A Lower-Bound Heuristic to Minimize Weighted Flowtime $\Sigma W_j C_j$ in the Open Shop

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Abstract: Scheduling jobs through machines with no specified order of machine processing poses more sequencing possibilities compared to the jobs which have precedence constraints. This open shop problem with deterministic processing times, and no preemption, with objective of minimizing flowtime can represent the cost of time that the job spends inside the production system. The difference between completion time and the time when the job was released is called the flowtime. For simplicity, it is assumed in this paper that all jobs were available for scheduling at time=0, and hence, flowtime is the completion time C_j of each job. The longer a job is in the system, a certain delay cost over the duration is ascribed by the job's weight. Pinedo (2008) showed that the minimizing weighted completion time $\Sigma W_j C_j$ sequencing problem is NP-hard, and, therefore, justifies the development of heuristic procedures to possibly shorten the schedule search process. The paper will present a schedule search heuristic that uses the relative values of processing times on the bottleneck and non-bottleneck machines to create an initial set of sequences on the bottleneck machine. A lower-bound operation would be presented that can be used to evaluate the attractiveness of an initial sequence for improvement efforts in a branch-and-bound procedure. The paper concludes by showing certain problem sets where complete enumeration was used to determine optimal sequences. The presented heuristic offers promising results with less computational effort.

Key Words: Open Shop Scheduling; Weighted Flowtime; Production Planning

1. INTRODUCTION

1.1 Open Shop Sequencing

The open shop scheduling problem in this paper assumes that all jobs are available for sequencing at the beginning of the planning period. There exists M machines that each job must go through in any order. Unlike the flowshop—where all jobs go through the same prescribed order—the open shop has more flexibility in machining order. Each job j (j=1,2.. k) has a deterministic processing time P_{ij} on machine i (i=1,2..M) and weight W_j. Once a job has begun, it cannot be interrupted until completion (i.e. non-preemptive schedule.)

Common open shop applications occur in testing and maintenance, where the order of which tested items can be processed may be of no consequence, as long as all items are tested. The inspectors would be the machines, while the machines to be maintained are the jobs. Another example would be teacher-class time-tabling. Teachers (the machines) need to be assigned to classes (jobs) but cannot have multiple instances of classes in the same time, but can teach the assigned classes in any order for the day.

1.2 Total Weighted Flowtime as Criteria

Flowtime is the time that a job spends in a shop. When all jobs are available/released at time t=0, flowtime is the completion time C_j of each job. Some jobs can be more important than others, hence a weight W_j may be assigned, representing the time-value of each job while waiting and being processed through the set of machines in the open shop. The sum of the weighted flowtime ΣW_jC_j can be thought of as the total cost of all jobs spending time in the shop.

Pinedo (2008) has shown that the objective of minimizing weighted completion times ΣW_jC_j is NP hard for more than one-machine setups. The open shop of interest assumes at least two machines, and qualifies as not easy to solve in polynomial time for search algorithms that use complete enumeration of relatively non-infinite sequences to test. This is the motivation for this paper's proposed heuristic.

2. HEURISTIC DEVELOPMENT

Each set of open shop jobs determines which machine has the highest utilization of processing time. The machine with the highest total processing times will be henceforth designated as the bottleneck machine. When this constraining resource is utilized well, we should be able to find a sequence of jobs on the other machines which would fit the non-interfering times of those jobs on the bottleneck machine. We could even say that the bottleneck machine sequence is the critical decision in the open shop. Consider the open shop problem shown in table 1. We could see that Machine M3 has the highest total processing time of 21 hours. When a correct sequence of jobs is made on Machine M3 then we could search for fitting sequences on the other machines.

Table 1. Open Shop processing times for illustration

Machine\Job	J1	J2	J3
M1	4 hrs	9	3
M2	3	0	8
M3	6	5	10
Weight W_j	1	2	3

A lower-bound for total weighted flowtime is proposed to assess if each bottleneck sequence may be useful for further sequences search. Such a lower bound would help in a branch-and-bound search procedure across the possible sequences. It is proposed in this paper that only the bottleneck machine's sequences should be generated and assessed using the lower bound procedure described below, to gather enough sense of whether to stop searching through not promising sequences, or to continue searching until no improvement can be made on an initial schedule.

The lower bound for the total weighted flowtime for a given sequence S on the bottleneck machine can be made using the following proposed steps:

1. Find the completion times for each job sequenced on the bottleneck machine.
2. If the job is first on the sequence, designate the lower bound for completion time for this job as the sum of all its processing times. This is so because we could schedule, at best, this job on the other machines as soon as it ends on the bottleneck machine. If the job is the last on the sequence, the lowerbound for completion time should be the time of completion in the bottleneck machine, which should theoretically be the sum of the processing times in said machine. The processing times in the other non-bottleneck machines can be allocated so that it can finish just prior to the beginning of this last job on the bottleneck.
3. For the jobs in the middle of the sequence, consider the other times on the other machines. If a time window is available on another machine whose processing times is less than the preceding jobs on

the bottleneck, then the lower bound for completion times should be the sum of the *other* machining times plus the completion time of job on the bottleneck machine. We assume that the completion times on the bottleneck is non-changing and remain uninterrupted, and that if any processing times cannot fit in the prior times, then completion should occur after the job is done on the bottleneck machine.

4. When the lower bound of completion times for each job is found, we can multiply these times to each job's respective weights, and determine the sum of these products to be the lower bound for the total weighted flowtime of this sequence of jobs.

To illustrate this lower bound determination, there exists 3!=6 possible sequences on M3 on the illustrative example. These are 1-2-3, 1-3-2, 2-1-3, 2-3-1, 3-1-2, 3-2-1. We can demonstrate the lower bound for the first sequence 1-2-3. Fig. 1 shows a partially completed schedule with Machine 3 having the sequence J1-J2-J3.

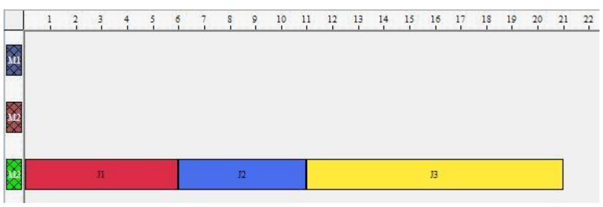


Fig. 1. Partially completed sequence for illustrative example for lower bound (LB) weighted flowtime determination

Job J1 ends at t=6, which means that the earliest completion time for J1 would be its total processing time 6+4+3 = 13. The last job J3 ends at time 21, and since its processing times on Machines M1 and M2 sum to 3+8=11, then the time window before J3 (t=0 to 11) is enough to fit the other machining times. J3 can finish at t=21. The middle job J2 has only one other processing of 9 hours in M1. Notice that the time before J2 begins on M3 is only 6 hours interval, then the processing time on machine M1 of job 2 P_{12}=9 hours cannot fit prior to M3 processing, it would therefore fit after M3 processing. Projected completion time would then be at t=20 hours (end at M3 t=11 plus 9 hours.) When the respective completion time (J1,J2,J3)=(13, 20, 21) are weighted (1,2,3), we would have a lower bound for weighted flowtime of (e.g. 13x1+20x2+21x3) 116.

Table 2 can show the projected LB weighted flow times for all the 6 possible permutations for M3. The scheduling heuristic procedure could now proceed to using the lowest possible lower bound (i.e. sequence 2-1-3 with LB=105) to generate the possible schedule shown in Fig. 2. Jobs 1,2,3 completed at time t=(18,14,21) with an actual total weighted flowtime of 109, slightly higher than the lower bound 105. One can survey the lower bounds on Table 2 and see that this 109 best actual value cannot be beaten by any of the other sequences.

Table 2. Lower bounds for all M3 sequences for illustrative example

	Completion times			
M3 Sequence	J1	J2	J3	Wtd Flowtime
123	13	20	21	116
132	13	21	24	127
213	14	14	21	105
231	21	14	23	118
312	16	21	21	121
321	21	15	21	114

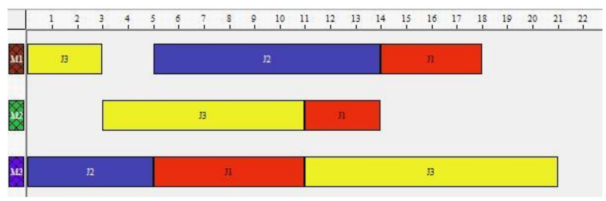


Fig. 2 Completed schedule for illustrative example. Actual total weighted flowtime=109 for the best sequence 2-1-3 on bottleneck M3

3. RESULTS AND DISCUSSION

We can make observations about the heuristics' search results in the illustrative problem to show how the best sequence was arrived at. The job sequence 2-1-3 corresponds to the shortest processing time (SPT) rule of minimizing flowtime on a single machine. The processing times in ascending order on M3 are (5-6-10) corresponding to the job sequence (2-1-3). The heuristic can further be hastened by using the SPT rule on the bottleneck machine, and try to see if other neighbourhood swap schedules would improve on the SPT sequence through evaluating the lower bounds of $\Sigma W_j C_j$ other sequences. When a partial enumeration of sequences have worst performances, then such sequences should not be considered further.

The drawback to this revision is that one does not know when to stop searching. When the number of jobs are palpably high, then complete enumeration cannot be a limit to the search. A partial enumerated search procedure could then be made on the first and last jobs, just to see if certain sequences are promising relative to the SPT sequence recommended first.

A second illustrative problem would strengthen the case for using SPT, as in the open shop problem shown in Table 3.

Table 3. Second illustrative example of Open Shop

Machine\Job	J1	J2	J3
M1	6 hrs	3	7
M2	2	5	10
Weight W_j	1	2	3

Bottleneck machine is M2, with total makespan of 17 hours. SPT recommends J1-J2-J3 sequence on M2. Such a sequence would have respective completion times at t=2 hrs, 7 hrs and 17 hrs.

The first on the sequence Job 1 can complete at the smallest total time of 2+6=8 hrs. The last job J3 can end at 17th hr, and a time interval of exactly 7 hrs is available prior to M2 starting time at t=7. This means that job 3 on machine M2 can begin as soon as it finishes at M1, with no apparent delay.

The middle job J2 has a non-bottleneck processing time of 3 hrs, and this will not fit in the 2 hour time window set by the first job J1, so machining in M1 for job J2 can only occur after M2 processing, ending at t=7. Completion lower bound is 7+3=10 hours.

Completion times for Jobs 1-2-3 is projected at times 8, 10 and 17. Respectively weighed by 1-2-3, and lower bound for the sumproduct $\Sigma W_j C_j$ is 79.

With only 3 jobs, the lower bounds for weighted flowtimes can be determined, and is shown on Table 4.

From the sequence 1-2-3 on Machine M2, we can generate a shop sequence like that on Fig. 3.

Table 4. Lower bounds for all M3 sequences for second illustrative example

M2 Sequence	J1	J2	J3	Wtd Flowtime
123	8	10	17	79
132	8	19	17	97
213	13	8	17	80
231	17	8	17	84
312	12	17	17	97
321	17	15	17	98

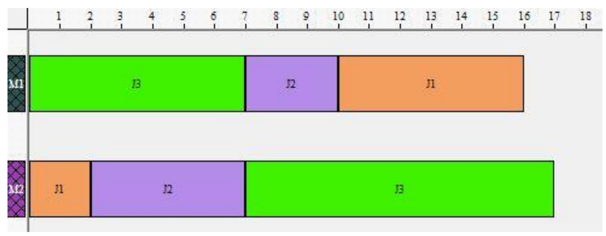


Fig. 3 Completed schedule for second illustrative example. Actual total weighted flowtime=86 for the sequence 1-2-3 on bottleneck M2.

Since the actual value of 86 may be dominated by the next best sequence from Table 4 which is 80 for sequence 2-1-3, the heuristic recommends generating the sequences for this second sequence. Fig. 4 is such a sequence.

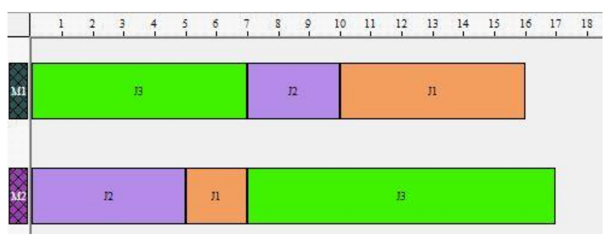
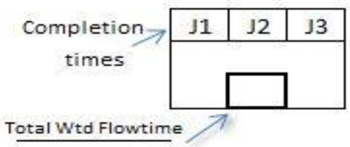


Fig. 4 Completed schedule for second illustrative example. Actual total weighted flowtime=87 for the sequence 2-1-3 on bottleneck M2.

A complete enumeration may be utilized to show the all the different sequences found for the 6^2= 36 possible sequences for this open shop. A enumeration is shown on Fig. 6. The heuristic found the optimal schedule with $\Sigma W_j C_j$=87.

Machine 1 Sequence (columns); Machine 2 Sequence (rows). Each cell: completion times J1 J2 J3 / Total Wtd Flowtime

Machine 2 Sequence	123	132	213	231	312	321
123	8 14 26 / 114	8 16 23 / 109	9 8 25 / 100	16 8 20 / 92	13 16 17 / 96	16 10 17 / **87**
132	8 31 26 / 148	8 28 23 / 133	9 17 19 / 100	25 17 19 / 116	13 22 17 / 108	16 22 17 / 111
213	8 9 26 / 104	8 16 23 / 109	11 8 26 / 105	16 8 20 / 92	13 16 17 / 96	16 10 17 / **87**
231	17 9 22 / 101	25 16 23 / 126	20 8 25 / 111	22 8 20 / 98	19 16 17 / 102	19 10 17 / 90
312	12 17 17 / 97	12 20 17 / 103	12 17 17 / 97	23 17 17 / 108	19 24 17 / 118	19 24 17 / 118
321	17 15 17 / 95	17 20 17 / 108	17 15 17 / 98	23 15 17 / 104	24 22 17 / 119	24 22 17 / 119



Two optimal sequences (M1: 321, M2: 123) = Found by heuristic
(M1: 321, M2: 213)

Fig. 6: Complete enumeration of 36 possible schedules

4. CONCLUSIONS

The proposed heuristic has rediscovered that the SPT rule that minimizes flow time for a single machine, may also be applicable to multiple machine problems like the open shop.

The lower-bound determination of a schedule's total weighted flowtime is helpful in making initial survey of promising schedules. The significant effort of generating a complete schedule from possible sequences may now be made with a more branch-and-bound rationale.

An area for further study for this study is to program a higher number of jobs and see how the heuristic fares in finding the optimal (or near-optimal) sequences. As such, the heuristic seems promising.

5. REFERENCES

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