A dynamic Leontief model with stochastic extensions for sustainable Jatropha curcas biofuel supply chain

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Abstract: Jatropha biofuel is an excellent candidate for an alternative renewable and clean energy source. However, an increasing number of countries and investors are losing interest in J. curcas mainly because of the challenges they encountered caused by ineffective organization and coordination of the entire supply chains. For this purpose, a time varying input-output model with stochastic extension had been presented which can be used to simulate the supply chain dynamics of J. curcas biofuel. The model incorporates various extensions to the basic Leontief input-output model such as non-square matrix requirement for the technology matrix, adapting behavior based on target and current production levels, a control matrix to intervene on undesirable dynamics, and a Gaussian stochastic parameter that simulates uncontrolled changes in the production capacity which may be brought up by natural calamities and market instabilities. Numerical simulations of two-sector case studies having behavioral matrix patterned after the broad interactions between various sectorial agents were able to replicate the general trends of J. curcas biodiesel supply chain. The coefficients of the control matrix used to eliminate oscillatory dynamics of the system made physical suggestions that are strikingly similar to the ones made by some researchers in the field. With the incorporation of stochastic extension, the model was able to make predictions about the relative extent on how different streams will be affected by sudden random changes that are brought into the system.

Key words: Jatropha curcas, stochastic input-output, Control Theory, supply chain, biofuel

1. INTRODUCTION

The International Energy Agency has a goal of reducing dependence on petroleum and coal by meeting more than a quarter of world demand for transportation fuel using biofuels and developing nations are engaged in research and production of fuels from certain plant materials (Tanaka 2011, Dufey 2007, de Fraiture et al. 2008, Pickett 2008).

Jatropha curcas, a flowering semi-evergreen shrub has attracted a lot of attention in recent years. Biodiesel derived from J. curcas seeds can be used alone or blended with petroleum diesel to run standard diesel engines. The methyl ester properties of J. curcas oil are similar to diesel fuel and transesterification of this oil can produce biodiesel that meets the specification standards of both the American Society for Testing and Materials and European Committee for Standardization (Kemp 2006, Abdelraheem et al. 2013). Jatropha biodiesel can be further processed into aviation turbine fuels. In the future, J. curcas fuel may find its application in aviation more than in land transportation since there are fewer aircraft than automobiles and far less complex infrastructure to deal with, as there are only a few hundred airport fueling stations around the world (Kanter 2008).

Economists and politicians saw a potential to generate livelihood opportunities through supply chain of J. curcas fuel (Gupta 2012). However, in recent years, there has been an increased concern over various social and economic effects of biofuel production and use. Being unable to deliver the foreseen profits, many reconsider venturing into this business and some even totally abandon it. Ever since foreign company investors entered Tanzania to venture into J. curcas plantation, many locals believe that the valuable land should instead be used to grow food. A report of WWF Tanzania have claimed that after years of doing business, no single job was created (Mutch 2010). In Kenya, researchers found biofuel crop to be of little economic benefit to farmers. Because of lack of returns, many farmers started to uproot their J. curcas plants to pave way for more profitable crops like maize (Ngotho 2012). The officials of the Energy Department of the Philippines admitted that J. curcas is not viable in the country, a couple of years after it started converting hundreds of hectares of marginal and agricultural lands and spending over a billion PHP (about 20 million USD) for the project (Cordon 2013).

It is important to take note that those countries who fail to establish a stable and sustainable J. curcas biofuel supply chain share similar pattern in their production. While significant emphasis was placed in the upstream production capacity, downstream layers of the supply chain such as oil pressing and diesel production have not been given enough investments. Without processing plants to convert seeds to biofuel of enough quantity and acceptable quality, many investors and farmers lose confidence in the project. These imbalance of priorities appears to result from two factors. First, J. curcas is generally simple to grow, making conversion of idle,
non-agricultural lands relatively easy. Second, scientific and technical research and development necessary for large-scale conversion of J. curcas seed into biodiesel did not meet public expectation, making investment in these areas relatively risky (Camiels 2007, Urtarte 2008, Cruz et al. 2009). It is unfortunate that biofuel, despite its successes in some countries, failed to achieve sustainability in developing countries. It is thus of major interest to develop models that can be used to analyze the dynamics of biofuel systems. These models can be used to device appropriate interventions that will drive the production of every sector toward desired path.

In this paper, we developed a dynamic input-output model of Cruz et al. with stochastic parameters for the analysis of biofuel supply chains. The paper incorporates the intersectoral connections within supply chains, which include technological, behavioral and control coefficients. The additional stochastic coefficients represents sudden changes in the dynamics of the system which may be brought by natural calamities such as typhoon and draught. Finally, semi-quantitative analysis of real world and theoretical scenarios limited to two sectors (upstream and downstream) are given to introduce the general features of the model.

2. MODELING FRAMEWORK

The most basic equation of Leontief static input-output model is in the form

\[ X = AX + Y \] (Eq. 1)

where \( X \) is the total output vector whose elements represents total output in each sector, \( Y \) is the net output vector whose elements represents net output in each sector, and \( A \) is the technology matrix which indicates how much of each sector went into the production of one unit of each product. The equation suggest that internal consumption of products are required in order for the sectors to function. In most economic applications, the goal is to determine the total amount of products \( X \) required to be produced in each sector to meet the desired demand \( Y \) given a predetermined technology matrix \( A \). Rearranging in Eq. (1) gives:

\[ X = (I - A)^{-1}Y \] (Eq. 2)

where \((I - A)^{-1}\) is known as the Leontief inverse. Notice that this form restricts the dimensions of the technology matrix to \( n \times n \). To eliminate this limitation, Cruz et al. (2009) introduced a modified input-output equation:

\[ X_{t+1} = (I - B_0T_0 + B_0K_0)X_t + B_0Z_t \] (Eq. 3)

where \( T_0 \) is the modified technology matrix that need not to be square, \( X_{t+1} \) and \( X_t \) represent the production vectors at time \( t + 1 \) and \( t \) respectively, \( Z_t \) is the set point vector that contains the desired number of output or input of each stream, \( B_0 \) corresponds to behavioral matrix which indicates the influence of the difference between \( Z_t \) and \( Y_t \) and \( K_0 \) can be thought of as the control matrix. This form of Leontief model eliminates the restriction in the number of rows that \( T_0 \) can assume offering more freedom and flexibility to input-output system models. A system that exhibits unfavorable dynamics such as instability or oscillation as a result of \((I - B_0T_0)\) can be controlled by setting appropriate values of control matrix \( K_0 \). It can be implied from Eq. (13) that \( K_0 \) has the same dimensions as \( T_0 \).

In this paper, we introduced a stochastic extension to the works of Cruz et al. to simulate the possibility of sudden and uncontrolled changes in the production capacity of sectors.

\[ X_{t+1} = (I - B_0T_0 + B_0K_0)X_t + B_0Z_t + S_tX_t \] (Eq. 4)

where

\[ \Delta B = \text{random changes in behavioral matrix} \]
\[ \Delta T = \text{random changes in technology matrix} \]
\[ \Delta K = \text{random changes in control matrix} \]
\[ S_t = \text{stochastic matrix.} \]

3. TWO - SECTOR SUPPLY CHAIN SIMULATIONS

The primary goal of this paper is limited to the presentation of a complex, evolutionary and non-deterministic model of effective supply chain design. No attempt has been made to develop an accurate matrix patterned after actual quantitative data. Instead, emphasis has been given to illustrate the model's main features, which can be summarized as follows: (1) ability to model the general behaviour of a supply chain, (2) ability to stabilize any oscillating and self-extinguishing systems using appropriate values of the control matrix, and (3) ability to predict the relative extent on how stochastic parameters affects each stream. Even though the values used in the model are hypothetical, the coefficients were chosen to reflect the overall characteristics of the sectorial production capacity and the behavior of agents in J. curcas supply chains, as implied by relevant statistical data, government legislations, researches and interviews in some developing countries particularly the Republic of Philippines and the United Republic of Tanzania. We apply the model in two-sector J. curcas biodiesel supply chain case studies.

For every scenario, the technology matrix and set point vector used is

\[ T = \begin{pmatrix} 1 & -2 \\ -0.1 & 1 \\ -0.5 & -0.2 \end{pmatrix} \]
\[ \begin{pmatrix} Z_0 = \begin{pmatrix} 0 \\ 100 \\ -60 \end{pmatrix} \end{pmatrix} \]

For the technology matrix, column 1 implies that 0.1 unit of final product and 0.5 unit of natural resource are needed to produce 1 unit of feedstock and column 2 implies that 2 units of feedstock and 0.2 unit of natural resource are needed to produce 1 unit of final product. For the set point vector row 1 indicates that it is desired to have a net feedstock output of zero, which means that the output from the upstream sector must be fully consumed in the downstream production row 2 shows that the target amount of final product output per unit time is 100 units, and row 3 indicates that the desired level of natural resource consumption is 60 units.

3.1 Scenario 1

The first scenario is developed based on the behaviors exhibited by the agents on the first few years of the
establishment of J. curcas supply chain. The behavioral matrix used in this system is

\[
B_0 = \begin{pmatrix}
-0.2 & 0.6 & 0 \\
-0.1 & 0.1 & 0
\end{pmatrix}
\]

\[B_{11}\] shows that the farm capacity increases in response to surplus of oilseed. This seemingly unreasonable behavior reflects the initial “hype” on J. curcas being a cash crop, able to yield two to nine times as much per hectare as compared to conventional crops (van Eijck and Romijn 2006). Much of the oilseed produced is used and sold to put up new farms. \[B_{12}\] indicates that the lack of biofuel final products at that time strongly influenced farmers and investors to further expand their plantation in anticipation of the foreseen profits on these crops. \[B_{21}\] and \[B_{22}\] reflects the relatively slower response of fuel processing sector expansion. Finally, \[B_{13}\] and \[B_{23}\] indicates that the upstream and downstream sectors are not influenced by natural resources availability. This values take into consideration the fact that the planning and decision making of the investors in both sectors are generally unaffected by the availability of land or water resources, since researches at that time claim that J. curcas can be cultivated easily by being able to thrive even in low-nutrient dry marginal soils (Abdelraheem et al. 2013).

\[
K = \begin{pmatrix}
0 & 0 \\
-0.5 & 0.8 \\
0 & 0
\end{pmatrix}
\]

Each non-zero coefficient of the control matrix represents outside interventions that can be applied to eliminate or reduce undesirable dynamics within the system. For the above matrix, \(K_{11}\) affects the behavior of the downstream sector by “encouraging” the sector to increase its processing capacity to compensate for the surplus of oilseeds. This may reflect research grants for the development of more efficient methods of oil pressing and transesterification and market-based interventions such as subsidies and tax-exemptions. On the other hand, \(K_{22}\) influences the same sector to increase its sensitivity in deficiency of final biofuel production. This may represent mandates for increased information sharing among downstream actors. Dynamic response of the production level and net flows are shown in Figs. 3 and 4, respectively. Comparison between these simulations and that of Scenario 1 shows a significant improvement in trend. At around \(t = 25\), the total upstream and downstream production approaches a steady state. On the other hand, at around \(t = 40\), the net flows assume steady state values.

We initially set the control matrix to be null to see how the supply chain evolve by itself based on the agent behaviors detailed above. The results of the dynamic input-output modeling using equation 3 are shown in figures 1 and 2, respectively. Figure 1 shows an imbalance in the production capability of upstream and downstream sectors. The production of final biofuel product cannot keep up with the production of oilseeds. After reaching a maximum production of about 630 units, the upstream production falls to zero at around \(t = 25\). Extrapolation of this trend shows that the general behavior of the system is oscillating and decaying quite fast. The system will eventually self-extinguish. This behavior matches the reported trends in Tanzania and Philippines. The inability of downstream sector to produce and sell final biofuel product causes a surplus of oilseeds. Over time, many investors and local farmers grew increasingly impatient of the lack of returns of their investments since oilseed in itself is not very profitable. In the Philippines, many have abandoned their investments and most farmers have uprooted their J. curcas crops. Eventually, the government itself withdrew all of its investments in J. curcas biodiesel production (Gatulala 2011).

For the next part, we try to stabilize the dynamics of the supply chain system using the control matrix \(K\):

\[
K = \begin{pmatrix}
0 & 0 \\
-0.5 & 0.8 \\
0 & 0
\end{pmatrix}
\]

Interestingly, the hypothetical values used in the control matrix strongly reflect the recommendations of some researchers in Philippines and Tanzania, as an answer to the
non-favorable trends in J. curcas in their respective regions. The proceedings of the Philippines' National Academy of Science and Technology workshop on biofuels, held on September 2007 stress the role that academia has to play for the success of J. curcas biofuel program. Suggested field of study includes manufacturing and fabrication of manually operated equipment for extraction of J. curcas oil, production of Jatropha methyl esters, other application of J. curcas oil and engine performance testing. The researchers also proposed creation of a centralized repository of information such as a website, which will help dissemination of information, promote awareness and answer some frequently asked questions (Culaba 2007). Diligent, a Dutch J. curcas biofuel investor company in Tanzania have suggested that sufficient attention to active and frequent communication with different actors along the supply chain is necessary to correct unstable and diverging supply chain system (Caniels 2007).

3.2 Scenario 2

In the previous scenario, the behavioral matrix was based on the observed behaviors of various actors in the supply chain. In this next scenario, a behavioral matrix was used to produce a highly stable, self-sustaining system. The behavioral matrix and control matrix that produced the stable biofuel supply chain is surprisingly simple:

$$B_0 = \begin{pmatrix} 0.2 & 0.2 & 0 \\ 0 & 0.1 & 0 \end{pmatrix}$$

The dynamic trends of the production and resource flows are shown in figures 5 and 6. Note that there is practically no oscillation. The production at the upstream and downstream sector increase over time until it reached a steady value. From the start, the net upstream output is zero which means that every feedstock produced in the upstream sector have been consumed in the downstream sector, as desired. A steady net biofuel production of 100 units will also be achieved over time. Even though consumption of natural resources is more than double than what was intended, the predictability of the dynamic trajectory will make it easier device a plan to compensate for this extra demands.

For two reasons, the behavioral matrix that produced these dynamics would not be very hard to physically replicate. First, only three of the six coefficients need to have non-zero values. It means that it is not necessary for one stream to fine tune its behavior based on the production of every other streams. Second, the values of the coefficients are not too extreme. Physically, this would imply that radical modification in a stream's production capability is generally not required every time that the production capability of other stream changes.

3.3 Effect of random changes in sectorial production capacity to the dynamics of the supply chain system

In this section we will apply the stochastic parameters described by equation 4 to generally stable systems to simulate sudden changes in the production of each sector. Physically, this may reflect changes brought by natural calamities, increase in demand, inflation and other related phenomena. We will investigate how each sector responds to these changes and how long would it take the entire supply chain system to re-establish equilibrium by itself. We will do it in two ways. First, we will create a setup where oversupply or undersupply in upstream and downstream production can happen at any time to study the extent on how each sector will be affected. Second, we will create a setup in which a single extreme change in production capacities of the upstream and downstream sector happen at a relatively early time in the
supply chain system’s time evolution and then examine how long it would take the system to establish equilibrium.

For the first setup, we will consider the generally stable systems produced in scenario 2. The model has been calibrated in such a way that there is a 10 percent chance that the coefficients of stochastic matrix $S$ will change its value from zero to a randomly generated number. We have used Gaussian distribution having a mean value of zero to equally simulate increase or decrease in production. The magnitude of the random change is equal to the quantity of previous production multiplied to a random value having a standard deviation of 0.1. The mean and standard deviation can be calibrated using the raw data. We run the simulations fifty times to determine if the supply chain will maintain its favorable dynamics and to see which of the product flows are most affected by these changes.

To demonstrate the overall effect of the stochastic parameters, three randomly chosen results were plotted against each other for each of the three scenarios. These was shown from Fig. 7 and Fig. 8. It can be seen that the total upstream production displays greater fluctuation in its values compared to the total downstream production. The farm production capacity also shows the greatest instability in the resource flows.

![Graph](image)

**Fig. 7.** Stochastic dynamics of upstream and downstream sector production capacities in Scenario 2

From these results, it can be implied that if there will be a sudden change in the production capacity of the supply chain, the farm production sector is the one who will be most affected. It is thus suggested to have a general plan to prepare for the unexpected oversupply or undersupply of biofuel feedstock. Practical and safe methods of storing J. curcas seeds must be developed. On a small scale, excess seeds may be manually pressed to be used in oil lamps, soap making, and cooking, similar to what was traditionally being done in African countries like Tanzania (Carrie 2007). Conventional manual harvesting is estimated to require a very large workforce, therefore mechanical or more efficient manual harvesting systems need to be urgently developed to prepare for sudden increase of demand (Abdelraheem 2013).

For the second setup, we consider a system having the following characteristics:

$$ B = \begin{pmatrix} -0.1 & 0.8 & 0.01 \\ -0.05 & 0.1 & 0.01 \\ 0 & 0.2 & 0 \end{pmatrix}, \quad K = \begin{pmatrix} 0 & 0 \\ -0.3 & 0.6 \\ 0.2 & 0 \end{pmatrix} $$

The model have been programed such that the upstream and downstream production will experience a one-time increase or decrease by an amount equal to its previous production value multiplied to random number having standard deviation of 25, which will result in one of four possible cases. Case 1 replicates the time evolution of systems where both upstream and downstream production have increased. Case 2 describes a situation where the production capacity of both the upstream and the downstream sector have declined radically. Case 3 depicts a scenario where the upstream production capacity have experienced considerable growth while the downstream production capacity have been reduced. and Case 4 models situations where there is a regression of production of the upstream sector despite a notable increase in the ability of downstream sector to process fuel. Simulations were done 100 times. The average of the differences between the productions at each time stage for every single trial and the difference between the dynamic trends of the production and resource flow of stochastic and non-stochastic system for each of the cases were determined. To show the dynamic trend of the difference between the ideal, non-stochastic system dynamics and the stochastic system dynamics, the graphs of the gross upstream and downstream production for each of the cases are shown in Fig. 9.

![Graph](image)

**Fig. 8** Stochastic dynamics of product and resource flows in Scenario 2

**Fig. 9.** Difference between stochastic and non-stochastic upstream and downstream sector production capacities

For each of the cases, the results of the simulation show that at about $t \approx 30$ after the sudden change of production, the difference between the production capacity of the stochastic and non-stochastic system approach zero, thus the system reestablishes equilibrium. This suggest that, a
generally stable system affected by random, abrupt change after a certain recovery time will resume stability by itself because of the adapting behavior of the model. The model presented made it possible to make crude predictions about the time needed by the system to recommence favourable dynamics. This feature can be applied in situations where the undesirable dynamics of a certain supply chain system can be solved by more than one possible solution. As discussed earlier, such solutions can be translated in a form of control matrix $K$. Since every solution will translate in a distinct set of coefficients of the control matrix, the system will be stabilized in different ways. The model may be helpful in comparing the long term effectiveness of each of these solutions before being implemented. This can be done by running simulations containing a single chaotic event and comparing the recovery time of the systems. The system that has the shortest recovery time is the most dynamically stable and the solution that produced that system is the most favorable.

4. CONCLUSION

The defining characteristic of this model is that each sector adjusts its production capacity based on the weighted sensitivity to the surplus or deficit of product and resource flows. It has been shown that the model is equally able to simulate supply chain trends and be used to generate logical predictions. The results of the simulations imply that aside from the physical linkages between the processes, the interactions between each sector plays a major role in defining the system dynamics. Numerical simulations have demonstrated that virtually all undesirable dynamics of the supply chain can be suppressed if not totally eliminated with appropriate coefficients of the control matrix. These coefficients made physical implications and suggestions that agreed with some of the recommendations by earlier researchers conducted in the Philippines and Tanzania. First, our model have shown that the unwanted dynamics may be corrected if the fuel processing sector catches up with the development of farm production sector, a conclusion that was similar to the once made by researchers which lead them to propose the need to increase the downstream sector production capacity in general and endorse further research in fields including seed harvesting and oil pressing methods, large scale transesterification and engine performance testing. Second, the simulations had demonstrated that if the sensitivity of the downstream sector actors to the deficiency of production within their own sector is increased, it will significantly improve the stability of the supply chain dynamics. This is similar to the researchers’ call for centralized information hub and increased frequency of active communication between different actors in the supply chain. Third, both the model and researchers agree that there is a need for an increased awareness in the amount of natural resources consumed especially in the processing sector. Finally, the numerical simulation and researchers both recommend that farmers venturing in J. curcas agriculture should start on a small scale since nascent biofuel supply chain evolve better if the upstream sector production will start slow, reducing the possibility of imbalance between upstream and downstream processing capacity. Running the simulations with stochastic extension predicts that in the event of sudden changes in production, which may be brought up by natural calamities and undesirable market trends, the farm production sector will be the one who will be most affected suggesting the need of practical and safe J. curcas seed storage methods and development of other applications of J. curcas oil, seedcake and its other by-products.

The model presented may also be applied in the analysis and development of practically any emergent supply chain. Suggested possible extensions include addition of new sectors, products and resource flows that will streamline the model to the kind of supply chain being studied. Technology coefficients calibrated after real world data will make the predictions more specific. Finally, static behavioral matrix may be replaced with agent based modelling which will more accurately simulate the interactions between supply chain actors over time.

6. REFERENCES


