Linearity in Stochastic Systems

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Abstract: Financial markets are generally taken to be stochastic in nature and that future values cannot be predicted from historical values. Unpredictable fluctuations are therefore characteristic of financial data time series. A feature indicating statistical independence of two variables is that their correlation vanishes. This study looked into the autocorrelation between the value of a financial system at time t, with its value at another time t + T. An investigation of 18 corporations in the Philippine Stock Exchange, 18 corporations in the New York Stock Exchange, and 6 foreign exchange pairs showed that the autocorrelation $\langle x(t)x(t+T)\rangle_t$ where averaging is over time t, does not vanish. Instead, it decreases linearly as a function of the interval T. Moreover, this linearity is persistent, whether the stochastic variable considered is the price, simple return, or the log return. As this linearity is surprising because it points to a degree of determinism in such stochastic systems, a deeper understanding of this feature is sought by delving into the conditions imposed by such a linearity on the probability density functions of the stochastic systems.

Key Words: stochastic system; autocorrelation; probability density function

1. INTRODUCTION

Financial markets are generally taken to be stochastic in nature. The fluctuations in prices was first modelled as a random walk by Bachelier (1900) leading to its characterization by a normal distribution. A closer study by Osborne (1960) however showed that it is the log return rather than the price that is normally distributed. Both cases, the simple and geometric Brownian motion, are Markov or memory-less processes. This means that the price at a given time is related only to the immediately preceding price and not the prices in the past. A way to test this independence is through the autocorrelation, as was done for example by Levich and Rizzo (1998).

Autocorrelation studies of stock markets in Europe has been done by Blandon (2007), Asian markets by Chang et al (1998), and Egan (2008) for the American markets. What started out as a study of the Philippine market turned up a curious result that led to a wider investigation that now included 36 Philippine and American stock issues in different sectors, and 6 foreign exchange pairs. As linear time-dependence is found in all these cases, this study also delved into the implications this might have on the stochasticity of the systems.

2. EMPIRICAL STUDY

Autocorrelation refers to the correlation of a time series with its own past and future values. It may be opertionally defined as

$$R(T) = \langle x(t)x(t+T)\rangle \qquad \text{(Eq. 1)}$$

where:

R : autocorrelation
x : stochastic variable
T : time interval
< > denotes time averaging

This study looked at 10-year daily closing prices spanning February 2004 to February 2014 for 36 corporations in the Philippine and New York Stock Exchange, as well as 6 currency pairs in the foreign exchange market. Calculation of the autocorrelation $R(T)$ for different time-intervals T revealed that the R decreases linearly with T. This is found to be true in all cases, whether the stochastic variable is taken to be the price, the simple return, or the log return. Fig. 1 shows a sampling of these results.

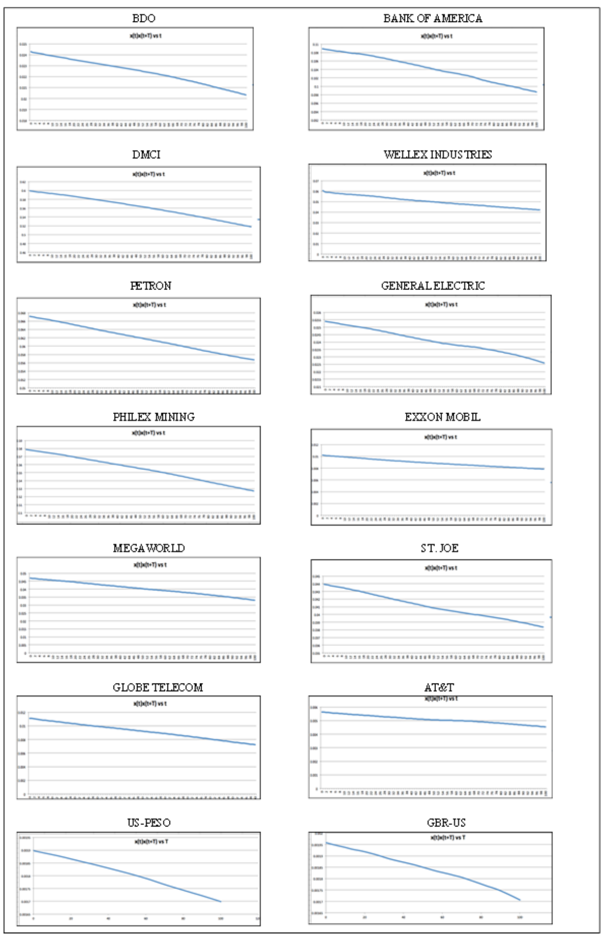

Fig. 1. Plots of autocorrelation vs time interval for some stock market issues and foreign exchange pairs.

These linear relations are curious as they are obtained from stochastic systems which are hardly expected to yield such deterministic relations. The fact that the lines are not exactly straight indicates that this is not a consequence of a general statistical law or an artifact of the definition of the autocorrelation. Rather, it is more likely that this is a characteristic arising from the dynamics of the systems.

3. THEORETICAL IMPLICATIONS

The autocorrelation defined by eqn. (1) is implemented by the averaging process

$$R(T) = \sum_{k}^{N-T} \frac{x(t_i)\,x(t_i+T)}{N-T} \qquad \text{(Eq. 2)}$$

where T is a discrete time interval. In the continuum limit,

$$R(T) = \int x(t)\,x(t+T)\,\rho(x,t,T)\,dx(t) \qquad \text{(Eq. 3)}$$

where ρ is the symmetrized probability density

$$\rho(x,t,T) = f(x,t)\,f(x,t+T) \qquad \text{(Eq. 4)}$$

and f is a probability amplitude.

The linear dependence of the autocorrelation to the time-interval T means that

$$R(T) = \sigma^2 - \alpha T \qquad \text{(Eq. 5)}$$

where α is the slope, and the y-intercept

$$\sigma^2 = \langle x(t)\,x(t)\rangle = \int x^2(t)\,f^2(x,t)\,dx(t) \qquad \text{(Eq. 6)}$$

is the variance if x is a martingale. If we let $t' = t + T$

$$x(t') \approx x(t) + \left.\frac{\partial x}{\partial t'}\right|_{t'=t} T + \frac{1}{2}\left.\frac{\partial^2 x}{\partial t'^2}\right|_{t'=t} T^2 \qquad \text{(Eq. 7)}$$

$$f(x,t') \approx f(x,t) + \left.\frac{\partial f}{\partial t'}\right|_{t'=t} T + \frac{1}{2}\left.\frac{\partial^2 f}{\partial t'^2}\right|_{t'=t} T^2 \qquad \text{(Eq. 8)}$$

Thus,

$$R(T) = \int x(t)x(t)f(x,t)f(x,t)dx(t)$$
$$+T\int x(t)f(x,t)\left.\frac{\partial(xf)}{\partial t'}\right|_{t'=t} dx(t)$$
$$+\frac{T^2}{2}\int \left.\frac{\partial^2(xf)}{\partial t'^2}\right|_{t'=t} dx(t) \qquad \text{(Eq. 9)}$$

A comparison with eq. (5) reveals that

$$\int \left.\frac{\partial^2(xf)}{\partial t'^2}\right|_{t'=t} dx(t) = 0 \qquad \text{(Eq. 10)}$$

and

$$\int x(t)f(x,t)\left.\frac{\partial(xf)}{\partial t'}\right|_{t'=t} dx(t) = -\alpha \qquad \text{(Eq. 11)}$$

But

$$\int x(t)f(x,t)\left.\frac{\partial(xf)}{\partial t'}\right|_{t'=t} dx(t) = \int \frac{1}{2}\left.\frac{\partial(xf)^2}{\partial t'}\right|_{t'=t} dx(t)$$
$$= \frac{1}{2}\frac{d}{dt}\int x^2(t)f^2(x,t)dx(t) = \frac{1}{2}\frac{d\sigma^2}{dt}$$

Eqn. (11) then implies that

$$\sigma^2 \propto t \qquad \text{(Eq. 12)}$$

which is the hallmark of a Wiener process.

4. CONCLUSIONS

A study of 42 financial systems showed that their autocorrelations $\langle x(t)x(t+T\rangle$ decrease linearly with the time interval T. This result seems to indicate some sort of determinism in what is generally characterized as stochastic systems. A theoretical analysis however reveals that such linearities are in fact characteristic of Wiener processes. This atudy therefore emprically shows that it is reasonable to assume that financial systems are Wiener processes provided that the stochastic variables are transformed into martingales. The stochastic financial processes can therefore be analyzed using Ito calculus.

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