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GALACTIC DYNAMICS OF SPIRAL GALAXIES FROM LINEARIZED GRAVITY

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Abstract: In this paper, we considered linearized gravity, a general relativistic approximation scheme in weak fields where the nonlinear terms of the spacetime metric can be ignored to study galactic dynamics. Equations of motion in galaxies are determined through the geodesic equations, which are in turn obtained using first-order correction to flat spacetime metric that were calculated from known galactic potentials.

Keywords: spiral galaxy, linearized gravity, general relativity, galactic dynamics, gravitation

1. INTRODUCTION

Spiral galaxies are composed of very dense bulge and a flat disk containing spiral arms. Stars and clouds at large distances from the bulge of spiral galaxies should be following Keplerian orbits, and therefore, their velocities should decrease with the distance. However, observations revealed that the velocities become constant at large distances from the galactic center (Rubin, V. C.; Peterson, C.J.; Ford, Jr, W.K., 1976). Furthermore, this has been also observed in other types of galaxies. These discrepancies are sometimes linked to dark matter that by some calculations account for 25% of the Universe's energy (Faber S.M., Gallagher, J.S., 1979). While there are various theories about dark matter, none has yet proven to be satisfactory.

Alternatively, modifying the Newtonian description of the galactic dynamics can address the issue. This has been attempted for example in modified Newtonian dynamics (MOND) and through non-Newtonian gravitational theories (Milgom, M.,1983; Bekenstein J.;Milgrom, M., 1984; Bekenstein, J. D., 2007). Most non-Newtonian gravity models are however speculative, while modified Newtonian dynamics is largely a semi-empirical approach. A direction that seems to have been largely unexplored is the application of the well-established Einstein's general theory of relativity in dealing with galactic dynamics.

General theory of relativity tells us that the presence of matter warps spacetime. In places where matter is not significantly large, the curvature is negligible and thus Newtonian gravity holds true. However, if the matter distribution is large enough, the curvature would be pronounced enough so that alteration to Newtonian gravity will be noticeable. It is from this point of view that we propose in this paper an alternative approach to galactic dynamics. In this work we applied linearized gravity to determine equations of motion of spiral galaxies through geodesic equations.

2. LINEARIZED GRAVITY

General relativity (GR) is a well tested theory of gravity (Will, C. M., 2006). To date, there is no evidence that suggests that it is not the correct theory of gravitation. But, Einstein's equation (1) is formidable to solve because one has to know either the metric of the system or the matter distribution beforehand, and neither is easily known.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu} \quad \text{(Eq. 1)}$$

However, if the fields are weak, treat Einstein's equation perturbatively and approximate the metric g, from the flat Minkowski space η such that

$$g_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu} \quad \text{(Eq. 2)}$$

where γ is a perturbation. Linearized gravity is an approximation to general relativity in which we substitute (2) in Einstein equation (1) and retaining only the terms linear in γ. The Christoffel symbol for the linear order in γ is given by

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2}\eta^{\rho\sigma}(\partial_{\mu}\gamma_{\nu\sigma} + \partial_{\nu}\gamma_{\mu\sigma} - \partial_{\sigma}\gamma_{\mu\nu}) \quad \text{(Eq.3)}$$

In trace-reversed form, the Einstein tensor to linear order is

$$G^{(1)}_{\mu\nu} = \partial^{\rho}\partial_{(\rho}\bar{\gamma}_{\nu)\rho} - \frac{1}{2}\partial_{\rho}\partial^{\rho}\bar{\gamma}_{\mu\nu} - \frac{1}{2}\partial^{\alpha}\partial^{\beta}\bar{\gamma}_{\alpha\beta}, \quad \text{(Eq. 4)}$$

with

$$\bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\gamma \quad \text{(Eq. 4)}$$

or

$$\bar{\gamma}_{\mu\nu} = \gamma_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\eta^{\alpha\beta}\gamma_{\alpha\beta} \quad \text{(Eq. 5)}$$

Applying the gauge transformation

$$\bar{\gamma}_{\mu\nu} \to \gamma_{\mu\nu} + \partial_{\mu}\xi_{\nu} + \partial_{\nu}\xi_{\mu}, \quad \text{(Eq. 6)}$$

the linearized Einstein's equation can be simplified as

$$G^{(1)}_{\mu\nu} = -\frac{1}{2}\partial_{\rho}\partial^{\rho}\bar{\gamma}_{\mu\nu} = 8\pi T_{\mu\nu} \quad \text{(Eq. 7)}$$

If we consider a rigid-body source, the components of energy-momentum tensor *T* are assumed to be zero, except for ν = μ = 0, Einstein's equation gives

$$\partial_\rho \partial^\rho \bar{\gamma}_{\mu\nu} = 0 \rightarrow \nabla^2 \bar{\gamma}_{\mu\nu} = 0 \qquad \text{(Eq. 8)}$$

or

$$\nabla^2 \bar{\gamma}_{\mu\nu} = -16\pi\rho \qquad \text{(Eq. 9)}$$

In the Newtonian limit, the perturbed metric may be determined from the potential ϕ through (Wald, . 1984)

$$\gamma_{\mu\nu} = \bar{\gamma}_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}\bar{\gamma} = -\left(4t_\mu t_\nu + 2\eta_{\mu\nu}t^\rho t_\rho\right)\phi \qquad \text{(Eq. 10)}$$

where

$$\phi = -\frac{1}{4}\bar{\gamma}_{\mu\nu}$$

satisfies Poisson's equation,

$$\nabla^2 \phi = 4\pi\rho \qquad \text{(Eq. 11)}$$

The motion of the bodies in this curved spacetime geometry may then be determined through the geodesic equation,

$$\frac{d^2 x^\mu}{d\tau^2} + \sum_{\rho,\sigma} \Gamma^\mu_{\rho\sigma}\left(\frac{dx^\rho}{d\tau}\right)\left(\frac{dx^\sigma}{d\tau}\right) = 0 \qquad \text{(Eq. 12)}$$

3. GALACTIC DYNAMICS BY LINEARIZED GRAVITY

In this paper, we considered the modified Newtonian logarithmic potential for spiral galaxies,

$$\phi(r) = -\left(\frac{GM}{r}\right) - \alpha GM \ln\left(\frac{r}{R}\right) \qquad \text{(Eq. 13)}$$

where α is a parameter related to reciprocal of the distance, L^{-1}, and R is an arbitrary constant. The logarithmic term suggests that in spiral galaxies, the energy and the radial pressure are much larger than the transverse pressure, this behaviour of a cosmic gas is very different compared with point-like objects (Soleng, H. H.,1995). Moreover, it has an advantage of giving a constant velocity to a Keplerian trajectories for large values of distances from the center of galaxies (Fabris, J. C.**,** Campos**,** J. P.**,** 2009).

Applying equation (8) and using the Minkowski metric in spherical coordinates,

$$ds^2 = -dt^2 + dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 \,, \qquad \text{(Eq. 14)}$$

the perturbations γ are calculated to be

$$\gamma_{tt} = -\frac{4GM}{r^3} + \frac{\alpha GM}{r^2} \qquad \text{(Eq. 15)}$$

$$\gamma_{rr} = -\frac{24GM}{r^3} + \frac{6\alpha GM}{r^2} \qquad \text{(Eq. 16)}$$

$$\gamma_{\theta\theta} = -\frac{4GM}{r} + 2\alpha GM \qquad \text{(Eq. 17)}$$

$$\gamma_{\phi\phi} = -\frac{4GM}{r}\sin^2\theta - 2\alpha GM \sin^2\theta \qquad \text{(Eq. 18)}$$

and other γ are zero. Then, using equation (3), the non-zero Christoffel symbols are

$$\Gamma^t_{tr} = \Gamma^t_{rt} = -\frac{6GM}{r^4} + \frac{\alpha GM}{r^3} \qquad \text{(Eq. 19)}$$

$$\Gamma^r_{tt} = -\frac{6GM}{r^4} + \frac{\alpha GM}{r^3} \qquad \text{(Eq. 20)}$$

$$\Gamma^r_{rr} = -\frac{36GM}{r^4} - \frac{6\alpha GM}{r^3} \qquad \text{(Eq. 21)}$$

$$\Gamma^r_{\theta\theta} = \frac{2GM}{r^2} \qquad \text{(Eq. 22)}$$

$$\Gamma^r_{\phi\phi} = \frac{2GM}{r^2}\sin^2\theta \qquad \text{(Eq. 23)}$$

$$\Gamma^\theta_{r\theta} = \Gamma^\theta_{r\theta} = -\frac{2GM}{r^4} \qquad \text{(Eq. 24)}$$

$$\Gamma^\theta_{\phi\phi} = -\frac{4GM}{r^3}\cot\theta + \frac{2\alpha GM}{r^2}\cot\theta \qquad \text{(Eq. 25)}$$

$$\Gamma^\phi_{r\phi} = \Gamma^\phi_{r\phi} = -\frac{2GM}{r^4} \qquad \text{(Eq. 26)}$$

$$\Gamma^\phi_{\theta\phi} = \Gamma^\phi_{\phi\theta} = \frac{4GM}{r^3}\cot\theta - \frac{2\alpha GM}{r^2}\cot\theta$$

(Eq. 27)

The corresponding geodesic equations are

$$\frac{d^2t}{d\tau^2} = -2\left[\frac{6GM}{r^4} + \frac{\alpha GM}{r^3}\right]\left(\frac{dt}{d\tau}\right)\left(\frac{dr}{d\tau}\right) \qquad \text{(Eq. 28)}$$

$$\begin{aligned}\frac{d^2r}{d\tau^2} = &-\left[-\frac{6GM}{r^4} + \frac{\alpha GM}{r^3}\right]\left(\frac{dt}{d\tau}\right)\left(\frac{dt}{d\tau}\right) - \left[-\frac{36GM}{r^4} - \frac{6\alpha GM}{r^3}\right]\left(\frac{dr}{d\tau}\right)\left(\frac{dr}{d\tau}\right) \\ &-\left[\frac{2GM}{r^2}\right]\left(\frac{d\theta}{d\tau}\right)\left(\frac{d\theta}{d\tau}\right) - \left[\frac{2GM}{r^2}\sin^2\theta\right]\left(\frac{d\phi}{d\tau}\right)\left(\frac{d\phi}{d\tau}\right)\end{aligned} \qquad \text{(Eq. 29)}$$

$$\frac{d^2\theta}{d\tau^2} = -2\left[-\frac{2GM}{r^4}\right]\left(\frac{dr}{d\tau}\right)\left(\frac{d\theta}{d\tau}\right) - \left[-\frac{4GM}{r^3}\cot\theta + \frac{2\alpha GM}{r^2}\cot\theta\right]\left(\frac{d\phi}{d\tau}\right)\left(\frac{d\phi}{d\tau}\right) \qquad \text{(Eq. 30)}$$

$$\frac{d^2\phi}{d\tau^2} = -2\left[-\frac{2GM}{r^4}\right]\left(\frac{dr}{d\tau}\right)\left(\frac{d\phi}{d\tau}\right) - 2\left[\frac{4GM}{r^3}\cot\theta - \frac{2\alpha GM}{r^2}\cot\theta\right]\left(\frac{d\theta}{d\tau}\right)\left(\frac{d\phi}{d\tau}\right) \qquad \text{(Eq. 31)}$$

4. DISCUSSIONS

Linearized Einstein's equation made the calculations of the geodesic equations workable. Initial results show that the geodesic equations are system of non-linear coupled differential equations, which indicate the complexity of the effect of perturbation term γ on the galactic dynamics of spiral galaxies using the logarithmic potential. Velocity curves for spiral galaxies may be obtained by solving this set of coupled differential equations.

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