

Accuracy Enhancement Performance of Least Mean-Squares Filter on Root-MUSIC-Extracted Direction-of-Arrival Estimates for Passive RFID Application

Reynaldo Ted L. Peñas II, MScEngg Pamantasan ng Lungsod ng Maynila theengineerpen@gmail.com

Abstract: The study aimed to enhance the accuracy of Root-MUSIC direction-ofarrival (DoA) estimates of a passive radio frequency identification (RFID) tag system with a reader that utilizes a two-element uniform linear array (ULA). The enhancement of accuracy was made by using an adaptive filter called least mean squares algorithm (LMS) to reduce the effect of noise and carrier leakage before extracting the DoA estimates through root-multiple signal classification (root-MUSIC) algorithm. Initially, through the use of a simulation in MatLab®, random complex signals from angle bearings negative 90 through positive 90 degrees are established, including noise and carrier leakage added and characterized as additive, white, Gaussian-distributed random variable. An LMS filter, with step sizes of 0.005, 0.002 and 0.001, was designed to reduce the inaccuracy of the estimates by filtering the distortion-afflicted complex signal obtained at the front end of the receiver of the model established. Results of the estimates were compared to the actual DoA of the tag by measuring the discrepancy in degrees as root-mean-square error. Observations have also been done in the case when signal-to-noise ratio (SNR) of the received signal was changed, or when the number of iterations of the filter was varied to show how the convergence of the estimates to the true bearing of the passive RFID tag behaved in accordance to the said variations. The LMS filter has been very helpful in reducing the error in extracting the estimates and the DoA estimates converged to its real value when the step size of LMS is 0.001.

Key Words: Direction-of-arrival, root-MUSIC, adaptive filtering, least mean-squares, passive RFID.

1. INTRODUCTION

Emitter localization has been a very challenging problem in sensor array signal processing systems and has been given attention to by researchers who have formulated algorithms to enhance the method in recent years. Location-finding methods, techniques and implementations have been deemed very helpful in networks where pinpointing the location of a mobile device or emitter, whether stationary or in motion, is one of their core functions. One example of a system that can benefit from the results of such innovations is the passive RFID tag system (Chawla and Ha, 2007). Unlike the active RFID system, the tag has no battery or power source from which it gets energy to transmit to a reader or



scanner. Rather, the tag depends on the signal emitted by the reader and the former elicits a response that uses the available power it can harness from the received signal so that the response may reach the reader. Such signal is transmitted back through backscattering. This paper illustrates the use of passive RFID tag system as a platform for the simulations done to obtain the location of a tag that is within the range of the RFID reader or sensor.

Analysis of the spectrum of the received signal has been one of the main emphases of the scenario in order to deal with the problem more effectively. Methods such as Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) (Paulraj et al, 1985), MUltiple SIgnal Classification (MUSIC) (Schmidt, 1986) and its more extensive version root-MUSIC, Maximum Likelihood (ML) (Fisher, 1971), Capon Algorithm (Capon, 1971), among others. This study employs the root-MUSIC algorithm in extracting the direction-of-arrival estimates of the passive RFID tag to a given sensor with a two-element uniform linear array (ULA) antenna system.

Several algorithms for estimation accuracy enhancement have been tried and tested to optimize localization systems. Each offers its own set of advantages in terms of ease of implementation, degree of resolution, speed of convergence, etc., and trades off with setbacks or disadvantages such as method complexity, sophistication in hardware implementation, memory limitations, etc. Adaptive filter algorithms, like Wiener (Wiener, 1942), Least Mean-Squares (LMS) (Widrow and Hoff, 1960), Recursive Least-Squares (RLS) (Plackett, 1950), Kalman (Kalman, 1960), among others, are used more frequently in such systems. The simulations in this study use the LMS filter algorithm to reduce the effect of noise in the deviations of tag location extraction through root-MUSIC.

2. METHODOLOGY

Ultra-High Frequency (UHF) RFID system, which was used as a platform model for this experimentation, extends the capability of using low frequency (LF) or high frequency (HF) (Dobkin, 2008). Although most of the application of RFID does not require longer read range, UHF RFID presents a flexibility of having to read tag within the near-field and also in the far-field whenever it is needed. This introduces a problem in DoA estimation since most of the high resolution algorithms for DoA estimation were based on far-field assumptions. At this point, the received signal model, to be used for DoA estimation when the tag is either in the far-field or near-field, was established.

2.1 FAR-FIELD AND NEAR-FIELD ASSUMPTION

For far-field assumption, the antenna array geometry shown in Fig. 1 was used. Note that since the signal source was in far-field, the wavefronts were assumed to be on a plane.

The backscattered signal from tag can be represented as

$$\tilde{x}(t) = \operatorname{Re}\left[s(t)e^{j2\pi f_c t}\right]$$
(1)



Figure 1. Antenna Array Geometry for a Far-Field Assumption

where Re[•] gives the real part of a function, s(t) is the complex envelope of the signal, f_c is the center frequency, and t is time (Haykin, 1995). The signal that impinges at the \dot{r} th antenna has an associated attenuation factor and propagation delay. Thus, the received backscattered signal from a single tag (the validity of assumption that the signal is from a single tag only is based on the anti-collision protocol of an RFID system in which only one tag responds to the reader's interrogation) can be expressed in the form

$$\widetilde{x}_{i}(t) = \operatorname{Re}\left[\left\{\alpha_{i}s(t-\tau_{i})e^{-j2\pi f_{c}\tau_{i}}\right\}e^{j2\pi f_{c}t}\right] + n_{i}(t) \qquad (2)$$
$$= \operatorname{Re}\left[\left\{\alpha_{i}s(t-\tau_{i})e^{-jp}\right\}e^{j2\pi f_{c}t}\right] + n_{i}(t)$$

where $n_i(t)$ is an additive white Gaussian noise afflicting the *i*-th antenna (i = 0, 1, 2, ..., m - 1, where m = no. of antenna), a_i is the attenuation factor of the backscattered signal.

From the array geometry shown in Fig. 1, the spatial frequency p in (2) can be written as a function of antennas' interspacing d, and DoA θ .

$$p = \frac{2\pi i d \sin \theta}{\lambda} \tag{3}$$



Imposing narrowband assumption, i.e., the bandwidth of the signal is much smaller than the reciprocal of the transit time of the wavefronts across antenna array $(B \ll 1/T)$ makes $s(t-\tau) = s(t)$,

thus, the low pass equivalent of the backscattered signal that impinges to the antenna array with interspacing d becomes

$$x_i(t) = \alpha_i s(t) e^{-jp} + n_i(t) \qquad (4)$$

For the near field assumption, the wavefronts were assumed to be spherical rather than plane. Using the antenna geometry for near-field assumption shown in Fig. 2, the spatial frequency can be expressed as (derivation adapted from (Aberbour et al, 2008) and (Wang et al, 2006)).

$$2\pi f_c \tau_i = \frac{2\pi f_c \left(r_i - r\right)}{c} = \frac{2\pi \left(r_i - r\right)}{\lambda} \tag{5}$$

where r_i denotes the distance of the signal source to the \dot{r} th antenna (i = 1, 2, 2N + 1), using the law of cosine then gives

$$r_i = r_v \sqrt{1 + \frac{m^2 d^2}{r^2} - \frac{2md\sin\theta}{r}}$$
(6)



Figure 2. Antenna Array Geometry for Near-field Assumption

where m = -N, ..., -1, 0, 1, ..., N. Using the binomial expansion theorem and assuming that d << r.

$$r_i = r \left(1 - \frac{md\sin\theta}{r} + \frac{m^2d^2\cos^2\theta}{2r^2} \right)$$
(7)

Thus, the backscattered signal impinges at the antenna array from a single tag is

$$x_i(t) = \alpha_i s(t) e^{-j(pm+qm^2)}$$
(8)

where $p = -\frac{2\pi d \sin \theta}{\lambda}$ and $q = \frac{\pi d^2 \cos^2 \theta}{\lambda r}$. Note that the

received signal is now a function DoA θ and of tag-toarray distance *r*. Hence, it is needed to perform a high complexity three-dimensional search to estimate

HCT-II-013

the DoA. In addition, most of the high resolution algorithm can only estimate the DoA of the signal. However, the proposed DoA estimation technique proposed in (Aberbour et al, 2008) and (Wang et al, 2006) showed that by using two antenna, the expression for the covariance matrix of the received signal when using far-field assumption is the same when the signal source is in the near-field, i.e., the covariance matrix is a function of the DoA θ alone. Using the two-antenna model as shown in Fig. 3 with m = 0, 1 for the first and second antenna respectively, (8) can be express in matrix form as

$$\mathbf{x}(t) = \mathbf{a}(\theta)\mathbf{s}(t) + \mathbf{n}(t)$$
⁽⁹⁾

where $\mathbf{x}(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$ is the received signal, $\mathbf{a}(\theta) = \begin{bmatrix} e^{-j(-p+q)} & e^{-j(p+q)} \end{bmatrix}^T$ is the steering vector, $\mathbf{n}(t) = \begin{bmatrix} n_1(t) & n_2(t) \end{bmatrix}^T$ is the AWGN vector. The covariance matrix \mathbf{R} is given as $\mathbf{R} = E \{ \mathbf{x}(t) \mathbf{x}^H(t) \}$

$$= \sigma_s^2 \begin{bmatrix} 1 & e^{j^2 p} \\ e^{-j^2 p} & 1 \end{bmatrix} + \sigma_n^2 \mathbf{I}$$

$$= \sigma_s^2 \mathbf{a}(\theta) \mathbf{a}^H(\theta) + \sigma_n^2 \mathbf{I}$$
(10)

where σ_s^2 represents the variance of the source signal power, σ_n^2 is the noise is the noise variance, **I** is the identity matrix, and $(\bullet)^H$ is the Hermitian transpose of a vector or matrix. Note the **R** is independent of *q*, and therefore depend only in the DoA of the signal which is similar to far-field assumption. This implies that a high-resolution DoA estimation algorithm such as root-MUSIC (Schmidt, 1986) can be used for estimating the DoA.



Figure 3. Two Element Antenna Array for Near-field DoA Estimation



2.2 ROOT-MUSIC ALGORITHM

For the case of a uniformly spaced linear array (Naidu, 2000) with inter-element spacing d, the m^{th} element of the steering vector $\mathbf{a}(\theta)$ may be expressed as:

$$\mathbf{a}_{m}(\theta) = \exp\left(j2\pi m\left(\frac{d}{\lambda}\right)\cos\left(\theta\right)\right), \quad m = 1, 2, ..., M$$
(11)

The MUSIC spectrum is an all-pole function of the form

$$P_{MUSIC}(\theta) = \frac{1}{\mathbf{a}^{H}(\theta)\mathbf{v}_{n}\mathbf{v}_{n}^{H}\mathbf{a}(\theta)} = \frac{1}{\mathbf{a}^{H}(\theta)\mathbf{C}\mathbf{a}(\theta)}$$
(12)

where $\mathbf{C} = \mathbf{v}_n \mathbf{v}_n^H$. Solving for the inverse of (12),

$$P_{MUSIC}^{-1} = \sum_{m=1}^{M} \sum_{n=1}^{M} \exp\left(-j\frac{2\pi md}{\lambda}\cos\theta\right) \mathbf{C}_{mn} \exp\left(j\frac{2\pi md}{\lambda}\cos\theta\right)$$
(13)

where \mathbf{C}_{mn} is the entry in the form of m^{th} row and n^{th} column of \mathbf{C} . Combining the two summations into one, it can be simplified as

$$P_{MUSIC}^{-1} = \sum_{n=1}^{M} \mathbf{C}_{l} \exp\left(j\frac{2\pi d}{\lambda}l\cos\theta\right)$$
(14)

where $\mathbf{C}_{l} = \sum_{m-n=l} \mathbf{C}_{mn}$ is the sum of the entries of **C** along the *l*th diagonal. Then a polynomial D(z) was defined as follows.

$$D(z) = \sum_{l=-M+1}^{M+1} C_l z^{-1}$$
(15)

Evaluating the MUSIC spectrum $P_{MUSIC}(\theta)$ becomes equivalent to the polynomial D(z) on the unit circle, and the peaks in the MUSIC Spectrum exist because the roots of D(z) lie close to the unit circle. Ideally, with no noise, the poles would have lied exactly on the unit circle at locations determined by the DOA.

In other words, a pole of D(z) at $z = z_1 = |z_1| \exp(j \arg(z_1))$ will result in a peak in the MUSIC spectrum at $\cos \theta = \left(\frac{\lambda}{2\pi d}\right) \arg(z_1)$.

Consider the covariance matrix given in (10), where the steering vector can be rewritten as

$$\mathbf{a}(\theta) = e^{-j(-p+q)} \begin{bmatrix} 1 & e^{-jp} \end{bmatrix}^T$$
$$= e^{-j(-p+q)} \begin{bmatrix} 1 & z \end{bmatrix}^T$$
$$= e^{-j(-p+q)} \mathbf{a}(z)$$
(16)

where $z = e^{-jp}$ and $\mathbf{a}(z) = \begin{bmatrix} 1 & z \end{bmatrix}^T$. The covariance matrix can also be represented or decomposed via singular value decomposition as

$$\mathbf{R} = \mathbf{v} \mathbf{D} \mathbf{v}^H \tag{17}$$

where **D** is a 2×2 diagonal matrix, whose diagonal elements are λ_1 and λ_2 . Assume that $\lambda_1 > \lambda_2$, then λ_2 corresponds to the power of the noise. It was noted that the eigenvectors of the noise subspace is orthogonal to the signal subspace, i.e., $\mathbf{a}^H(\boldsymbol{\theta})\mathbf{v}_2 = 0$. The Root-MUSIC (Schmidt, 1986) spectrum is

$$P_{root} = \frac{1}{\mathbf{a}^{H}(\theta)\mathbf{v}_{2}\mathbf{v}_{2}^{H}\mathbf{a}(\theta)} = \frac{1}{\mathbf{a}^{H}(z^{-1})\mathbf{v}_{2}\mathbf{v}_{2}^{H}\mathbf{a}(z)}$$
(18)

The spectrum is maximized if *z* is

$$z_1 = z_2 = e^{-jp}$$
(19)

The roots were found to be identical to element $\mathbf{R}(2,1)$ of the covariance matrix \mathbf{R} , then the DoA of the source can be obtained by

$$\theta = -\sin^{-1} \left(\frac{-\lambda \arg\left[\mathbf{R}(2,1) \right]}{2\pi d} \right)$$
(20)

where λ is the wavelength of the signal, **arg**() returns the argument or angle of the complex variable it contains, and *d* is the distance between the two elements.

2.3 LEAST MEAN-SQUARES FILTER ALGORITHM

The Least Mean-Squares algorithm is an adaptive filter algorithm that utilizes a gradientbased method of steepest descent. Considering the Wiener (Wiener, 1942) filter

$$\hat{d}(n) = \sum_{k=0}^{p} w(k) x(n-k)$$
⁽²¹⁾

where w(n) is the unit sample response of the finite impulse response (FIR) Wiener filter that produces the minimum mean square estimation of a desired process d(n), x(n) is a wide-sense stationary process data. The error in estimates is given by $e(n) = d(n) - \hat{d}(n)$, then the coefficients of the filter to minimize mean-square error $E_1^{(|e(n)|^2)}$ are obtained by solving the Wiener-Hopf (Hayes, 1996) equations

$$\mathbf{R}_{x}\mathbf{w} = \mathbf{r}_{dx} \tag{22}$$

where \mathbf{R}_x is the input data matrix and \mathbf{r}_{dx} is the result after using the filter coefficients to converge the data to their desirable values. However, if x(n) and d(n) are non-stationary processes, then the filter coefficients that minimize $E_1^{d} | e(n) |^2$ will be dependent on the *n*domain, and the filter will be time-varying

$$\hat{d}(n) = \sum_{k=0}^{p} w_n(k) x(n-k)$$
(23)



where $w_n(k)$ is the value of the *k*th coefficient of the filter at a given time interval *n*. As a vector expression

$$d(n) = \mathbf{w}_n^{\mathsf{T}} \mathbf{X}(n)$$

(24)

$$\mathbf{w}_{n} = \begin{bmatrix} w_{n}(0), & w_{n}(1), & \dots, & w_{n}(p) \end{bmatrix}^{T} \quad (25)$$

is the coefficient vector of the filter at time *n*, and

$$\mathbf{x}(n) = \begin{bmatrix} x(n), & x(n-1), & \dots, & x(n-p) \end{bmatrix}^{T} (26)$$

Throughout time, it is necessary for the filter to obtain the optimum values for its coefficients. Hence, this problem may be simplified, if the need for \mathbf{w}_n to minimize the mean-square error at each time *n* has to be considered, by instead choosing a coefficient update equation of the form

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \Delta \mathbf{w}_n \tag{27}$$

where $\Delta \mathbf{w}_n$ is a correction applied to the coefficients of the filter \mathbf{w}_n at time *n* to form a new set of coefficients \mathbf{w}_{n+1} for the next time interval n + 1. Assuming that \mathbf{w} and \mathbf{x} are complex, then the gradient is the derivative of $E\{\|e(n)\|^2\}$ with respect to \mathbf{w}^* .

$$\nabla \xi(n) = \nabla \left\{ \left| e(n) \right|^2 \right\} = E \left\{ \nabla \left| e(n) \right|^2 \right\} = E \left\{ e(n) \nabla e^*(n) \right\}$$
(28)

and

where

$$\nabla e^*(n) = -x^*(n) \tag{29}$$

it follows that

$$\nabla \xi(n) = -E\{e(n)\mathbf{x}^*(n)\}$$
(30)

Thus, with a step-size of μ , the steepest descent algorithm is obtained

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu E\left\{e(n)\mathbf{x}^*(n)\right\}$$
(31)

which leads to the least mean squares algorithm.

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu e(n) \mathbf{x}^*(n-k)$$
(32)

2.4 ADAPTIVE UNIFORM LINEAR ARRAY DESIGN

Considering a general beam-former uniform linear array with N isotropic elements, the following figure is conceived for this study. However, this paper only focuses on using a two-element ULA. The inclusion of the LMS filter is then integrated to enhance the accuracy of the estimates.

The output of the beam-former y(n) is then compared to the desired signal to be received and the difference between the two is the error that must be fed into the LMS Update Algorithm for the purpose of optimizing the filter coefficients for the next time interval.



Figure 4. The proposed adaptive beam-former ULA with LMS

3. RESULTS AND DISCUSSION

3.1 SIMULATION OF DOA ESTIMATION

Simulations are executed through the use of computer software MatLab® to analyze the performance of the least-mean square filter in enhancing the accuracy of extracting DoA estimates. In the following cases, the signal from the source tags were impinging from various angles relative to the broadside of the two-element uniform linear array antenna system designed at the reader device. The spacing between the elements was half a wavelength of the working frequency of a passive RFID tag system, $f_c = 915$ MHz, and the distance of the source tag from the array was irrelevant and was assumed to be fixed given that it was still within the scope of the reader's signal reception area. Noise was added to the modeled received signal from the tag, and was characterized as additive, white and Gaussian distributed with zero mean and unity variance. The range of signal-to-noise ratio used was limited from -5 dB to 5 dB since passive RFID systems operate on relatively low-power applications. Thus, the eigenvalues of the covariance matrix formed were computed and the highest value obtained is approximately 7. This indicates an upper limit ($0 < \mu$ $< 2/\lambda_{max} = 2/7$) for the step-size that can be used for the LMS filter in order to achieve convergence of the DOA estimation (Hayes, 1996). The step sizes chosen to be analyzed in this experiment are $\mu = 0.005$, 0.002 and 0.001. The number of samples was held fixed at 100 snapshots.

Table 1. Comparison of Actual and Computed DoA



ANGLE (degrees)	Method	STEP-SIZE (µ)	DOA Estimate	% Error
-45	root-MUSIC only		-50.3186	1.181911111
	root-MUSIC with LMS	0.005	-50.3145	1.181
		0.002	-45	0
		0.001	-45	0
0	mot-MUSIC only		0.9113	0.9113
	root-MUSIC with LMS	0.005	0.9027	0.9027
		0.002	-2.72E-07	2.717E-07
		0.001	-2.72E-07	2.717E-07
30	root-MUSIC only		26.0187	13.271
	IDOE-MUSIC with LMS	0.005	26.0285	13.23833333
		0.002	30,0006	0.002
		0.001	30	0
60	root-MUSIC only		56,7907	5.348833333
	root-MUSIC with LMS	0.005	56.7996	5.334
		0.002	60	.0
		0.001	60	0

A bigger step-size used for the LMS filter reduced a very small amount of error in the DoA estimates while a smaller one totally eliminated the effect of noise in the received signal, therefore resulting to a more accurate DoA estimate.

The pseudospectrum obtained through the use of root-MUSIC algorithm was presented and analysis was done in comparison with the procedure that includes the addition of the least-mean square filter algorithm to mitigate noise significantly so that the DOA estimates extracted from the received signal becomes more accurate.

The following figures 5 through 8 present the pseudospectrum of the angle bearings presented in Table 1. The peaks of each magnitude in dB were indicative of the poles of (12) of each case, which were extracted by the root-MUSIC algorithm before converting into angle bearings in degrees.



Figure 5. Pseudospectrum at $\theta = -45^{\circ}$



Figure 6. Pseudospectrum at $\theta = 0^{\circ}$



Figure 7. Pseudospectrum at $\theta = 30^{\circ}$



Figure 8. Pseudospectrum at $\theta = 60^{\circ}$

The results of the simulation in this section used the standard deviation error formula, also



known as the root-mean-square error (RMSE), given as

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} \left(\theta - \hat{\theta}_i\right)^2}$$
(33)

where N is the number of samples or snapshots, θ is the true DoA and $\hat{\theta}$ is the estimated DoA of the *i*th sample. The angle θ was held fixed at 30° and the number of samples was still at 100 snapshots of received data. Signal-to-noise ratio was varied from – 5 dB to 5 dB.



Figure 9. Convergence of DoA Estimation in Varying SNR

The DoA estimates were virtually error-free when LMS algorithm is used with a $\mu = 0.001$ before extracting the angle. The step-size was small enough to eliminate the effect of noise in the received signal despite the channel was very noisy which was indicated by a negative SNR.

The number of samples had also been varied to study the convergence of DoA estimates as they are extracted using root-MUSIC algorithm.



Figure 10. Convergence of DoA Estimation in Number of Samples

Employing the LMS filter significantly helped in the convergence of the DoA estimates as the number of sample or snapshots increases. Adjusting the step-size to a smaller one increased the resolution of the accuracy. Even with just one sample, using LMS filter with $\mu = 0.001$ can totally eliminate the effect of noise and distortion. Hence, the DoA extracted converged immediately to its true value when the filter, with the right step-size, was used.

4. CONCLUSIONS

In this study, a passive RID tag system was considered and the direction-of-arrival of a tag was estimated through the use of root-MUSIC algorithm. A two-element ULA was designed at the front-end of the reader to compute for the DoA using a beam-former. The least mean-squares algorithm was used to mitigate the effects of noise in the received signal before extracting the angle using root-MUSIC. Comparison was done between the methods of using root-MUSIC algorithm only to that of implementing an LMS filter to reduce the effect of noise before determining the DoA through root-MUSIC. The variation in step-size in using the filter was also considered and results were compared and analyzed. It was found out through this work that the use of adaptive filter algorithm Least Mean Squares Algorithm, with the right step size that is small enough ($\mu = 0.001$) for the output to converge to the true value, can significantly reduce, if not eliminate, the effect of noise that can distort the DoA estimate. If the step-size is not small enough, the convergence can still be achieved



by increasing the number of samples or increasing the signal-to-noise ratio of the received signal by powering up the back-scatter signal received from the passive tag.

5. ACKNOWLEDGMENTS

The author would like to express thanks and gratitude to Prof. Michael Gringo Angelo R. Bayona for his unwavering guidance and enlightening advices from the commencement of this work until its completion, and to Prof. Rhandley D. Cajote for his unselfishness and perseverance in sharing his unparalleled wisdom and knowledge to his students in his Digital Signal Processing class.

6. REFERENCES

- Aberbour, L., Craeye, C. & Decostre, A. (2008, July). Low profile compact-slot antenna array for RFID applications: Backscatter-tag identification and localization. Antennas and Propagation Society International Symposium, 2008. AP-S 2008. IEEE, vol. 1, no. 4, p. 5-11.
- Capon, J. (1969). High-resolution frequencywavenumber spectrum analysis. Proceedings of the IEEE, vol. 57, p. 1408-1418.
- Chawla, V. & Ha, D. S. (2007, September). An Overview of Passive RFID. IEEE Applications & Practice, p. 11–17.
- Dobkin. D. M. (2008). The RF in RFID: Passive UHF RFID in Practice. Newnes, Ed. Elsevier Inc.
- Fisher, R. A. (1971). The goodness of fit of regression formulae, and the distribution of regression coefficients. J. Roy. Statist. Soc. 85, p. 597-612, Bennett.
- Hayes, M. H. (1996). Statistical Digital Signal Processing and Modeling. Wiley.
- Haykin, S. (1995). Adaptive Filter Theory. Prentice-Hall, Inc., Upper Saddle River, NJ, USA.
- Kalman, R. E. (1960). A New Approach to Linear Filtering and Prediction Problems. Trans. ASME, J. Basic Eng., 82, p. 35-45.
- Naidu, P. S. (2000). Sensor Array Signal Processing. CRC Press, Boca Raton, Florida.
- Paulraj, A., Roy, R. & Kailath, T. (1985). Estimation Of Signal Parameters Via Rotational Invariance Techniques (Esprit). Nineteenth Asilomar Conference on Circuits, Systems and Computers, p. 83–89.

Presented at the DLSU Research Congress 2014 De La Salle University, Manila, Philippines March 6-8, 2014

- Plackett, R. L. (1950). Some Theorems in Least Squares. Biometrika, 37, p. 149-157.
- Schmidt, R. O. (1986, March). Multiple Emitter Location and Signal Parameter Estimation. IEEE Trans. Antennas Propagation, Vol. AP-34, p. 276-280.
- Wang, J., Amin, M. & Zhang, Y. (2006). Signal and array processing techniques for RFID readers. Wireless Sensing and Processing, Proc. of SPIE.
- Widrow, B. & Hoff, M. E. (1960). Adaptive switching circuits. Proc. Of WESCON Conv. Rec., part 4, p. 96-140.
- Wiener, N. (1942, February). The interpolation, extrapolation and smoothing of stationary time series. Report of the Services 19, Research Project DIC-6037 MIT.