Application of Response Surface Methodology: Optimum Mix Design of Concrete with Slag as Coarse Aggregate

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Abstract: The optimum mix design of slag in concrete is one of the best ways in identifying which mixture will yield high compressive strength without compromising good behavior and significance of each variable in every compressive strength test when a certain percentage of slag is being mixed in concrete. To determine the mix design that will yield the optimum compressive concrete strength, response surface methodology (RSM) is explorer in this study.

RSM is an optimization tool explored in the study because it interprets experimental results even in a non-linear response surface manner and it provides sufficient experimental interpretation as part of the conclusive result [1]. It has modern optimization features that can be useful in most complicated experimental design. Its most important applications are in the field where variables have potential significance in predicted system behavior called response. The combination of factorial application and modern experimental design has outstanding contribution in optimizing experimental procedures in a reduced number of studies and the response is easy to interpret.

RSM was used on the data obtained from laboratory experiments conducted by the researchers. The experiments conducted include the influencing factors: slag percentage (50%, 75%, and 100%), curing period (14 days, 21 days, and 28 days), and types of cement (1P, I, and IP), and the interaction effects of these factors in compressive strength test are analyzed in this paper through response surface methodology. The responses of each specimen have showed significant increase in attained strength with respect to the control specimens.

Key Words: concrete; slag; optimization; Response Surface Methodology; aggregate; Design of Experiment

1. INTRODUCTION

The construction industry is growing rapidly. The use of concrete as a construction material is in great demand, thus requiring the industry to create a wide range for its building components. In order to meet the increasing demand of these components, it is necessary to adapt waste material recycling to compensate for the lack of natural resources and to obtain alternative ways of conserving the environment.

Concrete is a widely used construction material worldwide. The raw materials are easily available and it does not require complex or expensive equipment to create. But due to its popularity and demand as a construction material, some of its component should have an alternative source.

Many research and study in engineering have been developed to use locally available materials for construction due to its economic problems [2,3]. As an attempt to innovate the construction material technology, studies about by-product waste, such as slag, is being developed as an alternative material for construction, both for horizontal and vertical purposes.

Slag is a by-product waste material from steel manufacturers. It is often being recycled, treated, or disposed. Since there are a lot of studies about slag's applications as substitute to various construction materials, manufacturers these days rarely dispose this waste; instead it is sold at a low cost.

Improper disposal of slag is the main problem in the industrial world. Steel makers produce a large amount yearly and it has been dumped unsuitably without proper implementation and remediation measures. It was then discovered that slag can be a hazardous element in the environment if not disposed appropriately. Due to its increasing demand, disposal of slag as solid waste material is a serious problem.

In addition, another environmental concern is raised, the production of coarse aggregates. In the absence of timber, demand for concrete increases. As expected, demand for aggregates increases too. Although gravel is the conventional coarse aggregate used to produce concrete, its increasing cost in the construction market and its geologic and geomorphic implications on gravel supply is becoming a concern. There was a forecast made by Dunne et. al [4] that the demand of gravel, could lead to scarcity of supply in every country and importation would eventually take place. The authors also discussed the constraints of the supply not only to gravel but also to sand in the river channels, which is a well-known source for these materials.

2. METHODOLOGY

Response Surface Methodology and Design of the Experiment, specifically, Box - Behnken Design were used as framework of the study.

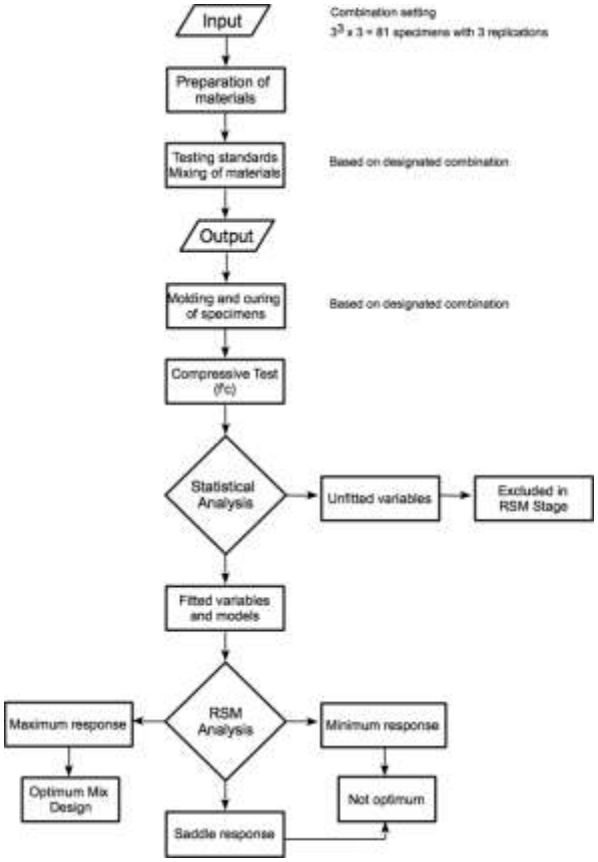


Fig. 1. Flowchart of the study

2.1 Factors and Levels

The low level values of the numerical factors are the lowest possible and acceptable level in each of the factors. 50% slag content and 14-day curing period could already attain concrete strength. The maximum levels where tested and were proven to achieve the desired quality for each concrete combination. Therefore exceeding these values will result to undesirable compressive strength.

	Factors	**Low Level**	**Middle Level**	**High Level**
Numerical Factors	Slag Content (%)	50	75	100
	Curing Period (days)	14	21	28

Categorical Factor	Cement (type)	1P	I	IP

Table 1. Values of each factor per level

Cement types are classified as Type 1P as an ideal blended cement, Type IP used for different type of aggregates (slag as a sample), and Type I as high early strength and shorter drying time cement.

2.2. Sampling Procedures and Runs

The performance of the different factors was evaluated independently using the runs randomly ordered by Design Expert software for Response Surface Design.

2.3. Experimental Procedures

To minimize the bleeding of the concrete in the experiment, 2 inches slump height was used for all combinations as the optimum slump. All batches were produced under good weather and clean environment to avoid impurities. The specimen preparation and testing standards are all in accordance with ASTM and AASHTO.

Curing and inspection for produced concrete were done right after the mixing process. Universal Testing Machine was used to measure the final compressive strength of each concrete mixes. Compression test was done right after the respective curing periods of each concrete mixes.

3. ANALYSIS

The experiment produced 81 concrete mixes at various levels of the three factors (Table 2).

Specimen	**Specimen Code**	**Curing Period**	**Compressive Strength (tons)**		
			1	**2**	**3**
1P 50%	1P50 14	14	27.1	29.0	31.4
1P 50%	1P50 21	21	59.5	55.3	55.0
1P 50%	1P50 28	28	40.7	47.2	41.6
1P 75%	1P75 14	14	18.0	23.6	20.1
1P 75%	1P75 21	21	52.1	58.3	54.0
1P 75%	1P75 28	28	54.8	54.2	55.4
1P 100%	1P100 14	14	25.7	26.4	19.8
1P 100%	1P100 21	21	52.5	44.9	47.6
1P 100%	1P100 28	28	46.7	52.8	53.4
I 50%	I50 14	14	26.0	20.9	25.4
I 50%	I50 21	21	55.2	51.8	60.2
I 50%	I50 28	28	60.3	61.8	60.9
I 75%	I75 14	14	22.4	21.0	18.6
I 75%	I75 21	21	58.0	56.9	52.2
I 75%	I75 28	28	63.1	58.4	48.1
I 100%	I100 14	14	46.0	42.0	46.8
I 100%	I100 21	21	49.9	50.3	50.1
I 100%	I100 28	28	50.3	53.2	55.0
IP 50%	IP50 14	14	34.7	36.4	30.3
IP 50%	IP50 21	21	41.3	43.7	38.4
IP 50%	IP50 28	28	42.7	43.8	43.1
IP 75%	IP75 14	14	35.3	44.0	39.2
IP 75%	IP75 21	21	47.5	48.7	44.5
IP 75%	IP75 28	28	52.8	51.9	53.6
IP 100%	IP100 14	14	32.3	46.2	30.1
IP 100%	IP100 21	21	46.1	46.8	49.3
IP 100%	IP100 28	28	50.8	43.2	45.9

Table 2. Result of the experiment
(Multiply 77.95 to convert in psi or 0.537465 in MPa)

3.1. Response Surface Formula

Formula = f ~ ct + SO (days, slagcont), data = cx
(Eq. 1)

Eq. (1) was used in analyzing response – surface model components after the experiment. The second – order response surface (SO) was used to capture the curvature immediately. Each type of cement has different analysis to relate the interaction between the slag content and curing period (Tables 3 to 5).

	Estimate	Std. Error	t-value	Pr(>|t|)	
Intercept	53.55278	12.40093	4.3184	0.0001675	***
cement type	0.62222	6.65238	0.0935	0.9261227	
days	9.02	0.99786	9.0394	6.197^{-10}	***
slag content	1.76667	3.79244	0.4658	0.6228104	
days : slag content	3.45167	0.81475	4.2365	0.0002098	***
days^2	-17.72917	1.57775	-11.237	4.393^{-12}	***
slag content^2	-1.35556	1.82183	-0.7441	0.4628249	

Table 3. Analysis of Type 1P cement using Eq (1)

	Estimate	Std. Error	t-value	Pr(>|t|)	
Intercept	61.54722	7.46878	8.2406	4.375^{-09}	***
cement type	-16.85556	8.48397	-1.9868	0.0564705	.
days	5.405	1.2726	4.2472	0.0002037	***
slag content	12.00556	4.83661	2.4822	0.0190923	*
days : slag content	0.14833	1.03907	0.1428	0.8874702	
days^2	-7.97083	2.01215	-3.9614	0.0004443	***
slag content^2	-4.99444	2.32343	-2.1496	0.0400625	*

Table 4. Analysis of Type IP cement using Eq (1)

	Estimate	Std. Error	t-value	Pr(>|t|)	
Intercept	37.95556	10.5543	3.5962	0.001183	**
cement type	17.98889	11.9888	1.5005	0.144304	
days	11.66333	1.79833	6.4856	4.242^{-07}	***
slag content	-6.46111	6.83471	-0.9453	0.352297	
days : slag content	-0.47667	1.46833	-0.3246	0.74779	
days^2	-13.5	2.84341	-4.7478	5.111^{-05}	***
slag content^2	3.81667	3.28329	1.1625	0.254529	

Table 5. Analysis of Type I cement using Eq (1)

Significant codes: 0 '***' | 0.001 '**' | 0.01 '*' | 0.05 '.' | 0.1 '' | 1

3.2. Analysis of Variance

The Analysis of Variance (ANOVA) indicates how the three factors affect the response. The analysis includes the first – order response surface (FO), two – way interaction (TWI), and pure quadratic (PQ) of each concrete mixes, the requirement for response surface 3D model. Tables 6 to 8 are the respective analysis of 3 types of cement.

	Dof	Sum Square	Mean Square	F - value	Pr (>F)
Cement type	1	71.38	71.38	3.5843	0.068347
FO (days, slag content)	2	2787.41	1393.70	69.9849	7.97^{-12}
TWI (days, slag content)	1	357.42	357.42	17.9479	0.0002098

	Dof	**Sum Square**	**Mean Square**	**F - value**	**Pr (>F)**
Cement type	1	391.02	391.02	6.0455	0.021505
FO (days, slag content)	2	3157.47	1578.73	24.4084	6.084^{-07}
TWI (days, slag content)	1	6.82	6.82	0.1054	0.7477903
PQ (days, slag content)	2	1545.40	772.70	11.9465	0.0001642
Residuals	29	1875.72	64.68		
Lack of fit	5	1588.79	317.76	26.5789	4.738^{-09}
Pure error	24	286.93	11.96		

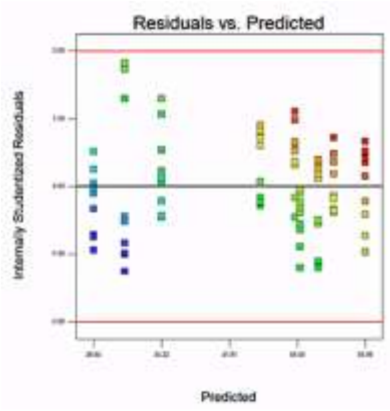
PQ (days, slag content)	2	2525.61	1262.81	63.4119	2.58^{-11}
Residuals	29	577.52	19.91		
Lack of fit	5	344.50	68.90	7.0966	0.0003358
Pure error	24	233.01	9.71		

Table 6. ANOVA Table of Type 1P cement

	Dof	Sum Square	Mean Square	F - value	Pr (>F)
Cement type	1	98.80	98.80	3.0505	0.0912996
FO (days, slag content)	2	793.72	396.86	12.2525	0.000139
TWI (days, slag content)	1	0.66	0.66	0.0204	0.8874702
PQ (days, slag content)	2	657.94	328.97	10.1565	0.0004538
Residuals	29	939.31	32.39		
Lack of fit	5	602.34	120.47	8.5799	9.019^{-05}
Pure error	24	336.97	14.04		

Table 7. ANOVA Table of Type IP cement

Table 8. ANOVA Table of Type I cement



F – value is a value of the test. It can be obtained by having the ratio between the variance of the group means and mean within the group variances. It was used to determine the significance of the test (FO, TWI, and PQ respectively). Unlike t – value, F – value should always be positive. The significance of these model terms can be evaluated if the value of Pr > F is equal or less than 0.0500. Since all models have significant lack of fit tests, it is a good indication of unbiased estimate or observation in the experimental phase.

3.3. Diagnostic Plots

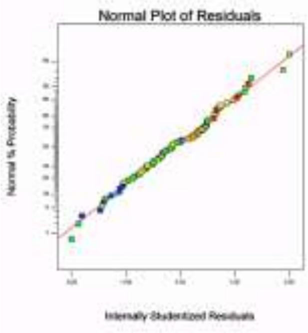


Fig. 2. Normal probability plot of the concrete experiment.

Diagnostic plots are useful to see whether assumptions are met. Figure 2 shows the normal probability plot of the residuals. As observed, there is no significant defection from the normal probability line and it can fairly conclude that the assumption of normality is satisfied.

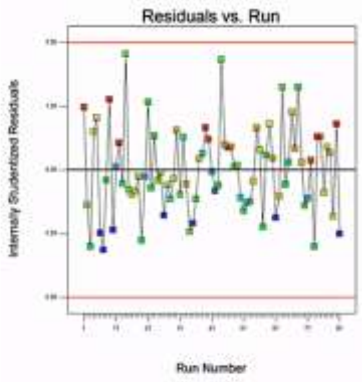


Fig. 3. Residual vs Run (Order) plot

Figure 3 is the Residuals vs. Run plot. There is no significant pattern or structure in the graph and the residual values did not exhibit significant patterns of increase or decrease as the run order is increased.

Fig. 4. Residual vs Predicted (Fits) plot

Figure 4 shows the Residuals vs. Predicted plot. The illustration exhibits no significant pattern of a "megaphone". This only means that when the predicted values increase, residual values show no sign of significant pattern of increase or decrease.

3.4. Response Surface Model

As observed in ANOVA tables, the interaction were considered not quite significant, thus, the optimum mix may be in the region between the middle and highest values of both curing period and slag content. The optimization tool of design expert software was used to find the optimal point on the response surface that will maximize the compressive strength of concrete.

The contour plots and 3D representation, in Figures 5 to 10, give ideas to the variation of strength when slag content and curing period vary.

3.4.1 Interpretation for Type 1P

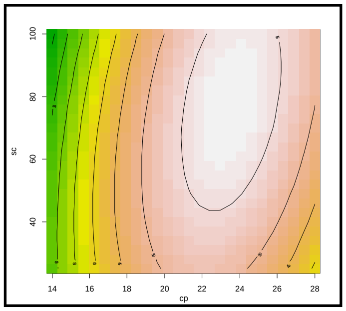
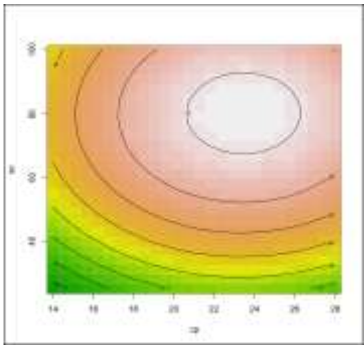


Fig. 5. Response Surface of Type 1P cement in region of optimum combination



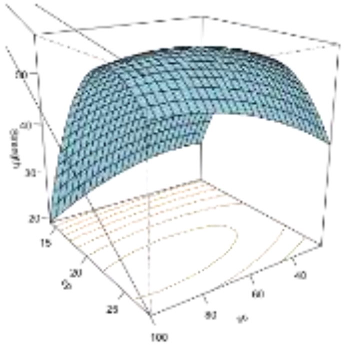


Fig. 6. 3D representation of contour plots of Type 1P

The predicted strength is given by the equation:

$Strength = 53.6 + 9.02\ days + 3.45\ (days*slagcont) - 17.7 days^2$ (Eq. 2)

Values	[1]	-1.175624	-17.909098
Vectors		[,1]	[,2]
	days	-0.1036956	-0.9946091
	slag content	-0.9946091	0.1036956

Table 9. Eigen Analysis for Type 1P

The operating condition in Type 1P is in the region where maximum slag content and low curing period lie. Its path of improvement moved downward until it reached the region of the optimum point below 80% slag content with more than 22 days curing period.

In Figs. 5 and 6, the intersecting stationary point in response surface is 0.362777 for days and 1.113511 for slag content while the intersecting stationary point in original units is 23.53944 for curing period and 77.83777 for slag content. Since the Eigenvalues are both negative (-1.175624 and -17.909098) as shown in Table 9, the stationary point in original units is now the optimal combination for Type 1P cement.

3.4.2 Interpretation for Type IP

Figure 7. Response Surface of Type IP cement in region of optimum combination

The predicted strength is given by the equation:
$Strength = 61.5 + 5.4\,days + 0.15\,(days*slagcont) - 7.97\,days^2$ (Eq. 3)

Values	[1]	-4.992597	-7.97268
Vectors		[,1]	[,2]
	days	-0.02489517	-0.99969007
	slag content	-0.99969007	0.02489517

Table 10. Eigen Analysis for Type 1P

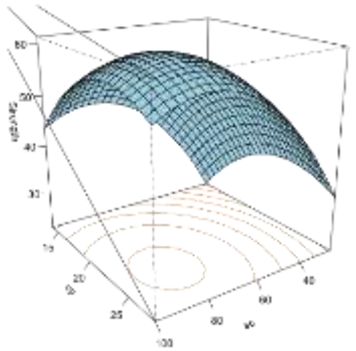


Fig. 8. 3D representation of contour plots of Type IP

In Type IP, the operating condition of response surface is the region where both slag content and curing period are low. It will start a good path of improvement where contours of constant response are moving upward until it reached the region of the optimum point more than 80% slag content with above 22 days curing period.

The stationary point in response surface of Type IP is 0.3502803 for days and 1.2070926 for slag content and the stationary point in original units is 23.45196 for curing period and 80.17731 for slag content. Table 10 shows the Eigenvalues of Type IP. Since the Eigenvalues are both negative, therefore, the stationary point in original units makes it the optimal combination for Type IP cement.

3.4.3 Interpretation for Type I

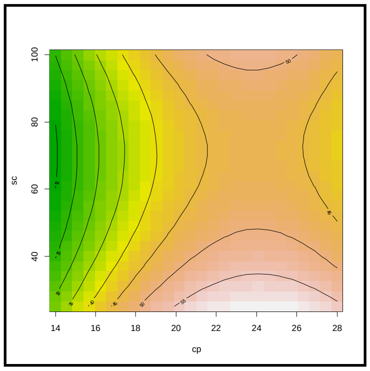


Fig. 9. Response Surface of Type I cement in region with saddle response

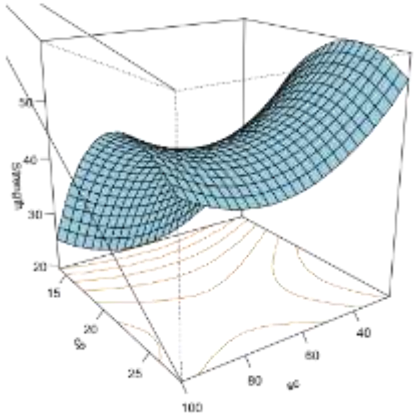


Fig. 10. 3D representation of contour plots of Type I

The predicted strength is given by the equation:

$$Strength = 37.96 + 11.66\ days - 0.48\ (days*slagcont) - 13.5\ days^2 \quad \text{(Eq. 4)}$$

Values	[1]	3.819946	-13.50328
Vectors		[,1]	[,2]
	days	0.01375933	-0.99905340
	slag content	-0.99905340	-0.01375933

Table 11. Eigen Analysis for Type I

Unlike the previous cement types, Type I exhibits a saddle response where the region of optimum cannot be located. The responses did not capture the possible combinations that would lead the operating condition to linear path of improvement.

Type I's stationary point in response surface is 0.4165729 for days and 0.8724468 for slag content. The stationary point in original units is 23.916101 for curing period and 71.81117 for slag content. Its resulting Eigenvalues are not negative, thus there is no optimal combination of slag content and curing period. However, the canonical path analysis was obtained to give an idea as to the possible combinations of the next phase of the experiment (Table 12). It was used to get potential combinations near the optimum region that is not included within the experiment result, thus requiring another mix calculation and subject to compressive test after the required curing period.

Looking at Table 12, the only relevant combinations are those from rows 9 to 14 because 1 to 8 combinations suggest slag content exceeding 100%, 15 to 16 combinations are below the lower limit (25%), and combinations 17-21 give negative percentages of slag content.

	Curing period	Slag content
1	23.436	196.800
2	23.485	184.300
3	23.534	171.800
4	23.576	159.300
5	23.625	146.800
6	23.674	134.300
7	23.723	121.800
8	23.772	109.300
9	23.821	96.800
10	23.870	84.300
11	23.919	71.800
12	23.961	59.300
13	24.010	46.825
14	24.059	34.325
15	24.108	21.825
16	24.157	9.325
17	24.206	-3.175
18	24.255	-15.675
19	24.304	-28.175
20	24.346	-40.675
21	24.395	-53.175

Table 12. Combinations from Canonical Path Analysis

4. Conclusion

This study proves that slag content, curing period, and cement types significantly affects the compressive strength of concrete.

The relationships of the three factors against the response (compressive strength) are not all linear. Slag content and curing period have a non-linear relationship and therefore should not be treated directly proportional against responses relative to the varying levels of the factors.

The uniaxial compression test results show that the compressive strength of the produced cylinders increases as the percentage of the substituted amount of slag to coarse aggregate increases. It increases by 39.18% of its compressive strength per 25% replacement of slag on its 28th day of curing, based on the average strength of 3 cylinders.

However, Type I cement exhibits saddle response in the analysis. The possible reason for its varying behavior is that there are a lot of directions leading to optimum region where stationary points could not meet. Therefore, the most useful action is to decide in which direction to explore further. The canonical path analysis decides which path of improvement (Table 12) to take that would lead to optimum combination.

Utilization of slag as partial substitute for coarse aggregate in the application of normal – strength concrete also helps the country, most specifically to the steel – making companies that produced EAF slag around 60 metric tons per hour.

The produced cylinder samples are considered as normal – strength concrete thus can be used in load – bearing structures. The specimens having 100% slag replacement to gravel with 28 days of curing cannot be predicted that would give an optimal combination with highest compressive strength. The analysis proved this null hypothesis incorrect when the optimum combination lies in the middle region.

The optimum combination for maximizing strength is in the region between the middle and highest values of the curing period and slag content, specifically, 21 days to 28 days and 75% to 100% respectively.

5. Acknowledgment

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