Dynamical Equation of the San Miguel Corporation Stock Log Return

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Abstract: Stock prices are generally held to be stochastic in nature and that it is impossible to predict the price value at any given time. Like any stochastic system however, the probability that the stock price takes on a specific value can be determined from its historical values, and this information can be used to characterize the system. The dynamics of San Miguel Corporation (SMC) stock price is studied through its log return based on closing prices over a 10-year period from 2003 to 2013. The statistical ensemble is constructed from 60-day sections of this dataset, so that the day-t ensemble is composed of the day-t stock price in each section. The dynamics of the system can then be determined from the time evolution of the corresponding time-dependent probability density function. The probability density function for each day-t for SMC is found to be generally best-fitted by the Log-Logistic distribution. The scale parameter α is found to have a significant deterministic dependence on time t, while the shape parameter β and the center parameter γ do not. Given these characteristics, it is then determined that the probability density function of the SMC stock price log return obeys the Liouville equation, or a diffusion-free Fokker-Plank equation. The drift is of the form x ln x. Comparison with the Feynman-Kac equation shows that the dynamics of the stock price can be modelled with a logarithmic potential.

Key Words: stock market; stochastic system; Fokker-Planck equation; log logistic distribution

1. INTRODUCTION

Stock markets are examples of systems that are inherently unpredictable because of the multitude of players that are involved, and that while the markets are influenced by the actions of the players, the players themselves are influenced by the movements in the markets. This feedback loop makes the system highly non-linear that may either bring the market to a settled state or to highly inflated states that ultimately bring about market crashes (Sornette, 2004). Moreover, most players in the markets do not have access to sufficient information and in totality, the actions of most players effectively constitute random motions. Fortunately, this randomness also makes the market susceptible to stochastic analysis, and it is these random movements that makes the efficient market hypothesis plausible (Weatherall, 2014).

Efforts to characterize the dynamics of stock markets are done not only in financial circles, but also in the field of econophysics, where the tools of physics are used to study such complex systems.

Among the many approaches are the study of probability distributions that characterize the market (Silva et al, 2004; Gu et al, 2008; Janairo, 2010; Tano, 2011; Leoncini, 2012), and temporal evolutions of markets (Chae et al, 2006; Yang et al, 2011; Kaw and Roleda, 2014). Related to these are models for price return distribution of Bucsa et al (2011) and a quantum model for stock market by Zhang and Huang (2010).

In this study, the dynamics of San Miguel Corporation (SMC) stock price log return from 2003 to 2013 is analysed by considering statistical ensembles constructed out of 60-day sections of the time series. The temporal evolution of the log return distribution is then determined from the time-dependence of probability distribution function (PDF) parameters within the 60-day period. With this, a Feynman-Kac model is then constructed by establishing the dynamical equation obeyed by the time-dependent PDF, and determining the corresponding potential.

2. METHODOLOGY

Dynamics of the San Miguel Corporation (SMC) stock is determined through the log return

$$x_i = \ln p_i/p_0 \qquad \text{(Eq. 1)}$$

where:

x_i = log return at day (i)
p_i = closing stock price at day (i)
p_0 = reference price

The mean closing price of the time series was used as the reference price p_0. To construct a statistical ensemble, the time series was divided into 60-day sections. The day-i ensemble is then composed of the day-i stock price in each 60-day section. The best-fit probability distribution for each day-i ensemble was determined using the statistical software Easyfit. The most commonly occurring best-fit probability density function (PDF) was then taken to be the representative PDF for the entire 60-day period. The time-dependence of the PDF parameters is determined from the values of the parameters for each day-i, best-fitted using a least-squares routine. The dynamical equation for the corresponding PDF is then obtained through the partial time and log-return derivatives of the PDF. The potential that defines the stochastic model is then obtained through a comparison of the obtained partial differential equation with the Feyman-Kac equation.

3. RESULTS AND DISCUSSION

The closing stock price of San Miguel Corporation from 2003 to 2013 is shown in Fig. 1. The corresponding log-return time series, using Eq. 1, is shown in Fig. 2.

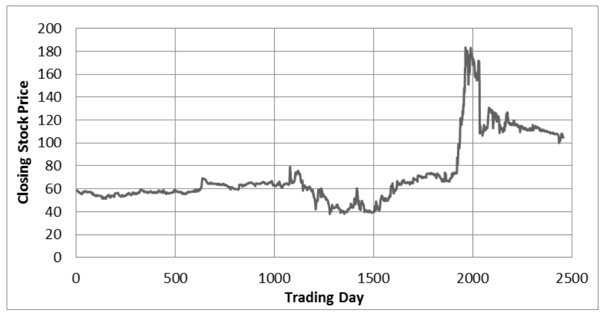

Fig.1. Closing stock price time series of San Miguel Corporation from 2003 to 2010

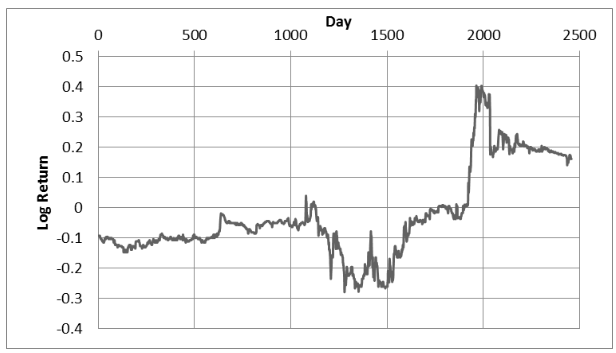

Fig.2. Log-return time series of San Miguel Corporation from 2003 to 2010

A sampling of 60-day sections of the log return time series in Fig. 3 shows that the sections are similar enough to be considered as elements of a statistical ensemble. With this, an empirical statistical distribution is obtained for each day-t of the 60-day period. Curve-fitting using Easyfit revealed that the most commonly-occurring best-fit probability density function (PDF) over the 60-day period is the log logistic distribution

$$f(x) = \frac{a}{b}\left(\frac{x-g}{b}\right)^{a-1}\left[1+\left(\frac{x-g}{b}\right)^{a}\right]^{2} \qquad \text{(Eq. 2)}$$

where:

α : scale parameter
β : shape parameter
γ : center parameter

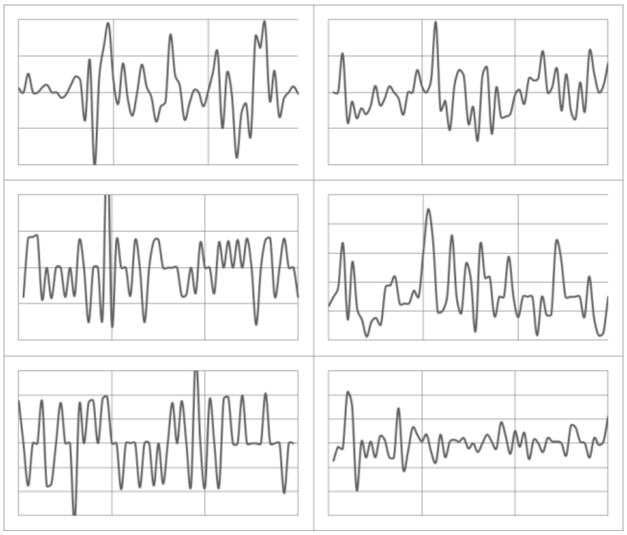

Fig. 3. Sampling of 60-day sections in the log return time series

The values of the distribution parameters for each of the 60 days is shown in Figures 4 to 6.

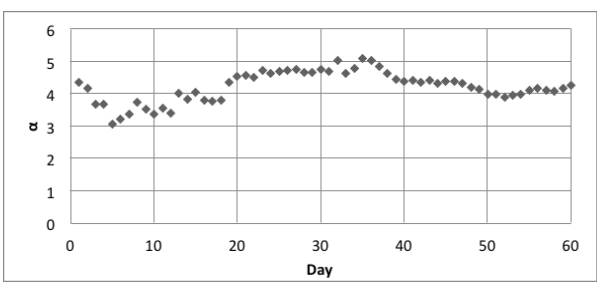

Fig. 4. The scale parameter α for each of the 60 days

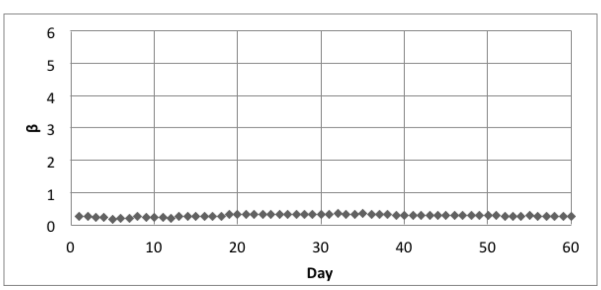

Fig. 5. The shape parameter β for each of the 60 days

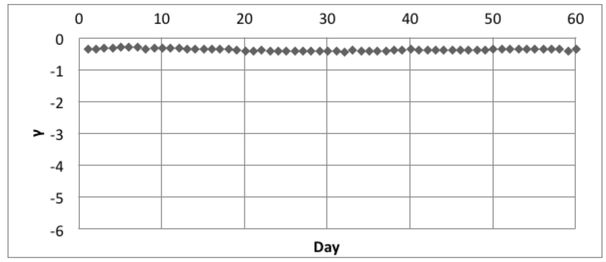

Fig. 6. The center parameter γ for each of the 60 days

Of the three parameters, only the scale parameter has significant time-dependence. Curve-fitting using the method of least squares shows that

$$\alpha(t) = -0.0011t^2 + 0.0779t + 3.245 \quad \text{(Eq. 3)}$$

Letting

$$z = \frac{x - \lambda}{\beta} \quad \text{(Eq. 4)}$$

it can be shown that

$$f = \frac{\alpha}{\beta} z^{\alpha-1}\left(1 + z^{\alpha}\right)^{-2} \quad \text{(Eq. 5)}$$

$$f' \equiv \frac{\partial f}{\partial x} = \frac{f}{\beta}\left[\frac{\alpha-1}{z} - \frac{2\alpha z^{\alpha-1}}{1+z^{\alpha}}\right] \quad \text{(Eq. 6)}$$

$$f'' \equiv \frac{\partial^2 f}{\partial x^2} = \left[\frac{\alpha-1}{z} - \frac{2\alpha z^{\alpha-1}}{1+z^{\alpha}}\right]\left[\frac{f'}{\beta} - \frac{f}{z\beta^2}\right] - \frac{2\alpha}{z\beta} f^2 \quad \text{(Eq. 7)}$$

$$\dot{f} \equiv \frac{\partial f}{\partial t} = \frac{\dot{\alpha}}{\alpha}\left[1 + \alpha\left(\frac{1 - z^{\alpha}}{1 + z^{\alpha}}\right)\ln z\right] f \quad \text{(Eq. 8)}$$

$$\ddot{f} \equiv \frac{\partial^2 f}{\partial t^2} = \frac{\dot{f}^2}{f} + \frac{\ddot{\alpha}}{\dot{\alpha}}\dot{f} + \frac{2\dot{\alpha}^2 \beta}{\alpha^2} z^{\alpha+1} f^2 \ln z \quad \text{(Eq. 9)}$$

where

$$\dot{\alpha} = \frac{d\alpha}{dt} = 0.0779 - 0.0022t \quad \text{(Eq. 10)}$$

A comparison of eqns. 6 and 8 revealed that

$$\dot{f} = \frac{\dot{a}}{a}\left[(1+\ln z) f + b f' z \ln z\right] \qquad \text{(Eq. 11)}$$

which may be recast as

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial x}\left(\frac{\dot{a}}{a} b f z \ln z\right) \qquad \text{(Eq. 12)}$$

This is a diffusion-free Fokker-Planck equation, also known as the Liouville equation:

$$\frac{\partial f}{\partial t} = -\frac{\partial \mu f}{\partial x} + \frac{\partial^2 D f}{\partial x^2} \qquad \text{(Eq. 13)}$$

where:

μ is the drift
D is the diffusion coefficient

The drift in this case is

$$\mu = -\frac{\dot{a} b}{a} z \ln z \qquad \text{(Eq. 14)}$$

The PDF dynamical equation (Eq. 12) can also be cast as a Feynman-Kac partial differential equation

$$\frac{\partial f}{\partial t} + \mu \frac{\partial f}{\partial x} + \tfrac{1}{2}\sigma^2 \frac{\partial^2 f}{\partial x^2} - V f + g = 0 \qquad \text{(Eq. 15)}$$

since eqn. 12 can be written as

$$\frac{\partial f}{\partial t} - \frac{\dot{a}}{a} b z \ln z \frac{\partial f}{\partial x} - \frac{\dot{a}}{a} b (1+\ln z) f = 0 \qquad \text{(Eq. 16)}$$

The SMC log return can thus be described stochastically by a log logistic probability density function, and the evolution of this PDF is governed by a time-varying potential

$$V(x,t) = \frac{\dot{a}}{a} b\left(1+\ln\frac{x-\gamma}{b}\right) \qquad \text{(Eq. 17)}$$

A plot of this potential is shown in Fig. 7.

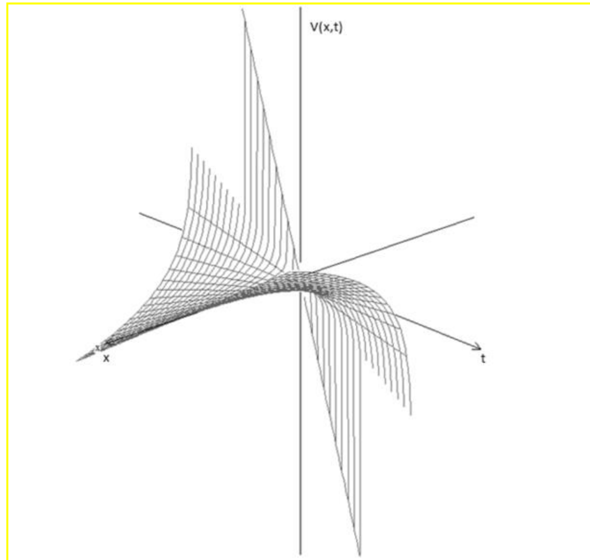

Fig.7. A plot of the potential that governs the evolution of the log-logistic probability density function that stochastically describes the SMC stock price log return.

4. CONCLUSIONS

It was shown in this study that the log return of the SMC stock price are distributed log logistically. Over a 60-day period, this distribution obey a diffusion-free Fokker Planck equation. Modelled using the Feynman-Kac equation, the evolution of the distribution is governed by a logarithmic potential. It would be interesting to find out if other blue-chip stocks behave similarly, and it would be worth knowing how other types of stocks behave,

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