00

The respective values of each respondent for the lower limit, modal value, and upper limit for each of nine (9) verbal judgments were obtained. Aggregation was done by computing for the geometric mean of all the values obtained. The aggregated lower limits, modal values, and upper limits collectively comprised the membership function for each of the verbal judgements. These membership functions were plotted to produce the calibrated fuzzy scale. The scale was compared to

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Saaty's 9 point scale, presented as the dotted lines in Figure 2.

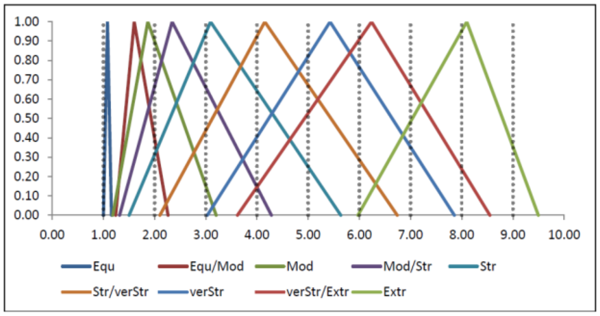

Figure 2. Calibrated Fuzzy Scale compared to Saaty's 9 Point Scale

The performance of the produced calibrated fuzzy scale was tested based on the paper of Harker and Vargas (1987). Table 4 shows the computed correlation coefficients (R) for both 9-point scale and calibrated fuzzy scale are measures of how close the computed relative distance to the actual relative distance through linear regression analysis. It was observed that the calibrated fuzzy scale is comparable and even suggest that it outperforms the 9 point scale. Thus, the calibrated fuzzy scale was justified to be a suitable scale in processing the verbal judgments in fuzzy AHP-based decision making.

Table 4. Initial validation of the calibrated fuzzy scale

City	*9 Point Scale*	*Calibrated Fuzzy Scale*	*Actual Normalized Distance*
Cairo	0.255	0.251	0.278
Tokyo	0.393	0.343	0.361
Chicago	0.036	0.051	0.032
San Francisco	0.123	0.134	0.132
London	0.165	0.184	0.177
Montreal	0.028	0.038	0.019
R	*0.990*	*0.992*	

The calibrated fuzzy scale was then used to derive the priorities based on the fuzzy preference programming described in Promentilla et al (2015). This methods approximates the solution ratios (weights) within the bounds of the fuzzy judgment while preserving the cardinal consistency by maximizing the λ which is indicative of the degree of satisfaction. The proposed method was then applied in prioritizing low carbon technologies in the Philippines.

3. ILLUSTRATIVE CASE STUDY

The decision structure used in this study is shown in Figure 3.

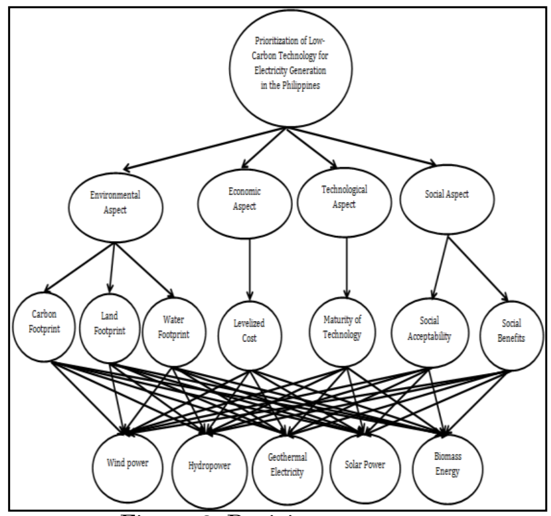

Figure 3. Decision structure

Expert Decision Makers (DM) were tasked to compare the priority of each criteria with respect to the goal, each sub-criteria with respect to its criteria and each alternative with respect to each sub-criteria using the verbal scale discussed in chapter 2. The judgements of one DM are shown in Tables 5 to 10.

Table 5. Verbal Judgement of Criteria with respect to Goal

WRT Goal	Environmental	Economical	Technological	Social
Environmental	1	-	Str	Mod
Economical	Mod	1	-	Mod
Technological	-	Str	1	Str
Social	-	-	-	1

Table 6. Verbal Judgement of Environmental Sub-Criteria with respect to Environmental Aspect

WRT Env	CFP	LFP	WFP
CFP	1	Mod	-
LFP	-	1	-
WFP	Mod/Str	VerStr/Extr	1

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Table 7. Verbal Judgement of Social Sub-Criteria wit respect to Social Aspect

WRT Social	Social Acceptability	Social Benefits
Social Acceptability	1	-
Social Benefits	Str/VerStr	1

Table 8. Verbal Judgement of Alternatives with respect to Maturity of Technology Aspect

WRT Maturity	Wind	Hydro	Geo	Solar	Biomass
Wind	1	Mod	Str	Str	-
Hydro	-	1	Mod	Mod	-
Geo	-	-	1	Equ/Mod	-
Solar	-	-	-	1	-
Biomass	Equ/Mod	Str	Str/VerStr	VerStr	1

Table 9. Verbal Judgement of Alternatives with respect to Sub-Criteria Social Acceptability

WRT Social Acceptability	Wind	Hydro	Geo	Solar	Biomass
Wind	1	Equ/Mod	-	-	-
Hydro	-	1	-	-	-
Geo	Mod/Str	Str	1	Equ/Mod	-
Solar	Equ/Mod	Mod/Str	-	1	-
Biomass	Str	Str/VerStr	Mod	Str/VerStr	1

Table 10. Verbal Judgement of Alternatives with respect to Sub-Criteria Social Benefits

WRT Social Benefits	Wind	Hydro	Geo	Solar	Biomass
Wind	1	-	-	-	-
Hydro	Equ/Mod	1	-	-	-
Geo	Str	Mod/Str	1	Equ/Mod	-
Solar	Mod/Str	Equ/Mod	-	1	-
Biomass	Str/VerStr	Str	Mod	Mod/Str	1

The verbal judgements were then converted into the calibrated fuzzy scale. The software LINGO was then used to compute for the priority weights of each matrix and also the final priority weights of each alternatives. The consistency (λ) was also computed to check for consistency. An example of the equations used in the non-linear programming for solving 3 crisp weights is shown in Figure 4.

The same equations were adapted to solve crisp weights of the pairwise comparison matrices described in Tables 2 to 5. The transformed pairwise comparison matrices with computed consistency and priority weight values are shown in Tables 11 to 16.

```
model:
!fuzzy number triangular distribution
(Lij, Mij, Uij) of judgement,

!upper right;
!Sub-criteria under ENVIRONMENTAL ASPECT
CF-1, LF-2, WF-3;
L12=0.06895 ; M12=0.19531 ; U12=0.54824 ;
L13=0.08107 ; M13=0.78029 ; U13=0.48706 ;
L23=0.00151 ; M23=0.00558 ; U23=0.02388 ;

!lower left;
L21=1/U12 ; M21=1/M12 ; U21=1/L12 ;
L31=1/U13 ; M31=1/M13 ; U31=1/L13 ;
L32=1/U23 ; M32=1/M23 ; U32=1/L23 ;

@free(lambda); !lambda for criteria;

max = lambda; !objective function
!lambda greater than 0 means
consistent judgement, otherwise, inconsistent;

w1 + w2 + w3 = 1;
w1 >= 0;
w2 >= 0;
w3 >= 0;
!upper right;

(lambda)*(M12 - L12)*w2 - w1 + L12*w2 <= 0;
(lambda)*(U12 - M12)*w2 + w1 - U12*w2 <= 0;

(lambda)*(M13 - L13)*w3 - w1 + L13*w3 <= 0;
(lambda)*(U13 - M13)*w3 + w1 - U13*w3 <= 0;

(lambda)*(M23 - L23)*w3 - w2 + L23*w3 <= 0;
(lambda)*(U23 - M23)*w3 + w2 - U23*w3 <= 0;

!lower left;
(lambda)*(M21 - L21)*w1 - w2 + L21*w1 <= 0;
(lambda)*(U21 - M21)*w1 + w2 - U21*w1 <= 0;

(lambda)*(M31 - L31)*w1 - w3 + L31*w1 <= 0;
(lambda)*(U31 - M31)*w1 + w3 - U31*w1 <= 0;

(lambda)*(M32 - L32)*w2 - w3 + L32*w2 <= 0;
(lambda)*(U32 - M32)*w2 + w3 - U32*w2 <= 0;


A12 = w1/w2;
A13 = w1/w3;
A23 = w2/w3;

end
```

Figure 4. Non-Linear Programming Equations for solving 3 crisp weights using LINGO software

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Table 11. CFAHP Matrix of Criteria with respect to Goal

WRT Goal	Environ.	Econ.	Tech.	Social	Priority Weights
Environ.	1	$\langle \frac{1}{3.20}, \frac{1}{1.87}, \frac{1}{1.18} \rangle$	$\langle 1.50, 3.09, 5.64 \rangle$	$\langle 1.18, 1.87, 3.20 \rangle$	0.1630
Econ.	-	1	$\langle \frac{1}{5.64}, \frac{1}{3.09}, \frac{1}{1.50} \rangle$	$\langle 1.18, 1.87, 3.20 \rangle$	0.2341
Tech.	-	-	1	$\langle 1.50, 3.09, 5.64 \rangle$	0.4895
Social	-	-	-	1	0.1134
λ max	0.3718				

Table 12. CFAHP Matrix of Environmental Sub-Criteria with respect to Environmental Aspect

WRT Environmental	CFP	LFP	WFP	Priority Weights
CFP	1	$\langle 1.18, 1.87, 3.20 \rangle$	$\langle \frac{1}{4.28}, \frac{1}{2.34}, \frac{1}{1.31} \rangle$	0.2422
LFP	-	1	$\langle \frac{1}{8.55}, \frac{1}{6.24}, \frac{1}{3.62} \rangle$	0.1155
WFP	-	-	1	0.6423
λ max	0.7403			

Table 13. CFAHP Matrix of Social Sub-Criteria with respect to Social Aspect

WRT Social	Social Acceptability	Social Benefits	Priority Weights
Social Acceptability	1	$\langle \frac{1}{6.74}, \frac{1}{4.15}, \frac{1}{2.10} \rangle$	0.1942
Social Benefits	-	1	0.8058
λ max	1		

Table 14. CFAHP Matrix of Alternatives with respect to Maturity of Technology Aspect

WRT Maturity	Wind	Hydro	Geo	Solar	Biomass	Priority Weights
Wind	1	$\langle 1.18, 1.87, 3.20 \rangle$	$\langle 1.50, 3.09, 5.64 \rangle$	$\langle 1.50, 3.09, 5.64 \rangle$	$\langle \frac{1}{2.26}, \frac{1}{1.60}, \frac{1}{1.24} \rangle$	0.2678
Hydro	-	1	$\langle 1.18, 1.87, 3.20 \rangle$	$\langle 1.18, 1.87, 3.20 \rangle$	$\langle \frac{1}{5.64}, \frac{1}{3.09}, \frac{1}{1.50} \rangle$	0.1590
Geo	-	-	1	$\langle 1.24, 1.60, 2.26 \rangle$	$\langle \frac{1}{6.74}, \frac{1}{4.15}, \frac{1}{2.10} \rangle$	0.1010
Solar	-	-	-	1	$\langle \frac{1}{7.86}, \frac{1}{5.43}, \frac{1}{3.03} \rangle$	0.0699
Biomass	-	-	-	-	1	0.4023
λ max	0.5712					

Table 15. CFAHP Matrix of Alternatives with respect to Sub-Criteria Social Acceptability

WRT Social Accept.	Wind	Hydro	Geo	Solar	Biomass	Priority Weights
Wind	1	$\langle 1.24, 1.60, 2.26 \rangle$	$\langle \frac{1}{4.28}, \frac{1}{2.34}, \frac{1}{1.31} \rangle$	$\langle \frac{1}{2.26}, \frac{1}{1.60}, \frac{1}{1.24} \rangle$	$\langle \frac{1}{5.64}, \frac{1}{3.09}, \frac{1}{1.50} \rangle$	0.1082
Hydro	-	1	$\langle \frac{1}{5.64}, \frac{1}{3.09}, \frac{1}{1.50} \rangle$	$\langle \frac{1}{4.28}, \frac{1}{2.34}, \frac{1}{1.31} \rangle$	$\langle \frac{1}{6.74}, \frac{1}{4.15}, \frac{1}{2.10} \rangle$	0.0814
Geo	-	-	1	$\langle 1.24, 1.60, 2.26 \rangle$	$\langle \frac{1}{3.20}, \frac{1}{1.87}, \frac{1}{1.18} \rangle$	0.1913
Solar	-	-	-	1	$\langle \frac{1}{6.74}, \frac{1}{4.15}, \frac{1}{2.10} \rangle$	0.1439
Biomass	-	-	-	-	1	0.4751
λ max	0.2485					

Table 16. CFAHP Matrix of Alternatives with respect to Sub-Criteria Social Benefits

WRT Social Benefits	Wind	Hydro	Geo	Solar	Biomass	Priority Weights
Wind	1	$\langle \frac{1}{2.26}, \frac{1}{1.60}, \frac{1}{1.24} \rangle$	$\langle \frac{1}{5.64}, \frac{1}{3.09}, \frac{1}{1.50} \rangle$	$\langle \frac{1}{4.28}, \frac{1}{2.34}, \frac{1}{1.31} \rangle$	$\langle \frac{1}{6.74}, \frac{1}{4.15}, \frac{1}{2.10} \rangle$	0.0799
Hydro	-	1	$\langle \frac{1}{4.28}, \frac{1}{2.34}, \frac{1}{1.31} \rangle$	$\langle \frac{1}{2.26}, \frac{1}{1.60}, \frac{1}{1.24} \rangle$	$\langle \frac{1}{5.64}, \frac{1}{3.09}, \frac{1}{1.50} \rangle$	0.1153
Geo	-	-	1	$\langle 1.24, 1.60, 2.26 \rangle$	$\langle \frac{1}{3.20}, \frac{1}{1.87}, \frac{1}{1.18} \rangle$	0.2405
Solar	-	-	-	1	$\langle \frac{1}{4.28}, \frac{1}{2.34}, \frac{1}{1.31} \rangle$	0.1666
Biomass	-	-	-	-	1	0.3977
λ max	0.5668					

Quantitative data was obtained from literature. Table 17 presents the obtained priorities of the alternatives with respect to the different quantitative sub-criteria. Priorities were obtained through normalization.

The final priority weights for the alternatives are then shown in Table 18. Note that these results were based from the aggregation of the individual judgments provided by the respondents of the survey.

Table 17. Normalized priorities from the quantitative sub-criteria.

RE	LCOE	Water Footprint	Land Footprint	Carbon Footprint
Biomass	0.24222	0.00547	0.00083	0.01717
Geothermal	0.27224	0.00892	0.00819	0.11229
Hydro	0.20594	0.02605	0.28738	0.49180
Solar	0.13449	0.25452	0.59141	0.01978
Wind	0.14511	0.70503	0.11220	0.35896

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Table 18. Priority weights of the alternatives.

	Biomass	*Solar*	*Geothermal*	*Hydro*	*Wind*
FINAL WEIGHTS	0.429	0.224	0.176	0.106	0.0659
Ranking	**1st**	**2nd**	**3rd**	**4th**	**5th**

4. CONCLUSIONS

A Calibrated Fuzzy Analytic Hierarchy Process (CFAHP) method that incorporates Fuzzy Set Theory with AHP was developed to prioritize low-carbon technology for electricity generation in the Philippines. The calibrated fuzzy scale was used in the fuzzy pairwise comparison matrix to address the vagueness involved in giving judgment. Criteria used for the prioritization were the Environmental, Economic, Technological, and Social aspects of the low-carbon technologies. These criteria were broken down into sub-criteria such as the levelized cost of Electricity, carbon, land and water footprints, as well as qualitative criteria such as maturity of technology, social acceptance, and social benefits. The alternatives to be prioritized were biomass, geothermal, solar, hydro, and wind power. For illustrative case study, the decision model prioritized the low carbon technologies for the Philippines from highest to lowest namely, Biomass, Solar, Geothermal, Hydro, and Wind.

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