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DEVELOPING STUDENTS' MATHEMATICAL UNDERSTANDING ON LINEAR EQUATIONS IN TWO VARIABLES USING A UBD UNIT PLAN

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Abstract: The purpose of the study was to identify the instrumental understanding and relational understanding (procedural and conceptual knowledge) developed by the students through paper-and-pencil exercises and performance task after the implementation of a unit plan inspired by the Understanding by Design, and the students' perceptions of their skills and proficiency before and after the implementation of the unit plan. The UbD unit plan was validated by eleven professional teachers. It was then carried out in a heterogeneous class of 40 students in a private laboratory school in Manila. The study used both qualitative and quantitative methods of research. The quality of the students' answers for both written exercises and performance task and the advice of the validators supported the qualitative analysis of the students' developed instrumental understanding and relational understanding (procedural and conceptual knowledge) through written exercise and performance task and students' perceptions of their skills and proficiency before and after the implementation of the unit plan. Descriptive statistics supported the quantitative analysis of determining if there are significant differences between the perceptions of the students on their skills and proficiency before and after the implementation of the unit plan. Results showed that the students have developed almost all the instrumental understanding, the rate of the students who developed relational understanding (procedural knowledge) was remarkably high, and the results on the students' developed relational understanding (conceptual knowledge) were promising. The z-statistic for differences between two proportions at 95% confidence showed that there are minimal significant differences between the students' perceptions of their skills and proficiency before and after the implementation of the UbD unit plan, and between the number of students who perceived that they could perform a certain skill and the number of students who were able to carry out the skill. The study recommends the following: engagement to more real world problems for the students to see the significance of studying mathematics and reinforcement of meaningful performance task as another means to gauge students' understanding.

Keywords: Understanding by Design, instrumental understanding, relational understanding, procedural knowledge, conceptual knowledge

1. INTRODUCTION

Teaching for understanding is one of the major issues in mathematics education. The challenge begins with how the lessons are prepared and ensured that students learn what needs to be taught in mathematics. Moreover, it is important that the acquired learning is retained and can be applied when confronted with a situation that calls for it. In a traditional classroom setting, students' learning in mathematics is measured on the scores of a paper and pencil test. This norm is very much limited though, in terms of gauging their understanding of the concepts since such tests would only gauge the learning of the students in the familiarization of the procedures, methods and rules in mathematics. Thus, this tends to result in short-term retention of learning. Knowledge and skills learned by the students do not guarantee understanding. People can acquire knowledge and routine skills without understanding the basis of its development or when to use them (Perkins, 1993).

Schools have always regarded understanding as a critical component of the mathematics classroom, yet teachers continue to struggle with meaningful ways to teach for mathematical understanding. Teachers will likely agree that understanding involves more than procedural knowledge and that it includes the ability to reason with and make sense of what is learned, but the lack of ways to design a classroom to facilitate understanding is central to the students' shortage of learning experiences.

Porteous (2004) defines understanding as comprehending. It is all about an individual making sense of some sort of attempted communication, or some sort of input. This implies that one of the ways to check the understanding of the students is their ability to give sense towards the lesson.

Quilter and Harper (cited in Johnston-Wilder, et al., 2011) emphasize that the main reason for students' difficulty is explained not in terms of the conceptual complexity of the subject matter, but in terms of its apparent irrelevance and/or the teacher's inability to provide learning experiences that would present it in a coherent, meaningful way. Teacher's failure to lead students to the realization of the significance of what they are doing would possibly discourage them to do the mathematics.

Mathematics should not be treated as a 'chameleon' (Johnston-Wilder, et al., 2011), that it fades away against the background of the real world as the teacher and the students go through with the subject. The teacher should have the ability to help students identify aspects of real-world situations where mathematics is relevant. Lee (cited in Johnston-Wilder, et al. 2011) stresses that pupils participate in mathematics when they develop new mathematical ways *by themselves* in order to organize their experiences or to reflect on the organization, strategies and concepts that they have already developed. This may include a search for patterns and consistency or an attempt to generalise or formalise procedures, make connections with the real-world situations and develop logical arguments to use to prove and share the results and outcomes that are discovered. Limjap (cited in Chua, 2009) concludes that for learning to be meaningful, the learners have to actively participate in the formation of mathematical concepts. There should be a regular linking of the concepts to real-life application and allot time for pupils to act as investigative mathematicians and problem-solvers. This will develop the students' capacity for seeing mathematics and provide opportunities for using mathematics in the world around them. Otherwise, if the teacher fails to do such, students will lose interest to do the mathematics.

Bizzarri (2000) states that the reason why students find math boring, difficult and hard to pursue because of the following reasons: (1) mathematics is taught in an unnatural way, and (2) mathematics is a practical discipline, but is taught in an abstract way. This implies that the failure to teach mathematics in a sensible and realistic manner makes the students lose interest. In the traditional classroom setting, the teacher teaches mathematics in a linear format. In order to understand the recent lesson then, the students should have understood the first lesson. Students are never given the "Big Picture" of the lessons (Fielding, 2006). But the "Big Picture" is not what the student understands and is being tested on, but merely the conceptual facets – as long as the student knows how to solve problems that follow an almost identical routine, it is assumed that the student understood the lesson very well. Students need to see that mathematics is important in everyday life. This is where the "Big Picture" can be enormously helpful in a mathematical subject. It is like showing the forest, instead of merely describing the trees one by one in order. Although some of the topics are very abstract, if the teacher is very successful in leading students to see it sensibly, they will be able to see that mathematics plays a vital role in holding our daily lives in place – from the food we eat to the work we do (Johnson, 2007). There is a need to help students see the significance of studying mathematics.

In the school year 2010-2011, the Understanding by Design (UbD) as a curriculum framework was formally implemented in the Philippines through the 2010 Secondary Curriculum. The principles in UbD are not new. This is all about backward planning where the designer of the unit plan should think first of learning outcomes before planning and organizing the appropriate teaching strategies and approaches and learning experiences. Steven Covey (2004) suggests that it is ideal to begin or start things with the end in mind. UbD contends on the surface learning and the numerous-overlapping lessons included in the syllabus and favors the deep understanding. It focuses on providing the students with the big ideas or big pictures for them to see the relevance of every lesson in the real-life set up. UbD highlights the enduring understanding – understanding that lasts. Ladrido is emphatic with the statement, "We teach by design, not by default" (2012). This implies that with UbD, students, through the well prepared lessons, visual aids, tasks and assessment tools, will see that everything prepared for them has a meaning and purpose or goal.

The newly-employed UbD as a curriculum framework is still a work in progress in terms of its efficiency. There is a great need to evaluate if the two years (SY 2010-2011 and 2011-2012) of formal implementation of UbD improved students' performance, engaged the use of critical and reflective thinking, and led to enduring understanding in mathematics. This is paramount most especially as DepEd Order No. 31 of 2012 directs the continuance of the use of UBD as a curriculum framework in the K-12 Basic Education Curriculum (BEC), effective in the school year 2012-2013.

With this frame of mind, the study seeks to investigate the effectiveness of a UbD Unit Plan in Algebra-linear equations in two variables in one section of first year high school. The section is a heterogeneous class, which is very much appropriate to check if the UbD is suitable for all types of learners. The research's objective is to evaluate the extent of the development of mathematical understanding (instrumental and relational understanding) of the students with the use of UbD. The end goal in mind is to be able to describe the usefulness of the UbD in the teaching for understanding. This information gathered can then be used to determine the Unit Plan's strengths and weaknesses and to identify aspects that need to be improved or maintained.

Algebra is chosen since most of the pre-requisite skills for higher mathematics are in this subject. Failure to establish and develop a good background in these pre-requisite skills would mean failure to deal with higher mathematics easily and efficiently.

The study investigated the instrumental understanding and relational understanding (procedural and conceptual knowledge) which the students developed after the implementation of the Understanding by Design (UbD) Unit Plan through written exercises and performance task.

The Ubd unit plan on linear equations in two variables was implemented for ten (10) consecutive meetings, one (1) hour every meeting, during the second grading period school year 2012-2013 to the forty (40) first year high school students in one heterogeneous class.

This study will be very significant to the schools, mathematics teachers, students and to the mathematics education community, in general. The schools will be having one of the means to assess the effectiveness of Understanding by Design (UbD) as a curriculum framework in mathematics. Strengths and weaknesses of UbD will be identified. Mathematics teachers will be able to see how the UbD works out in the development of students' instrumental understanding, procedural and conceptual knowledge and relational understanding. From this, they will be able to design teaching strategies, performance tasks and assessment tools which will raise the level of students' understanding of mathematics. Students will be engaged more on learning experiences that enhance their mathematical understanding and be able to see the relevance of doing the mathematics.

2. METHODOLOGY

The study focused on the acquisition of the instrumental understanding and relational understanding (procedural and conceptual knowledge).

The study was supported by the constructivist theory by structuring, monitoring, and adjusting activities which facilitated the learning process.

The sources of information were the subjects of the study from whom certain documents were gathered.

Pseudonyms were used to conceal the identity of the subjects and other individuals involved in the study.

Research Design

The study employed the descriptive research design. Qualitative and quantitative data were gathered using a) self-assessment checklist, b) instrumental understanding checklist for written exercises and performance task, c) relational understanding (procedural knowledge) checklist for written exercises and performance task, and d) relational understanding (conceptual knowledge) for written exercises and performance task.

Participants

The study was conducted in a heterogeneous class of forty (40) first year high school students during the second grading period of school year 2012-2013 in a private laboratory school in Manila. The researcher invited three (3) professional teachers to observe the class during the implementation of the UbD unit plan to assure its proper execution.

Research Procedure

The researcher developed a UbD unit plan on linear equations in two variables through the help of the eleven experts in the field of mathematics and education (see Appendix I): (1) Evaluator 1, a doctoral degree candidate who has been a chairperson for 5 years in a learner-centered college in Manila and has been teaching for 20 years in the same school, (2) Evaluator 2, a master's degree holder who has been a chairperson in a learner-centered exclusive for girls college for 6 years and has been teaching for 17 years in the same school, (3) Evaluator 3, a doctoral degree candidate of education major in curriculum and instruction who has been a coordinator in a learner-centered exclusive for girls college for 8 years and has been teaching for 21 years in the same school, (4) Evaluator 4, a master's degree holder who has been teaching in a learner-centered college in Manila for 21 years, (5) Evaluator 5, a master's degree holder who has been teaching for 10 and 2 years in universities in Quezon City and Manila respectively, (6) Evaluator 6, master's degree holder who has been teaching in a university in Manila for 5 years. (7) Evaluator 7, a master's degree holder who has been teaching in a private university for 4 years and has been a supervising teacher for 2 years. (8) Evaluator 8, a master's degree holder who has been a coordinator for 9 years and has been teaching for 27 years in a private school in Bulacan. (9) Evaluator 9, a master's degree holder who has been teaching for 31 years in a public school in Manila, (10) Evaluator 10, a master's degree holder who has been teaching for 32 years in a science high school in Manila, and (11) Evaluator 11, a master's degree holder who has been teaching for 5 years in a public school in Quezon City.

The researcher implemented the Ubd unit plan for 10 consecutive meetings (2 weeks). Three teachers/subject matter experts: Evaluator 6, Evaluator 9, and Evaluator 10 were invited to observe the class to guarantee proper implementation of the unit plan. On the first day of the class, the students received a self-assessment checklist (Appendix A) – a list of skills where the students have to mark competencies which they think they could carry out. This checked the students' perceptions in terms of their skills and proficiency before the implementation of the unit plan. A similar checklist was also given on the 7^{th} day of the meeting, before taking the written chapter exercises. This confirmed if there were changes in the students' perceptions in terms of their skills and proficiency after the implementation of the unit plan. On the same day, the class took paper-and-pencil exercises (Appendix B) which served as a summative assessment. This measured the developed instrumental and relational understanding (procedural and conceptual knowledge) of the students in terms of the mastery of the content. The instrumental understanding and relational understanding (procedural and conceptual knowledge) checklists for written exercises identified what were the specific instrumental understanding and relational understanding (procedural and conceptual knowledge) of the students. The checklists' results were also used to verify if the students' perceptions of their skills and proficiency were present in the written exercises. On the 8th day of the implementation, the class was given a performance task (Appendix C). The class was divided into 5 groups with 8 members each. The class was given instructions on the performance task that was carried out for two days. The students were reminded that the presentation will be on the 10th day, the last day of the implementation of the unit plan. The performance task served as a summative assessment. The task gauged the developed instrumental understanding and relational understanding (procedural and conceptual knowledge) of the students. The instrumental understanding and relational understanding (procedural and conceptual knowledge) checklists (Appendices D, E, & F) were utilized to identify the developed instrumental understanding and relational understanding (procedural and conceptual knowledge) of the students through the performance task.

Instrumentation

All the instruments were developed by the researcher through the help and proper guidance of his mentor and through the carefully taken and respected suggestions and comments of the research panel.

All the skills and competencies from the checklists were adapted from Wiggins' and McTighe's 1998 book, *Understanding by Design*, published by Merrill Prentice Hall.

Self-Assessment Checklist

The first instrument, the self-assessment checklist (Appendix A), consisted of twenty-seven (27) questions involving the skills on linear equations in two variables which were to be developed. The students were asked to place a check (√) mark on the space provided if they think that they can carry out the given skill. Otherwise, they will place a cross (x) mark. This checked the students' perceptions of their skills and proficiency on linear equations in two variables before and after the implementation of the UbD unit plan.

Paper-and-Pencil Exercises

The second instrument, written exercises (Appendix B), consisted of five parts: a) I- IV containing routinary exercises and b) V containing non-routine problems selected from different sources. These were validated by three experts in the field. Test I consisted of fifteen items about plotting points and identifying the coordinates of points on the rectangular coordinate plane. Test II contained one item about sketching the graph of a linear equation in two variables. Test III, consisting of four items, was focused on the slope and equation of a line. Test IV, containing eight questions, was about linear relationships – identifying the dependent and independent variables. The tests I-IV measured the instrumental understanding and procedural knowledge of the students. The Test V consisted of two non-routine problems. Problem 1 was a semblance of a real-world situation involving the application of linear equation in two unknowns in identifying the linear relationship of two variables. Problem 2 was a non-routine problem involving slopes. The problem focused on identifying which of the two parking services is more affordable given certain situations. It was given to test if the student had a full grasp of the concepts involved. The test V of the activity measured the procedural and conceptual knowledge of the students. This paper-and-pencil activity determined the instrumental and relational understanding (procedural and conceptual knowledge) developed by the students after the implementation of the UbD unit plan.

Performance Task

The third instrument, the performance task (Appendix C), was about planning a birthday party for a friend. The students were asked to manage birthday invitations, foods, birthday decorations and entertainment. The students had to manage a budget of Php10,000 for the things they were assigned to prepare. This measured the developed instrumental understanding and relational understanding (procedural and conceptual knowledge) of the students after the implementation of the UbD unit plan.

Instrumental Understanding Checklist

The fourth instrument, the instrumental understanding checklist (Appendix D), consisted of seven (7) instrumental understanding items which the students should have developed through written assessments and performance tasks. This identified the instrumental understanding developed by the students after the implementation of the unit plan. This was also used to check if the students' perceptions of their skills and proficiency in terms of instrumental understanding were demonstrated in the written exercises and performance task.

Relational Understanding (Procedural Knowledge) Checklist

The fifth instrument, the relational understanding (procedural knowledge) checklist (Appendix E), consisted of nine (9) procedural knowledge items which were to be developed by the students through written assessments and performance task. This determined the procedural knowledge, one of the knowledge that the students had to build up in order to attain relational understanding, which the students should have developed after the implementation of the unit plan. This was also used to check if the students' perceptions of their skills and proficiency in terms of procedural knowledge were demonstrated in the written exercises and performance tasks.

Relational Understanding (Conceptual Knowledge) Checklist

The sixth instrument, conceptual knowledge checklist (Appendix F), consisted of eighteen (18) conceptual knowledge items which the students should have developed through written assessments and the performance task. This determined the conceptual knowledge, a type of knowledge that the students had to build up in order to attain relational understanding, which the students should have developed after the implementation of the unit plan. This was also used to check if the students' perceptions of their skills and proficiency in terms of conceptual knowledge are demonstrated in the written exercises and performance tasks.

Data Analysis

Two copies of self-assessment checklists involving the skills and competencies on linear equations in two variables were used for the following: (1) to check the students' perceptions of their skills and proficiency before the implementation of the UbD unit plan, (2) to check the students' perceptions of their skills and proficiency after the implementation of the UbD unit plan, and (3) to check if the students' perceptions of their skills and proficiency on linear equations in two variables were demonstrated through written and performance tasks after the implementation of the unit plan.

Checking for Significant Differences

Z-statistic for differences in two proportions was used to determine significant differences between (1) students' prior knowledge and students' perceptions of their skills and proficiency after the implementation of the UbD unit plan, (2) number of students' who perceived they could carry out a certain skill and number of students who attained the said skill.

Three checklists (instrumental understanding checklist, relational understanding (procedural knowledge) checklist, and relational understanding (conceptual knowledge) checklist) involving the skills and competencies on linear equations in two variables were used for the following: (1) to determine the developed instrumental understanding and relational understanding (procedural and conceptual knowledge) through paper-and-pencil exercises, and (2) to determine the developed instrumental understanding and relational understanding-procedural and conceptual knowledge- through performance tasks after the implementation of the UbD unit plan.

The scores on the three checklists for both paper-and-pencil exercises and performance task were attained based on the quality of the answers of the students. The operational definitions of instrumental understanding and relational understanding (procedural and conceptual knowledge) and the suggestions and comments of subject matter experts: Evaluator 1, Evaluator 4, and Evaluator 5 were carefully taken and considered in deciding if a student has demonstrated evidence of the development of a certain understanding based from his answers on the exercises.

Paper-and-Pencil Exercises

For paper-and-pencil exercises, the researcher, evaluator 1, evaluator 4 and evaluator 5, carefully and intelligently discerned and set a cut-off score based from the operational definition of instrumental understanding, relational understanding (procedural knowledge) and relational understanding (conceptual knowledge) to determine the number of the students who attained a certain skill. They decided that only those students who got 80% – 100% for each of the following parts of the written exercises have acquired the certain skill, since this range is classified as approaching proficiency, proficient and advanced in the level of proficiency of Department of Education's DepEd Order (DO) No. 31, s. 2012.

Performance Task

For performance task, since the class was divided into five groups with eight members each, then this indicates that if a certain group failed to carry out a certain skill, then each of the members of the group is also affected. The scores were gathered through the quality of the groups' presented product following the performance task.

3. RESULTS AND DISCUSSION

This chapter presents and discusses the results of the study. It should be recalled that the purposes of the researcher were: to find out the students' instrumental understanding and relational understanding (procedural and conceptual knowledge) developed through written exercises after the implementation of the unit plan, to find out the students' instrumental understanding and relational understanding (procedural and conceptual knowledge) developed through performance task after the implementation of the unit plan, and to determine the students' perceptions of their skills and proficiency before and after the implementation of the UbD unit plan on linear equations in two variables. Findings will be presented in the same order as the research problems listed.

The Students' Developed Instrumental Understanding and Relational Understanding (Procedural and Conceptual Knowledge) Through Written Exercises

The researcher used the following checklists; (1) instrumental understanding checklist, (2) relational understanding (procedural knowledge), and (3) relational understanding (conceptual knowledge) after the students have answered the paper-and-pencil exercises. The exercises were checked intelligently and analyzed qualitatively to know who among the forty (40) students have developed each of the following instrumental understanding and relational understanding (procedural and conceptual knowledge).

Tables 1, 2 and 3 show the number of students who were able to meet the cut-off scores set by the researcher, evaluator 1, evaluator 4 and evaluator 5.

Table 1 shows the developed students' instrumental understanding (see Appendix D) through written exercises. It shows the number of students who built up each of the following instrumental understanding after analyzing their answers qualitatively.

Table 1: *The Students' Developed Instrumental Understanding Through Written Exercises*

Instrumental Understanding	**Number of Students who Attained Instrumental Understanding**	**Percentage**
1) Plot points on a coordinate plane	40	100.00%
2) Add, subtract, multiply, and divide positive and negative real numbers	32	80.00%
3) Solve problems involving real numbers	30	75.00%
4) Simplify and evaluate algebraic expressions, using commutative, associative, and distributive properties as appropriate	38	95.00%
5) Add and subtract linear expressions	36	90.00%
6.a) Define a variable	40	100.00%
6.b) Write linear equations	30	75.00%
6.c) Solve linear equations	30	75.00%
6.d) Solve slopes	34	85.00%
7) Use the addition and multiplication properties of equality to solve one- and two-step linear equations	38	95.00%

The percentage of the students who were able to develop the instrumental understanding ranged from 75.00% to 100.00%. Majority of the students have developed the skills of plotting points on a coordinate plane (100.00%), defining a variable (100.00%), simplifying and evaluating algebraic expressions (95.00%), adding and subtracting linear expressions (90.00%), and using addition and multiplication properties to solve one- and two-step linear equations (95.00%). There are noticeable skills which need to be given attention: the skills about solving problems involving real numbers, writing and solving linear equations and slopes, all of which had a rate range of 75.00% to 80.00% with students who developed the instrumental understanding.

Some of the students who failed to demonstrate strong evidences of the development of instrumental understanding in terms of the skills about solving problems involving real numbers and linear equations were Ma. Khristine, Danica, Kumar and Pauline.

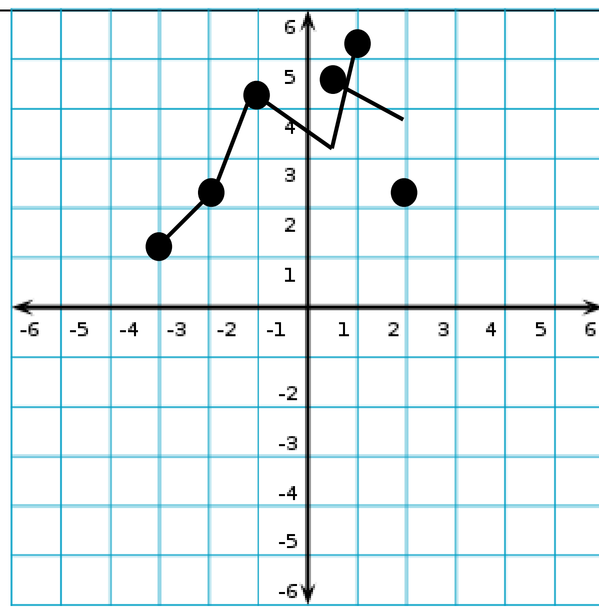
These four students namely share a common mistake – the performing of operations involving real numbers, which is a pre-requisite skill in order to carry out more complex computations such as solving linear equations. In Test II, students were asked to construct linear equations using the given table of values. Since these students were not able to master the skill of performing operations involving real numbers, it followed that they did not get the Test II correctly.

The following show the solving strategies of Ma. Khristine. This shows how the inability to master pre-requisite skills affects the acquisition of more complex skills.

Ma.Khristine's solution in Test II

Complete the table and graph. Connect the points on the graph.

	x	y	Ordered Pair
1)	-3	2	(-3, 2)
2)	-2	3	(-2, 3)
3)	-1	5	(-1, 5)
4)	0	-5	(0, -5)
5)	0.5	-4.5	(0.5, -4.5)
6)	1	6	(1, 6)
7)	2	3	(2, 3)



exercises about slopes.

Julian did not answer anything on test III about slopes. He explained, "*Sir, 'di ko alam eh, absent ako*.". It may be the main excuse for skipping the items on slopes but there was one thing which manifested evidence that the student did not acquire mastery of skills about the cartesian coordinate system which may lead to failure to acquire understanding on slopes. Here is the solution strategy that Julian used to start solving for the slope at Test III, item #1.

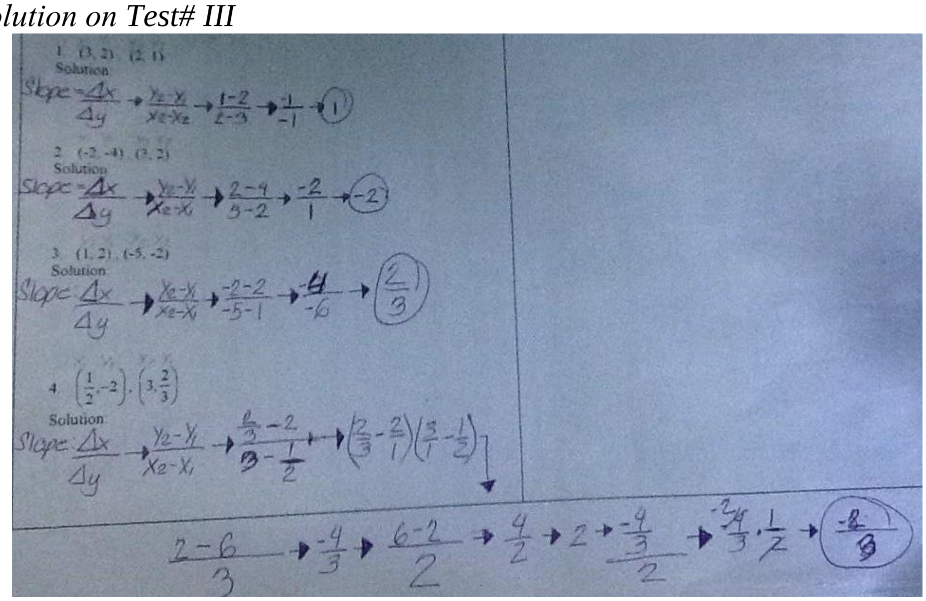
Julian's solution in Test III Item #1
X_1 X_2 Y_1 Y_2
2. (3 , 2) , (2, 1)

In Julian's solution, it shows that he thought that given two points on a line, the first ordered pair is the x_1 and x_2 while the second ordered pair is the y_1 and y_2. In this problem, it should be noted that since he did not know that an ordered pair consists of x- and y- values, the abscissa and ordinate, he may encounter more problems as the lesson goes on just like in this slope problem.

Danica and Bridget may have attempted to solve slope problems but tend to stop after substituting the values. Danica did not answer item #3, while Bridget did not answer item# 4. The problems in their solution strategies are similar with Julian. They thought that given two points on a line, the first ordered pair is the x_1 and x_2 while the second ordered pair is the y_1 and y_2. Both were able to answer item #1 since it does not contain any negative real numbers.

The following are the solution strategies of Pauline to solve slope problems. Pauline may have solved the first slope problem but her solutions for the four items on slopes need to be given attention.

Pauline's solution on Test# III



Her solutions show that she has not mastered the instrumental understanding of solving for slope. It is important to recall that instrumental understanding is acquired when one is capable to replicate the procedure, method and strategies presented by its mentor. For slope with formula $\frac{(y_2 - y_1)}{(x_2 - x_1)}$, we get the quotient of the differences of the y-values and x-values. It implies that whether the values of y_1 and x_1 are negative or positive numbers, subtraction is still the operation to be used. Her solutions show that Pauline may have answered item #1 maybe because the values involved are all positive integers, while she got wrong answers for the next three problems because she did not recognize that the minus signs in the formula are the operations that you have to perform to solve for the slope and do not refer to the signs of the given coordinates. She may have done performing operations involving real numbers correctly, but the mere fact that she did not substitute the values correctly led her to make the wrong answers.

The following are the solution strategies of John. John's problems in his solutions focus on the wrong substitution of values and performing operations involving real numbers. His first problem is similar with Pauline where he thought that the minus signs in the formula of slope refer to the negative signs of the coordinates given the points. The second is considered a crucial problem because most of the lessons in mathematics involve performing operations especially if the problem involves negative real numbers; failure to master this basic skill would lead a student not to acquire higher skills. He got the item #1 on slopes correctly because all the given coordinates are positive, but was not able to solve the next three values correctly due to lack of skill on performing operations involving real numbers.

John's solution in Test III, Item #1 1. (3 , 2) , (2, 1) $= \frac{1-2}{2-3} = \frac{-1}{-1} = 1$
John's solution in Test III, Item #2 2. (-2 , -4) , (3, 2) $= \frac{2-4}{3-2} = \frac{2}{1} = 2$
John's solution in Test III, Item #3 3. (1 , 2) , (-5, -2) $= \frac{-2-2}{-5-1} = \frac{0}{6} = 0$

John's solution in Test III, Item #4

4. $\left(\frac{1}{2},-2\right)$, $\left(3,\frac{2}{3}\right)$ = no answer

There are some points that should be given attention in the results for table 1. Since some students were not able to develop succeeding skills due to failure to master its requisite, this indicates that teachers need to monitor students' performance from time to time to guarantee that no one is left behind. This deed may be tiresome to the educators but this would be beneficial to both parties. For the teachers, there will always be a good continuous flow of discussion of lessons because this will prevent re-teaching of the previous topics. And at the same time, teachers would always have assurance that none of the students is left behind. And for the students, they will always have interest to engage themselves in the learning for they are equipped with skills pre-requisite to the lesson to be taken.

Table 2 shows the students' relational understanding (procedural knowledge) (see Appendix E) developed through written exercises. It shows the number of students who built up each of the following relational understanding (procedural knowledge) after checking students' answers.

Table 2: *The Students' Developed Relational Understanding (Procedural Knowledge) Through Written Exercises*

Relational Understanding (Procedural Knowledge)	**Number of Students who Attained Procedural Knowledge**	**Percentage**
1) Plot points on a coordinate plane	34	85.00%
2.a) Represent relations from tables	33	82.50%
2.b) Represent relations from graphs	33	82.50%
2.c) Represent relations from formulas	33	82.50%
3.a) Describe relations from tables	19	47.50%
3.b) Describe relations from graphs	19	47.50%
3.c) Describe relations from formulas	19	47.50%
4) Add, subtract, multiply, and divide positive and negative real numbers	32	80.00%
5) Solve problems involving real numbers	30	75.00%
6) Simplify and evaluate algebraic expressions, using commutative, associative, and distributive properties as appropriate	38	95.00%

7) Add and subtract linear expressions	38	95.00%
8.a) Define a variables	40	100.00%
8.b) Write linear equations	30	75.00%
8.c) Solve linear equations	30	75.00%
8.d) Solve slopes	25	62.50%
9) Use the addition and multiplication properties of equality to solve one- and two-step linear equations	38	95.00%

It is important to recall that procedural knowledge is attained when the following questions were addressed: (1-a) What is the goal of the procedure?, (1-b) Up to what can this procedure do?, (1-c) What sort of outcome should I expect?, (2-a) How do I execute the procedure?, (2-b) What are some other procedures I could use instead?, (3) Why is the procedure effective and valid?, (4) How will I able to verify my answers?, and (5) When is this certain procedure 'best' to use? (Hasenbank, 2004). This implies that if one of these questions is not addressed, then the learning acquired is most likely the instrumental understanding rather than the procedural knowledge.

The rate of the students who developed each of the following procedural knowledge ranged from 47.50% to 100.00%. Majority of the students developed the skills of defining a variable (100.00%), simplifying and evaluating algebraic expressions (95.00%), adding and subtracting linear expressions (95.00%), and using addition and multiplication properties of equality to solve one- to two- step linear equations (95.00%). Some skills got an average rate ranging from 75.00% - 85.00% with the skills on writing and solving linear equations given a problem (75.00%), performing operations involving real numbers (80.00%), and representing a relation from tables, graphs and formulas (82.50%). Relational understanding (procedural knowledge) which need to be given attention are on the describing relations from tables, graphs, and formulas (47.50%),

Some of the remarkable errors which show strong evidences that certain skills were not developed like the procedural knowledge on plotting points were demonstrated by Bridget, Danica, Izel, and Justin.

The following are the solutions of Bridget and Danica. The Test I, Letter B of the written exercises asks the students to determine the coordinates of the point plotted on the rectangular coordinate plane. This will test the students if they have acquired the skill of doing the reverse of plotting the points on the coordinate plane. Their solutions demonstrated that they were still confused on the positive and negative values of the x- and y- axes. They may have known that the x-axis is the horizontal number line and y-axis is the vertical number line, but they did not know where the positive and negative values of the two axes are.

Bridget's solution for Test II, Letter B		
	Bridget's answer	*Correct Answer*
Item #14	(7, 1)	(-7, 1)
Item #15	(4, 3)	(-4, -3)

Danica's solution for Test II, Letter B		
	Danica's answer	*Correct Answer*
Item #9	(4, 0)	(0, 4)
Item #10	(6, -6)	(-6, 6)
Item #11	(1, -7)	(-7, 1)
Item #15	(-3, -4)	(-4, -3)

Izel and Justin shared a common mistake – they did not know that in an ordered pair, the first value is the abscissa/x-coordinate and the second value is the ordinate/y-coordinate. And the position of these two values cannot be interchanged since points on rectangular coordinate plane refer to positions/locations. The change of its values would also imply change on its location. For example, a point (-5, 5) lies in Quadrant II while point (5, -5) lies in Quadrant IV. This misconception should immediately be addressed in order to for the students to acquire succeeding skills such as graphing linear equations using table of values, x- and y-intercepts and slopes.

Izel's solution for Test II, Letter B		
	Flores's answer	*Correct Answer*
Item #12	(4, 0)	(0, 4)
Item #13	(6, -6)	(-6, 6)
Item #14	(1, -7)	(-7, 1)
Item #15	(-3, -4)	(-4, -3)

Justin's solution for Test II, Letter B		
	Bugarin's answer	*Correct Answer*
Item #9	(-3, 4)	(4, -3)
Item #10	(3, 10)	(10, 3)
Item #14	(1, -7)	(-7, 1)
Item #15	(-3, -4)	(-4, -3)

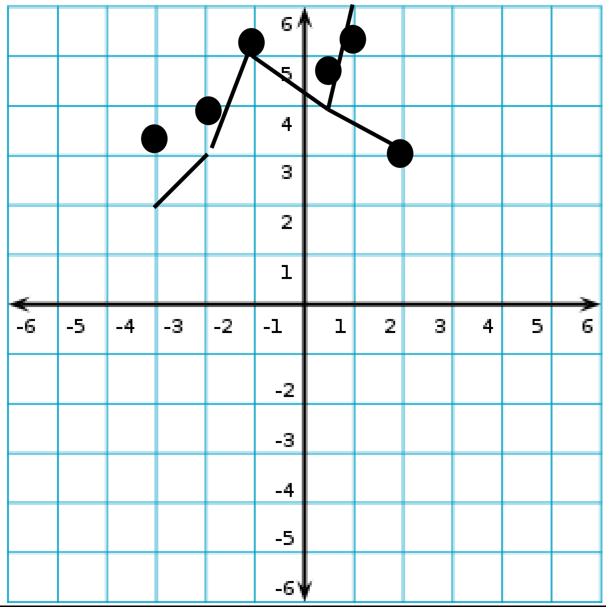
Moreover, there were 21 students who failed to demonstrate strong evidences of the development of relational understanding (procedural knowledge) in terms of the skills about describing relations from tables, graphs and formulas. Some did not describe their sketched graphs, while others were not able to give reasonable descriptions. Some of these were Ma. Khristine, Kyle, and Mae.

The following are the ways on how Ma. Khristine sketched and described the graph of a linear equation. It is good to recall that since she was not able to get the correct coordinates of the points due to an incorrect execution of operations involving real numbers, she did not also get the graph of the linear equation correctly. Also, what is interesting was on the way she described her sketched graph. She wrote that "the graph is zigzag and not proportional". It is in one of the questions on procedural knowledge acquisition framework that the student who has acquired procedural knowledge should have learning on verifying the correctness of answers. She should know that a concept may have multiple representations of knowledge. If Ma. Khristine has attained procedural knowledge in terms of sketching the graphs and solving linear equations, she should have recognized that she can check her answers by substituting the values of x and y to the given linear equation and check if the answer after performing operations will arrive to a correct equation.

Ma.Khristine's solution in Test II

Complete the table and graph. Connect the points on the graph.

	x	y	Ordered Pair
1)	-3	2	(-3, 2)
2)	-2	3	(-2, 3)
3)	-1	5	(-1, 5)
4)	0	-5	(0, -5)
5)	0.5	-4.5	(0.5, -4.5)
6)	1	6	(1, 6)
7)	2	3	(2, 3)



The next shows Kyle's solution strategies on graphing. Kyle's sketched graph may be correct in terms of the plotted points and the idea that he has to connect the points to create a line. But the line he drew did not have arrowheads, which are an essential part of a line because they indicate that the line can be extended up to its desired length. What Kyle has drawn is a segment, a set of points with end points or an undefined term in geometry which cannot be extended. This shows that Kyle has not acquired the procedural knowledge about sketching the graph of a linear equation. A student who acquired procedural knowledge to the said skill should have at least remembered that it is important to place arrowheads to the line even without knowing the reason behind placing arrowheads. Moreover, Kyle may have a point that the graph lies across the third and fourth quadrants, but he did not recognize that since lines can be extended up to the desired length, then the line will also cross the first quadrant. This shows that he may have acquired the procedural knowledge of describing the graph but the understanding is still limited because of the failure to foresee that the line can be extended. This implies the importance of the acquisition of the higher knowledge for relational understanding which will be tackled on the foregoing discussions of results.

Kyle described the graph for Test II this way:

The graph lies across the third and fourth quadrants because of the points that lie on the y-axis are all negative.

The following focus on the way Mae sketched and described the graph. The error in Mae's graph is similar with Kyle; she forgot to place arrowheads in the line. In terms of describing the graph, she made a mistake on identifying the orientation of the line. She thought that the line is decreasing from left to right, which is the other way around. The lines should be increasing from left to right. A student who acquired procedural knowledge should have at least learned on how to identify the orientation of the graph even without the understanding on how to interpret it.

Mae described the graph for Test II this way:

The trend of the graph is said to be decreasing. This means that the value of the independent variable x increases while the value of the variable y decreases.

However, there are students who were able to demonstrate acquisition of the procedural knowledge on describing relations from tables, graphs and formulas. Some of these are Jan, Trissa, Marie Eugenie, and Mel.

The following show the answers of Jan and Trissa about describing the graph, as the test instructions indicated. The acquisition of procedural knowledge on describing relations from tables, graphs and formulas is evident once a student attained the instrumental understanding on solving linear equations, plotting points, and sketching the graph of a linear equations, and is capable on identifying the orientation of the graph and give additional information even without understanding what it is all about. Jan and Trissa have similar answers. First, they have shown evidence of the developed instrumental understanding on solving linear equations, plotting points and sketching the graph of a linear equation. Then, they described that the graph is increasing from left to right. This supports what Skemp (cited in Macnab and Cummine, 1986) says that instrumental understanding may not be understanding at all since it is more on rules without reasons and meaning. But instrumental understanding may serve as the starting point of the students to achieve higher learning. Jan and Trissa may not know the meaning of the graph which is increasing from left to right, but the acquired skill may serve as their foundation for the development of higher learning.

Trissa described the graph for Test II this way:

The line is going upwards to the right meaning it increases.

The foregoing shows the way Marie Eugenie and Mel described their sketched graphs. Same with Jan and Trissa, as they were able to acquire the instrumental understanding needed, they could describe most likely the graph. For these two students, they wrote that the graph has a direct relationship since as the x-value increases, the y-value increases also. They may not have expounded their answers, but this shows a good start in order to carry out higher learning. This procedural knowledge would probably leads the students to link this lesson to other mathematical concepts.

Jan described the graph for Test II this way:

Marie Eugenie described the graph for Test II this way:

Positive.Direct relationship.

Mel described the graph for Test II this way:

The line indicated direct relationship, therefore x- and y- values increase.

The following show students who have developed instrumental understanding on solving for slopes but did not acquire procedural knowledge on the said skill. A student who acquired instrumental understanding on solving slopes has an ability to substitute the values on the formula and perform the indicated operations. But with an inability to anticipate that after performing the operations the answers still need to be simplified, there is an indication that the student did not attain the procedural knowledge on solving slopes. This would mean that the student was not able to address the following questions: (a) What sort of outcome should I expect?, and (b) How do I execute the procedure? which are part of the framework for the acquisition of the procedural knowledge. Some of these students were Jhana, Danel, Ma. Patricia, Ma. Khristine, Mel, Trissa, Patrick, and Caryl. These students have a common mistake– the simplification of the fractions. They may have substituted the values on the formula and performed the operations correctly which shows the attainment of the instrumental understanding on solving slopes, but since they did not recognize that the answers still need to be simplified then it demonstrates that they have not achieved the procedural knowledge on solving slopes.

Jhana, Danel, Ma. Patricia, Ma. Khristine, Mel, Trissa, Patrick, Caryl's solution in Test III, Item #2

2. (3 , 2) , (2, 1)

$$= \frac{1-2}{2-3} = \frac{-1}{-1}$$

Jhana, Danel, Ma. Patricia, Ma. Khristine, Mel, Trissa, Patrick, Caryl's solution in Test III, Item #2

3. (1 , 2) , (-5, -2)

$$= \frac{-2-2}{-5-1} = \frac{-4}{-6}$$

Jhana, Danel, Ma. Patricia, Ma. Khristine, Mel, Trissa, Patrick, Caryl's solution in Test III, Item #2

4. $\left(\frac{1}{2}, -2\right)$, $\left(3, \frac{2}{3}\right)$

$$= \frac{\frac{2}{3} - (-2)}{3 - \frac{1}{2}} = \frac{\frac{2+6}{3}}{\frac{6-1}{2}} = \frac{\frac{8}{3}}{\frac{3}{2}}$$

There are remarkable points on the results which are important to be given attention. First, the acquisition of relational understanding (procedural knowledge) may be affected by student's mastery of the lower learning which is the instrumental understanding. Second, the results show indication that students are not used to thinking of other venues or alternatives to solve problems. And pupils are not used to verifying if their answers are correct. These things need to be given attention for it shows how students limit themselves only to few methods of solving problems which may hinder to acquire understanding of the significance of doing the mathematics. There are ways or methods in solving a problem which are very comprehensive and long to perform while there are some which will require you minimum step solutions. Students should be engaged to all those methods in order for them to appreciate how mathematics problems can be addressed by using different methods and in order for them to appreciate the effort of searching for patterns to discovery new ways of solving a problem.

Table 3 shows the students' relational understanding (conceptual knowledge) (see Appendix F) developed through written exercises. It shows the number of students who were able to carry out each of the following relational understanding (conceptual knowledge) after analyzing their answers qualitatively.

It is essential to discuss the way the researcher determined who among the students has developed each of the following conceptual knowledge. Skemp (1989) explicates that conceptual knowledge consists of relationships that connect a number of mathematical ideas or concepts. This ability is evident once a student can create connections between ideas, facts and procedures that are generally accepted. The ideas, facts or procedures are collected to form a basic concept, and these concepts are connected with other concepts to form new and more complex concepts. It is knowledge rich in relationships, concepts such as square, square root, linear equation, slope, derivative and the like with the inclusion of understanding. Conceptual knowledge is described as that which is part of a network comprised of individual pieces of information and the relationships between these pieces of information. It is a connected web of knowledge, a network in which the linking relationships are as prominent as the discrete bits of information.

One of the evaluators of the Ubd Plan, Evaluator 4, says "*Ang bata na may conceptual knowledge dapat marunong i-connect yung mga lessons sa math sa totoong buhay*." (A student who has acquired conceptual knowledge knows how to connect the lessons in math into real life). He gives emphasis that a student who acquired conceptual knowledge is capable to clearly connect the concepts in mathematics to real-world set up. He has the ability to link how mathematics can represent real problems. Based from his statements, it shows that a student who was able to solve a real-world problem but failed to explain the connection of his answer to the context of the problem did not acquire conceptual knowledge. A student should have the ability to see the application of his answer to the real-world situation.

The following show the explanation of Evaluator 1, one of the evaluators of the UbD unit plan. She explains, "*Mahirap i-assess yung conceptual knowledge sa written exam kasi nga madalas ang gusto lang ng bata makuha 'yung tamang sagot*." (It is difficult to gauge conceptual knowledge through written exercises because most of the time, students only aim to get correct answers). She continues, "*Pero kapag nakapag-explain yan kung ano yung connection nung answer 'nya 'dun sa problem, nakuha niya yung conceptual knowledge*." (But once a student has able to explain the connection of his answer to the given problem, he has acquired conceptual knowledge). What she said shows that conceptual knowledge is very difficult to assess through paper-and-pencil tests. There should be another authentic assessment to use to measure the acquired conceptual understanding; assessment which would test the ability of the students to link their learning or understanding to real world scenarios. This supports the content of Philippines' Department of Education DepEd Order No. 31, s 2012. In the memo, the DepEd would like to give emphasis that the proficiency of the students can in fact be measured by providing students with tasks which deal with transfer of learning. These tasks should challenge the students to apply their learning or understanding in real-life situations and to develop confidence and self-efficacy in doing the task on their own. With this means, students are given challenge to bring into reality the lessons they learned from the subjects. Teachers should focus on assessing the ability of the students to demonstrate evidence of learning, not just by merely paper-and-pencil exercises because such assessment tools limit students' capability to show their acquired learning. This might address the needs of the students to master not only on the knowledge and skill areas in mathematics, but also on how it is connect to the real world scenarios.

Through careful consideration of suggestions and comments of the Evaluator 1, Evaluator 4 and Evaluator 5, the researcher had the foregoing results.

Table 3: *The Students' Developed Relational Understanding (Conceptual Knowledge) Through Written Exercises*

Relational Understanding (Conceptual Knowledge)	Number of Students who Attained Conceptual Knowledge	Percentage
1) Build new mathematical knowledge through problem solving	9	22.50%
2) Solve problems that arise in mathematics and in other contexts	27	67.50%
3) Apply and adapt a variety of appropriate strategies to solve problems	27	67.50%
4) Monitor and reflect on the process of mathematical problem solving	9	22.50%
5) Recognize reasoning and proof as fundamental aspects of mathematics	27	67.50%
6) Make and investigate mathematical conjectures	27	67.50%
7) Develop and evaluate mathematical arguments and proofs	12	30.00%
8) Select and use various types of reasoning and methods of proof	9	22.50%
9) Organize and consolidate their mathematical thinking through communication	12	30.00%
10) Communicate their mathematical thinking coherently and clearly to peers, teachers and others	9	22.50%
11) Analyze and evaluate the mathematical thinking and strategies of others	9	22.50%
12) Use the language of mathematics to express mathematical ideas precisely	9	22.50%
13) Recognize and use connections among mathematical ideas	9	22.50%
14) Understand how mathematical ideas interconnect and build on one another to produce a coherent whole	9	22.50%
15) Recognize and apply mathematics in contexts outside of mathematics	9	22.50%
16) Create and use representations to organize, record and communicate mathematical ideas	9	22.50%
17) Select, apply, and translate among representations to solve problems	9	22.50%
18) Use representations to model and interpret physical, social, and mathematical phenomena	12	30.00%

Very few of the students were able to demonstrate evidences of the development of the relational understanding (conceptual knowledge), the rate ranged from 30.00% to 67.50%. The conceptual knowledge in which most of the students are able to develop are the skills on solving problems that arise in mathematics and in other contexts (67.50%), applying and adapting a variety of appropriate strategies to solve problems (67.50%), recognizing reasoning and proof as fundamental aspects of mathematics, making and investigating mathematical conjectures (67.50%). The conceptual knowledge which show noticeable results are the skills on building new mathematical knowledge through problem solving, monitoring and reflecting on the process of mathematical problem solving, selecting and using various types of reasoning and methods of proof, communicating their mathematical thinking coherently and clearly to peers, teachers and others, analyzing and evaluating the mathematical thinking and strategies of others, using the language of mathematics to express mathematical ideas precisely, recognizing and using connections among mathematical ideas, recognizing and applying mathematics in contexts outside of mathematics, creating and using representations to organize, record and communicate mathematical ideas, selecting, applying, and translating among representations to solve problems, and using representations to model and interpret physical, social, and mathematical phenomena. All of which got a rate of 22.50%. The results support what Evaluator 1 said regarding the difficulty to assess the developed conceptual knowledge of the students because their aim is to simply get the correct answers. There were 27 students who were able to get the answers correctly in Test V which deals with the application of linear equations in two variables. However, there were only 12 students who were able to see the context of their answers; 9 of which were able to give complete solutions, answers and explanations, and 3 were able to get the answers by simply sighting a concrete example which they linked to the given problem. The other 15 got the answers correctly but either they only gave short and very vague explanations or totally did not give explanations to their answers. In these results, it shows that these students were able to model real world situations using mathematics but were unsuccessful to give meaning to their answers and relate them to the problem.

The foregoing discusses the answers of the 9 students who were able to demonstrate strong evidence of the development of the conceptual knowledge on linear equations in two variables. They were Franz, Emmalyn, Madeline, Nicole, Kristine, Shaira, Patrick, Trissa, and Kyle.

The solutions of Franz, Emmalyn, Madeline, Nicole, Kristine, and Shairafor Test V, Problem #2 were almost the same. Test V, Problem #2 is about identifying which of the parking services is more affordable; the parking service ABC or XYZ. These 9 students were able to identify the linear equations of the two given parking services, sketch the graph of linear equations, and give meaning to their answers. They explained that if you are going to park for only 1 hour, then you better choose the parking service XYZ because the fee will only be amounting to Php45. If you are planning to park your car for 2 hours, then you may choose any of the parking services since both fees will be amounting to Php60. But, if you are planning to park for more than 2 hours, then the parking service ABC is more affordable. The students used the results on the table of values to identify which is more affordable. They were able to see the relation of the result to address the problem.

In case of Patrick, Trissa, and Kyle, their solutions are almost the same with the other 6 students, but instead of using the results on the table of values, they used the concepts of the slopes to identify which is more affordable. This is another venue to address the problem. They have recognized that the line which is steeper/higher value of slope would mean that the price is higher. Their solutions and answers show the acquisition to relate one concept to the other.

Franz, Emmalyn, Madeline, Nicole, Kristine, Shaira, Patrick, Trissa, and Kyle's solutions for Test V, Problem #2

Let x be the hour

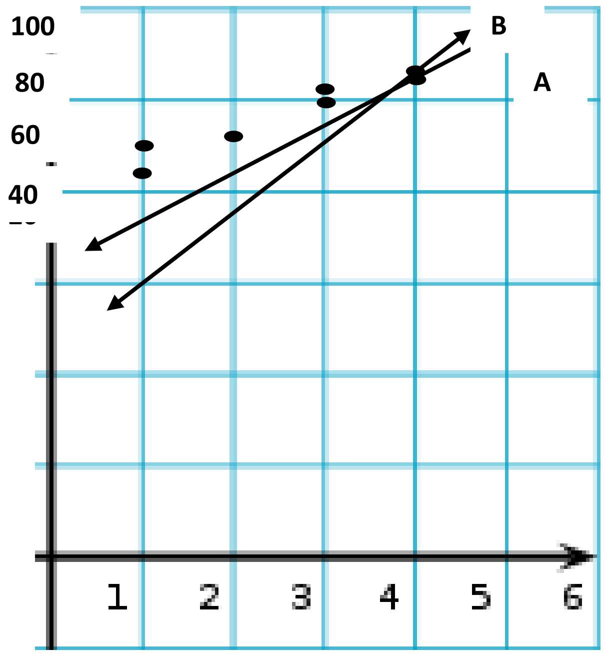
Let y be the charge/fee

A. Parking Service ABC: y = 40 + 10x

	x	y	Ordered Pair
1)	1	50	(1, 50)
2)	2	60	(2, 60)
3)	3	70	(3, 70)
4)	4	80	(4, 80)

B. Parking Service XYZ: y = 30 + 15x

	x	y	Ordered Pair
1)	1	45	(1, 45)
2)	2	60	(2, 60)
3)	3	75	(3, 75)
4)	4	90	(4, 90)



The following show the solutions of the 9 students. Their solutions and answers show that they were able to connect the lesson on linear equations in two variables to the real world problems which this lesson can address. They were able to see the context of the problem and show that mathematics can model real world problems

Franz's conclusion for Test V, Problem #2:

If you only want to park for 1 hour, choose parking service XYZ. If you want to park for 2 hours, then choose any of the two, but if you want to park for more than 2 hours choose ABC.

Emmalyn's conclusion for Test V, Problem #2: If less than two hours, I prefer the Parking Service of XYZ but more than that I prefer Parking Service ABC.
Madeline's conclusion for Test V, Problem #2: If I'm going to park for a short time only which is 1 hours, parking service XYZ is what I prefer because it (it's) cheaper. But if I have to park several hours, parking service ABC is more practical.
Nicole's conclusion for Test V, Problem #2 I would prefer to go to Parking Services ABC with the Php40 flat surcharge with Php10/hr because it's cheaper than Parking Services XYZ. If you park for more than two hours, they maybe the same but if you park more than two hours, it's better to go for parking services ABC. Don't be fooled by the smaller amount of flat rate for XYZ. Compute for more hours to know which is better.
Kristine's conclusion for Test V, Problem #2: I choose XYZ if only one hour. I can choose either of the two parking services saying that I will stay in the parking for 2 hours. More than two hours, I choose ABC.
Shaira's conclusion for Test V, Problem #2: I choose ABC if I will stay for more than 2 hours, if not I choose XYZ. If I stay for less than 2 hours, I choose XYZ.
Patrick's conclusion for Test V, Problem #2: I would prefer the parking service ABC because it is cheaper per hour than the parking service XYZ. If you would compare the two graphs of the two parking services you would see that the parking service XYZ has a steeper graph than the parking service ABC. This means that service ABC has a cheaper price.

Trissa's conclusion for Test V, Problem #2: It is said that steeper the line the higher the price. That means parking service ABC is cheaper compare to parking service XYZ, unless you will park for less than two hours.
Kyle's conclusion for Test V, Problem #2: Parking service ABC should be picked because it costs less than service XYZ. The more the graph is close to the y-axis, the costly the expense (the more expensive it will get).

The following show the solutions and answers of Justin, Izel and Julian, the three students who are considered as learners who are starting to build up and attain more conceptual knowledge. These three students were able to link their answers to address what is being asked in Test V, Problem #2, but they were not able to show complete mathematical solutions. They only sighted some examples which will justify their answers. They might have achieved the skill of developing and evaluate mathematical arguments and proofs, organizing and consolidating their mathematical thinking through communication, and using representations to model and interpret physical, social and mathematical phenomena but these three students are still into the developing stage to achieve other conceptual knowledge – they have not anticipated that drawing their conclusion out of one or two concrete examples that would satisfy the problem is not enough. It’s possible that there is a counter example that might invalidate their conclusions. They should have represented the parking rates as linear equations before they drew inferences from the results. But what these three students have done show a good starting point to acquire more conceptual knowledge.

Justin’s conclusion for Test V, Problem #2: I prefer the parking service ABC because it is more affordable since it has a surcharge of Php10 per hour. For example if you exceed at 2 hours, the parking fee for ABC would be equivalent to Php20. While if you would prefer the parking service XYZ, exceeding 2 hours would charge you Php30 of parking fee which would be more expensive than the parking service ABC.
Izel’s conclusion for Test V, Problem #2: I prefer Parking Services ANC because of its surcharge of Php10/hr. Most likely, I will be parking my car for 5 hours or more or for my (the) whole day. I computed that if I parked my car for 5 hours (5 x 10 = 50 + 40 = 90), my parking bill would only be P90. While If I chose parking services XYZand will park my car for 5 hours (5 x 15 = 75 + 30 = 105), then my parking bill would cost P105. So its (it’s) and wiser to choose the parking that has lower surcharge for an hour though with a higher flat rate than choose the parking that has a higher surcharge for an hour though with a lower flat rate.

Julian's conclusion for Test V, Problem #2:

I would choose parking service ABC because it would be cheaper as the number of hours increases.

Parking Service x = no. of hours

ABC:

40 + 10x =

40 + 10 (5hours) = 90

XYZ:

30 + 15x

30 + 15 (5hours) = 105

One of the remarkable points which should be given attention is on the poor results on table 3. The results seem to show an indication that there is a need for students to be more engaged with the learning experiences which will enhance their abilities to connect and relate lessons in mathematics in real world scenarios. Mathematics educators should be focusing more on the applications of the lessons they are teaching in order for the students to see the implications and reasons of studying and doing math.

The Students' Developed Instrumental Understanding and Relational Understanding (Procedural and Conceptual Knowledge) Through Performance Task

The researcher used the checklists of instrumental understanding and relational-understanding (procedural and conceptual knowledge) after the students have presented and submitted the performance task.

It is important to recall that the performance task was about organizing a birthday party (Appendix C). The forty (40) students were divided into five groups with eight members each. This indicates that if a certain group failed to carry out a certain skill, then each member of the group is also affected. The class was given two days to accomplish the performance task and one day to present the work. Each of the following submitted performance tasks were checked intelligently and analyzed qualitatively to know who among the groups have developed each of the following instrumental understanding and relational understanding (procedural and conceptual knowledge).

The Students' Developed Instrumental Understanding (see Appendix D)
Through Performance Task

The rate of the students who developed instrumental understanding through performance task ranged from 60.00% to 80.00%. The percentage is low compared to the developed instrumental understanding through written exercises since the students were graded as groups.

Four out of five groups (80.00%) were able to demonstrate evidence of the acquisition of the first to fifth instrumental understanding which are (1) Plotting points on a coordinate plane, (2) Adding, subtracting, multiplying, and dividing positive and negative real numbers, (3) Solving problems involving real numbers, and (4) Simplifying and evaluating algebraic expressions, using commutative, associative, and distributive properties as appropriate, and (5) Adding and subtracting linear expressions; Three groups (60%) exhibited attainment of the sixth-a and b instrumental understanding which are (6.a) Defining a variable, and (6.b) Writing linear equations; And four groups (80.00%) were able to show signs of the acquisition of the sixth-c and d and seventh instrumental understanding which are (6.c) Solving linear equations, (6.d) Solving slopes, and (7) Using the addition and multiplication properties of equality to solve one- and two-step linear equations.

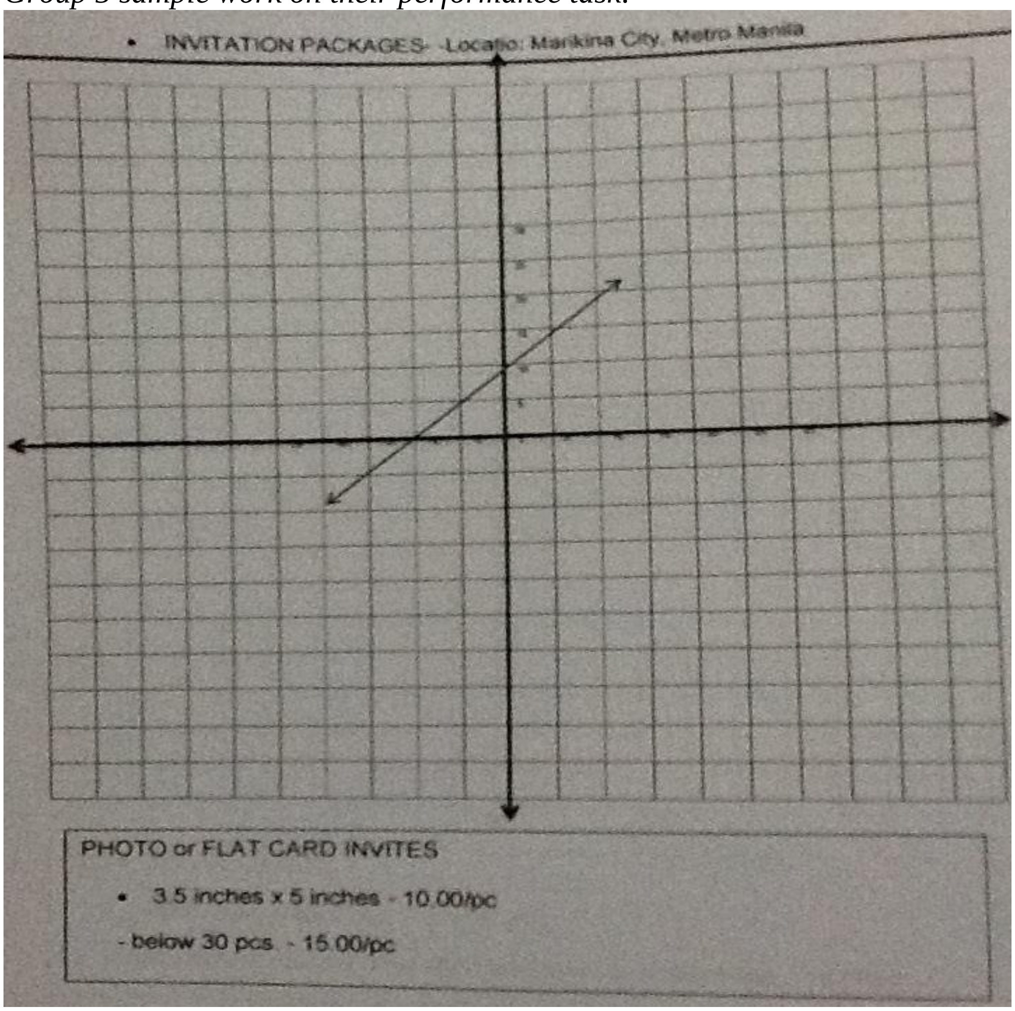
There were three groups whose works manifested weak evidences of the development of the instrumental understanding through the performance task. But the actuality that these groups have shown good starting points on the manifestation of the attainment of instrumental understanding, then they some part of their work were still given fair consideration. Those were groups 5, 2, and 3.

The foregoing shows the work of the group 5. In the written exercises, five members of the group got low scores. The quality of their work supports that these students may have not really acquired the instrumental understanding on linear equations in two variables. Their work is quite alarming. They did not execute the task correctly. They did not get anything right aside from looking for packages for the things that they needed to manage.

The performance task assesses the students' ability to demonstrate evidence of understanding (Wiggins &McTighe, 2005). This is one of the effective assessment tools to check if students can apply what they have learned in the real-world set up. Wiggins & McTighe (2005) explicates that the performance task would gauge if students really understood the lessons as these contend with the real purpose of taking up the lesson. This shows that the students may have solved some problems on the written exercises involving the instrumental understanding correctly, but they may have not known the application of such. The work of group 5 indicates that they did not achieve the unit plan's goal of learning to model appropriate real-world situations using linear equations in two variables. The group 5 made the mistake from plotting the points on the rectangular coordinate plane given their chosen packages to choosing which of the packages is the most affordable for each of the following requested things that they have to manage.

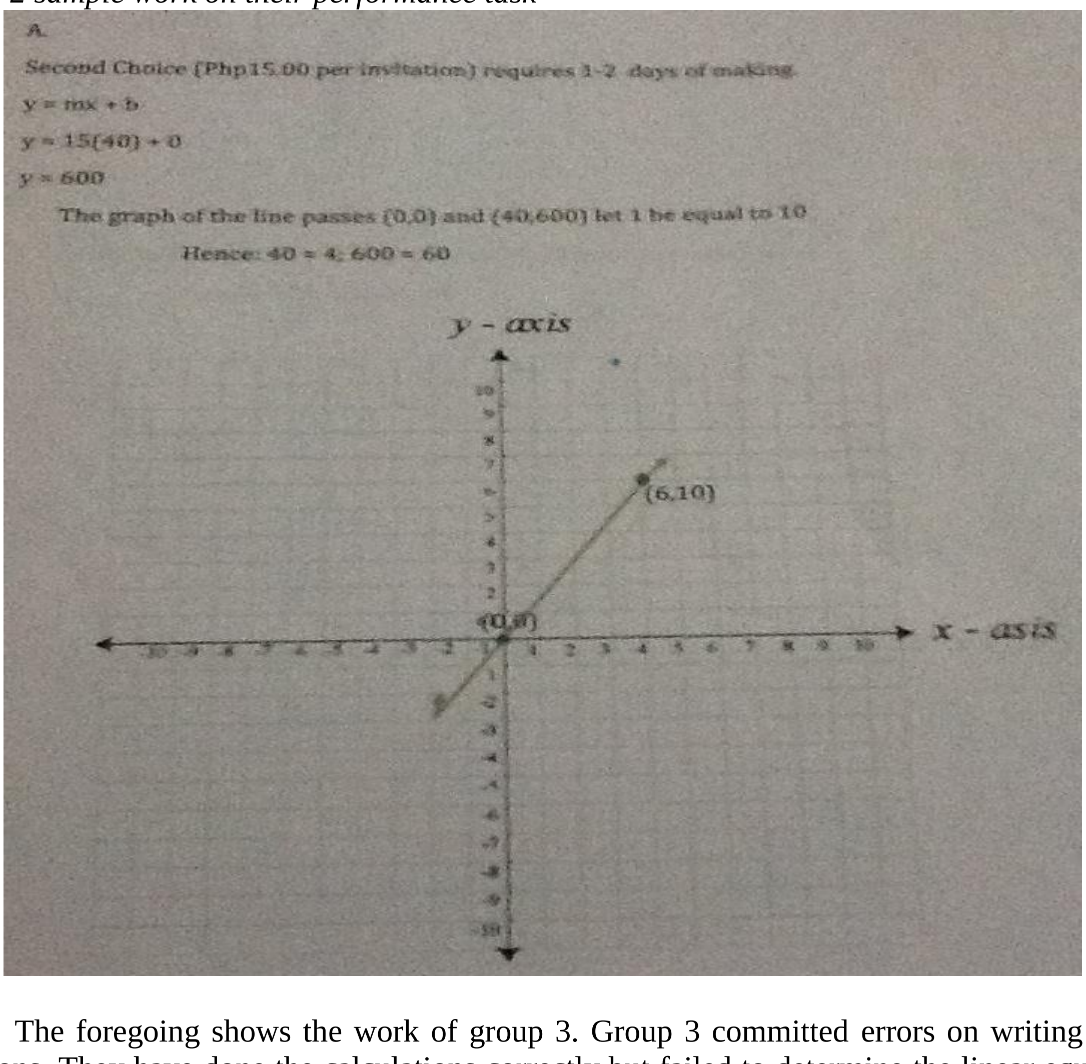
In the sample work of group 5 on their performance task, the sketch of the graph is incorrect. Assuming that the value of x is 0, the value of y should be zero. So that means that one of the points of the linear equation should be the origin with coordinates (0, 0). The group said that they used software but could not remember the name, so they thought that the graph was correct. The problem was they did not check if the graph is correct by before submitting the performance task; checking by sketching the graph of their equations manually. They totally relied on the on the use of the program, without anticipating that might commit error as they utilize the math software. This failure indicates strong evidence of the inability to apply the acquired instrumental understanding of the members in the written exercises. This shows that these students got used to routinary means of assessing their acquired learning. And a little change of checking students' learning challenged the group. The students may have failed to exhibit development of instrumental understanding through performance task especially in terms of first to fifth skills but this activity was a good way of engaging them to assessments and challenges which are innovative and authentic. This would address what Bizzarri (2000) stated about the main reasons of students' discontinuity in mathematics; (1) mathematics is taught in unnatural way, and (2) mathematics is a practical discipline but taught as an abstract one. This may serve as an eye opener to students to see the subject as sensible and realistic.

Group 5 sample work on their performance task:



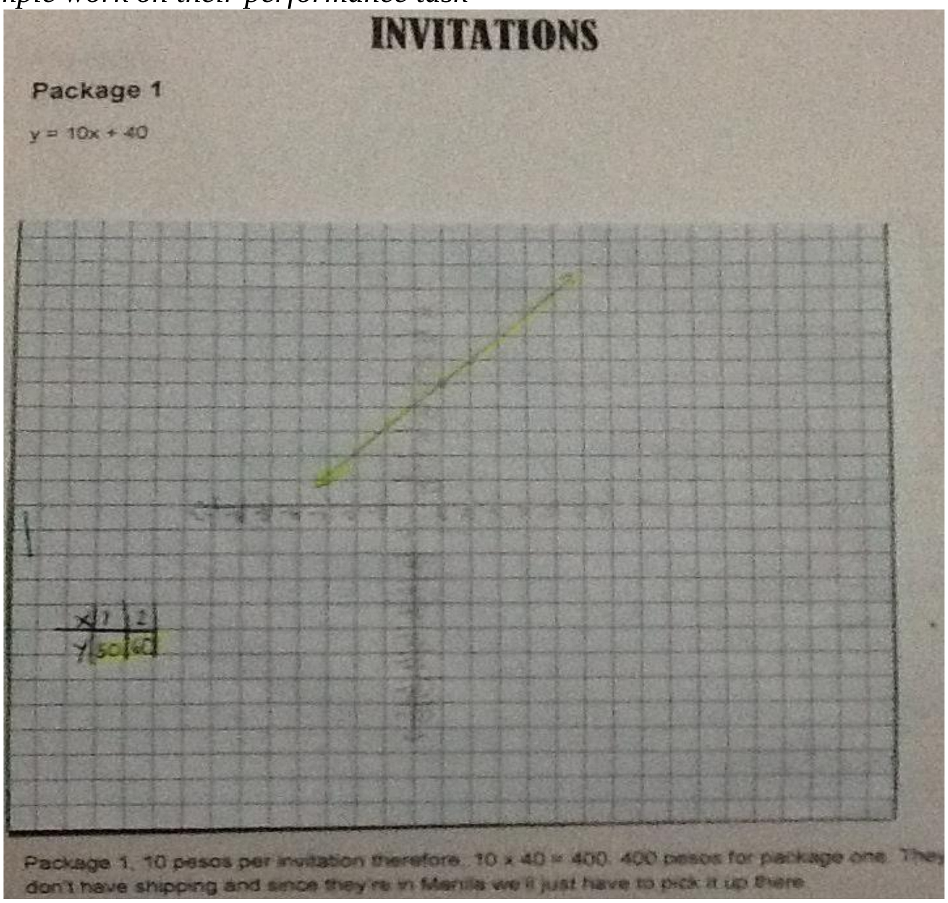
The following show the work of group 2. Group 2 was able to demonstrate evidence of the development of the instrumental understanding through performance task. The only problem was they did not place labels for their assigned values for x and y. Their work has shown that they lack the skill of defining a variable. They did not anticipate that a variable may represent any value. They should have specified that the x-values represent the quantity and the y-values represent the price.

Group 2 sample work on their performance task



The foregoing shows the work of group 3. Group 3 committed errors on writing linear equations. They have done the calculations correctly but failed to determine the linear equations for each of the following chosen packages. Because of this, they have carried errors over to the next tasks as well. This shows that instrumental understanding may be considered a short term understanding, but it should also be given attention for this will lead to higher learning. Instrumental understanding serves as the base for the acquisition of more complex skills.

Group 3 sample work on their performance task



In the sample work of group 3, since package #1 quotes 10php per invitation and there will be no discounts whatever quantity they avail, then the linear equation should be y = 10x. The group's answer was y = 10x + 40.

The Students' Developed Relational Understanding (Procedural Knowledge) (see Appendix E) Through Performance Task

The rate of the students who developed relational understanding (procedural knowledge) ranged from 60.00% to 80.00%.

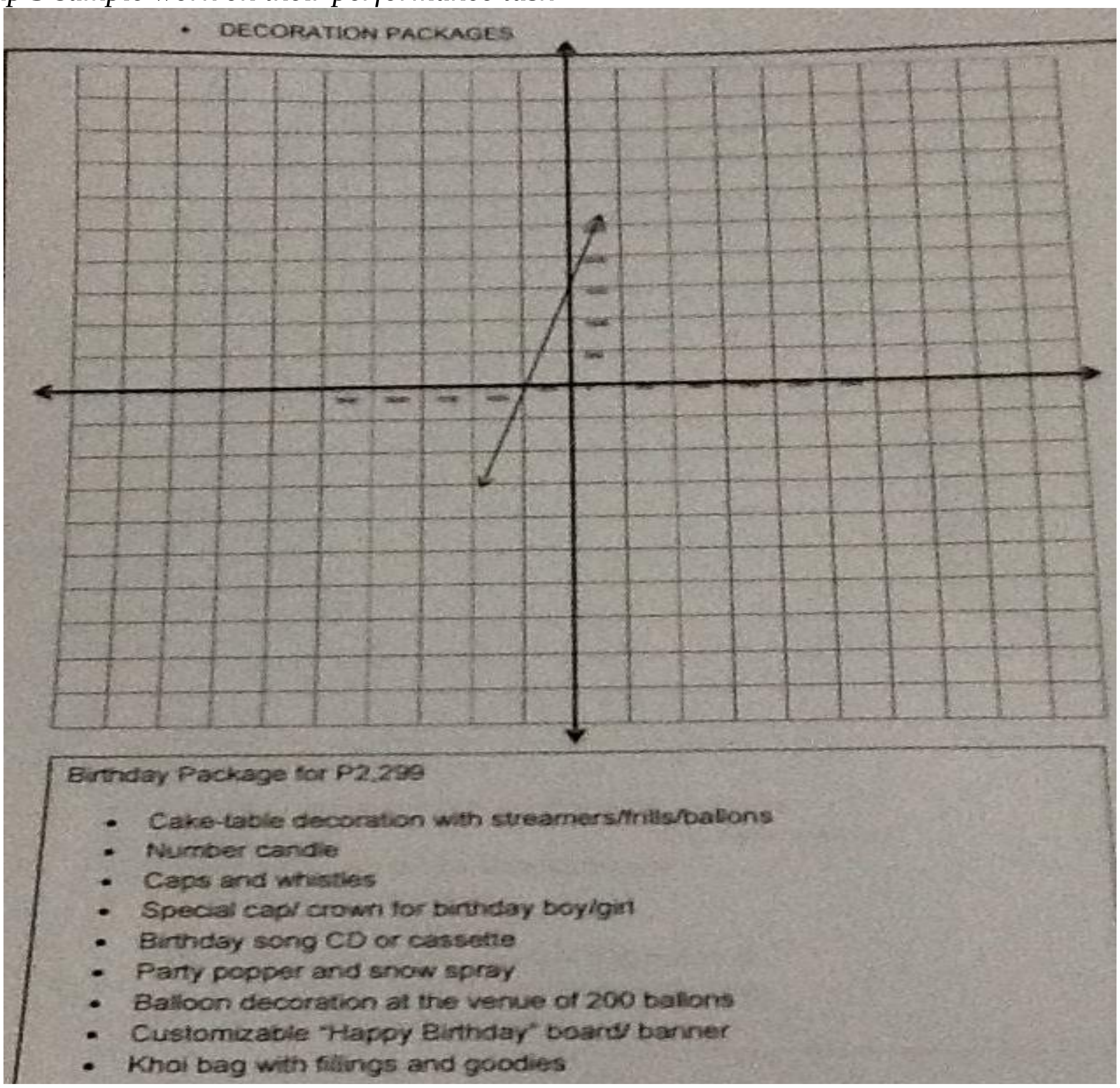
Four groups (80.00%) were able to demonstrate acquisition of the first relational understanding (procedural knowledge) which is (1) Plotting points on a coordinate plane; Three groups (60.00%) acquired the second-a to third-c relational understanding (procedural knowledge) which are (2.a) Representing relations from tables, (2.b) Representing relations from graphs, (2.c) Representing relations from formulas, (3.a) Describing relations from tables, (3.b) Describing relations from graphs, and (3.c) Describing relations from formulas; Four groups (80.00) attained the fourth to seventh relational understanding (procedural knowledge) which are (4) Adding, subtracting multiplying, and dividing positive and negative real numbers, (5) Solving problems involving real numbers, (6) Simplifying and evaluating algebraic expressions, using commutative, associative, and distributive properties as appropriate, and (7) Adding and subtracting linear expressions; Three groups (60.00%) acquired the eight-a and eight-b relational understanding (procedural knowledge) which are (8.a) Defining a variable, and (8.b) Writing linear equations; Four group (80.00%) developed the eight-c to ninth relational understanding (procedural knowledge) which are (8.c) Solving linear equations, (8.d) Solving slopes. And (9) Using the addition and multiplication properties of equality to solve one- and two-step linear equations.

It is important to take note that a student who developed relational understating (procedural knowledge) should have the ability to think of ways to verify his answers. From this statement alone, Groups 5, 2, and 3 again were not able to demonstrate strong evidences to some of the following procedural knowledge, one of the knowledge that a student should attain in order to achieve relational understanding.

For group 5, since they have committed errors from the very beginning, then it follows that they also have committed errors on the more complex tasks which will measure the higher learning. Recalling the steps to determine students' acquisition of the relational understanding (procedural knowledge), (1-a) What is the goal of the procedure?, (1-b) Up to what can this procedure do?, (1-c) What sort of outcome should I expect?, (2-a) How do I execute the procedure?, (2-b) What are some other procedures I could use instead?, (3) Why is the procedure effective and valid?, (4) How will I able to verify my answers?, and (5) When is this certain procedure 'best' to use? (Hasenbank, 2004), it shows that the group 5 was not able to carry out some of these. The work of group 5 demonstrates that they may have not developed relational understanding (procedural knowledge) through performance task because they did not confirm if the points and graphs of their linear equations on their work are correct. They have not foreseen that there is a chance that the software will provide wrong illustrations if they have committed errors in entering the values into the software applications.

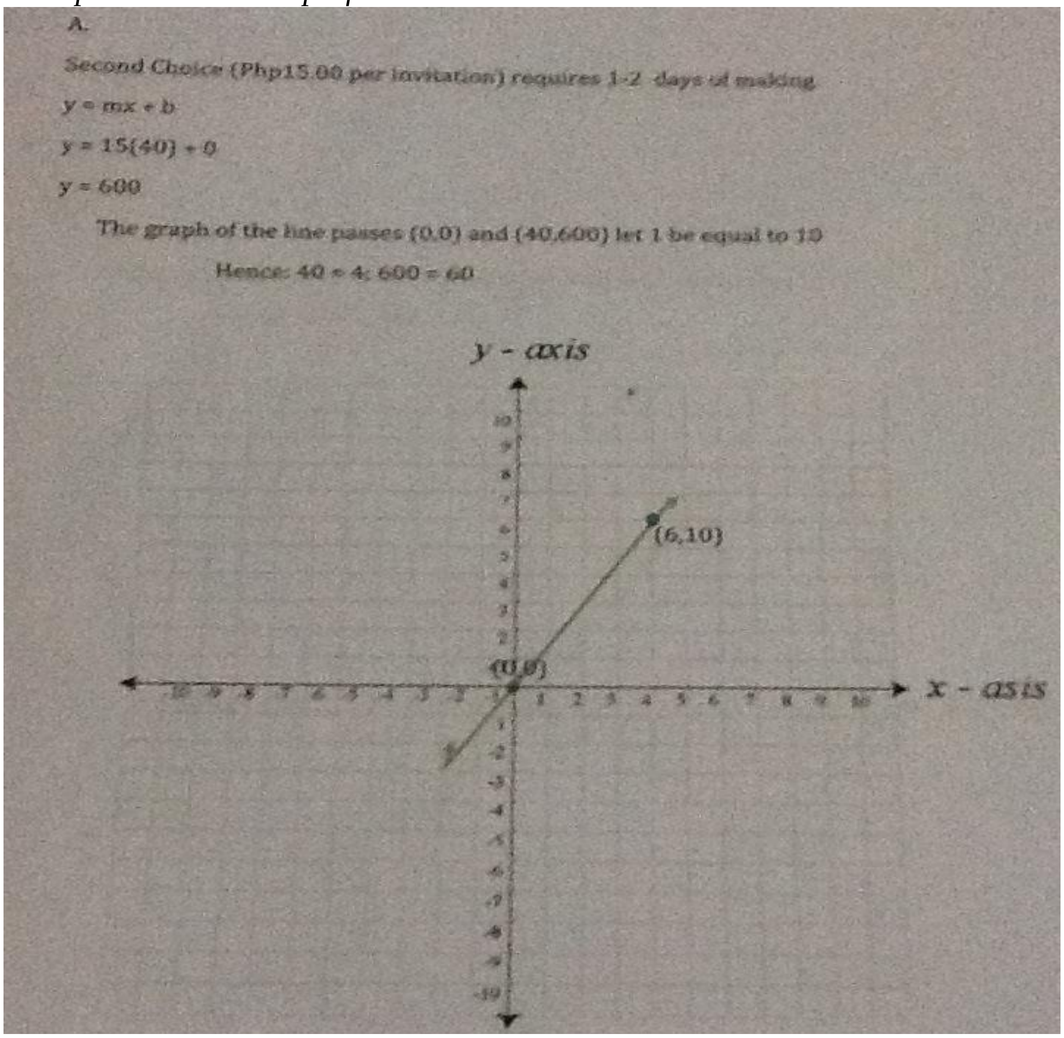
The foregoing shows another sample work of group 5. It shows an incorrect graph of their chosen decoration package. Since the package is amounting to Php2,299, that means once that the group would not avail the package then they will not be paying anything; this situation may be represented with the point (0, 0). Then again, the origin should be part of the points on the line. If the group would avail two of the said packages with the condition that they will not be given any discounts, then they would be paying Php4,598 – this can be represented with the point (2, 4598). The group should have at least checked if their graphs were correct.

Group 5 sample work on their performance task



The following shows another sample work of Group 2. A student who acquired procedural knowledge has an ability to anticipate up to what extent is a certain procedure can do. In the procedure of sketching the graph of a linear equation, it can lead the student up to the identifying the direction of the graph. The group should have at least described each of their work; what is with the graph, what are the values of x and y stand for, and the like.

Group 2 sample work on their performance task



A.

Second Choice (Php15.00 per invitation) requires 1-2 days of making

y = mx + b

y = 15(40) + 0

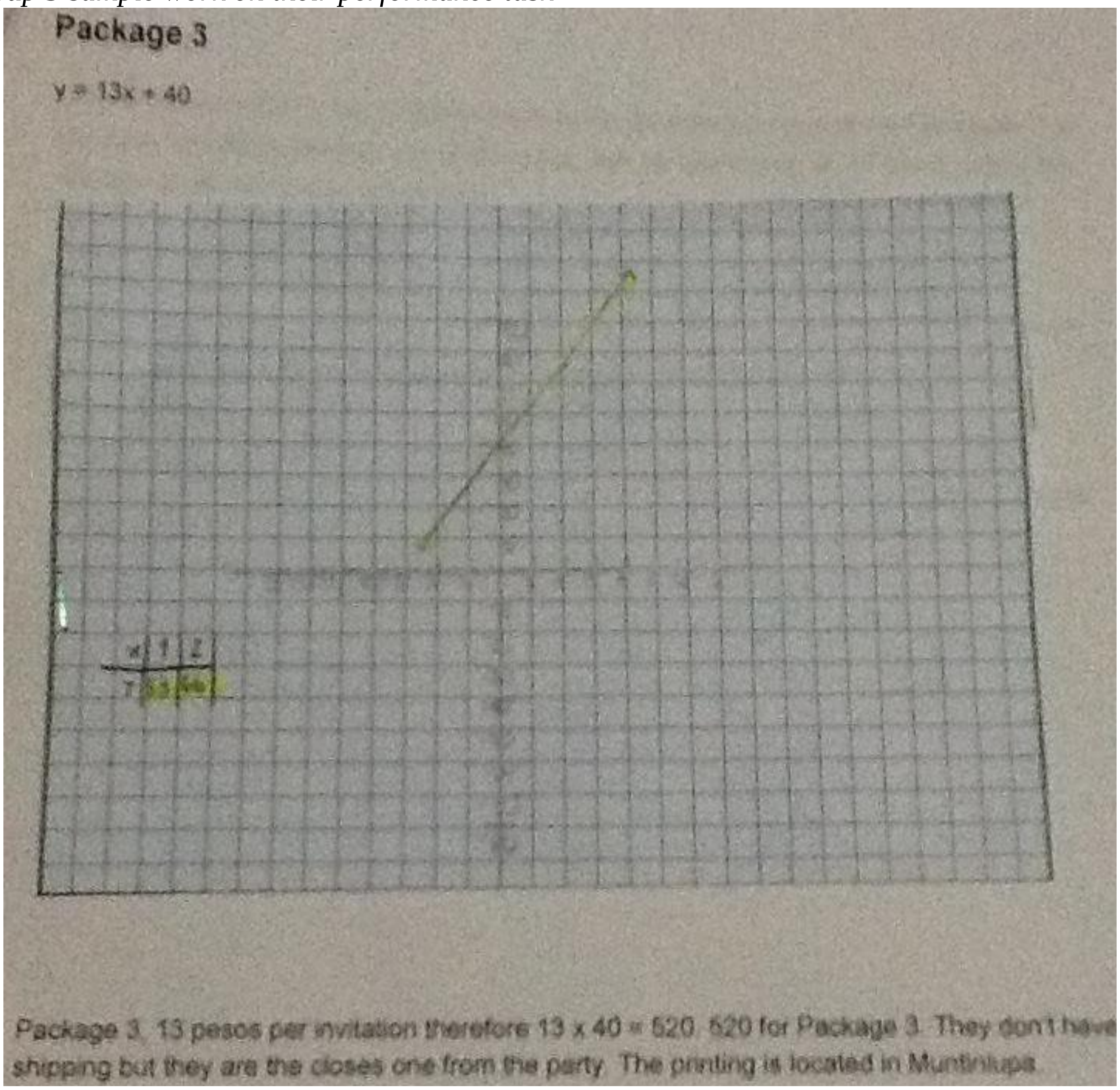
y = 600

The graph of the line passes (0,0) and (40,600) let 1 be equal to 10

Hence: 40 = 4; 600 = 60

The following shows another sample work of Group 3. This group encountered a similar problem with group 2, but this time with the samples. In the group's sample work, they gave an explanation every after the presentation of the graph. They stated "Package 3 (invitation package), 13 pesos per invitation therefore 13 x 40 = 520, 520 for Package 3. They don't have shipping but they are the closest one from the party. The printing is located in Muntinlupa". Since the group constructed a table of values, they should have anticipated that if they avail one invitation piece, then they will be paying an amount of Php13; which is not equal to what they have written on their work. They should have at least checked if the graphs of their chosen packages matched with their explanations below.

Group 3 sample work on their performance task



The Students' Developed Relational Understanding (Conceptual Knowledge) (see Appendix F) Through Performance Task

The rate of the students who developed each of the following relational understanding (conceptual knowledge) through performance task was 60.00%. This implies that three out of five groups were able to demonstrate evidence of the acquisition of all the relational understanding (conceptual knowledge) which are (1) Building new mathematical knowledge through problem solving, (2) Solving problems that arise in mathematics and in other contexts, (3) Applying and adapt a variety of appropriate strategies to solve problems, (4) Monitoring and reflecting on the process of mathematical problem solving, (5) Recognizing reasoning and proof as fundamental aspects of mathematics, (6) Making and investigating mathematical conjectures, (7) Developing and evaluating mathematical arguments and proofs, (8) Selecting and using various types of reasoning and methods of proof, (9) Organizing and consolidating their mathematical thinking through communication, (10) Communicating their mathematical thinking coherently and clearly to peers, teachers and others, (11) Analyzing and evaluating the mathematical thinking and strategies of others, (12) Using the language of mathematics to express mathematical ideas precisely, (13) Recognizing and use connections among mathematical ideas, (14) Understanding how mathematical ideas interconnect and build on one another to produce a coherent whole, (15) Recognizing and apply mathematics in contexts outside of mathematics, (16) Creating and using representations to organize, record and communicate mathematical ideas, (17) Selecting, applying, and translating among representations to solve problems, and (18) Using representations to model and interpret physical, social, and mathematical phenomena.

Identifying students who developed relational understanding (conceptual knowledge) through performance task is easier compared to paper-and-pencil test since this assessment tool required students to comprehensively explain their answers. Students are more driven to put meaning to their answers. Students who can see the context of their answers and relate them to real-world situations have developed conceptual knowledge. They are students who can see the connection of one concept to another.

The foregoing shows the work of the groups who were able to develop conceptual knowledge. And these were the groups 4, 1 and 3. One group totally manifested the acquisition of the conceptual knowledge, while the other two have developed conceptual knowledge but needed improvement on the procedural knowledge and instrumental understanding.

The work of group 4 was quite impressive. The group has recognized that they may represent their chosen packages for everything that they were assigned to manage as mathematical equations. And at the same time, despite that they were not given instruction that the best thing to do to really compare which among the chosen packages is the most affordable by sketching all the graphs in one coordinate plane, they were able to anticipate such action to address the situation. What the group did was everything that they were assigned to manage – they sketched the graphs of their chosen packages after expressing them as mathematical equations then they identified which line was the least steep. The group was also able to give clear explanations on why they have chosen a particular package. The group was able to demonstrate the acquisition of conceptual knowledge for they were able to see the connection of the situation and how mathematics can address it. They used mathematics to solve the given real world problem. The group recognized the ability of mathematics to model real world problems. What manifested in this group was the evidence that they were able to develop both instrumental understanding and relational understanding.

Group 4 sample work on their performance task

FOOD

Let:
x = number of guests
y = price per person

Option 1: PIZZA HUT
Offer: a slice of family hawaiian pan pizza, spaghetti with meat sauce, & a glass of pepsi for P149 per person

x	10	20	30	40
y	1,490	2,980	4,470	5,960

y = 149x
y = 149(10)
y = 1,490

y = 149x
y = 149(20)
y = 2,980

y = 149x
y = 149(30)
y = 4,470

y = 149x
y = 149(40)
y = 5,960

Option 2: JOLLIBEE
Offer: spaghetti, regular fries, regular softdrink, and sundae for P128 per person

x	10	20	30	40
y	1,280	2,560	3,840	5,120

y = 128x
y = 128(10)
y = 1,280

y = 128x
y = 128(20)
y = 2,560

y = 128x
y = 128(30)
y = 3,840

y = 128x
y = 128(40)
y = 5,120

Option 3 : KENNY ROGERS ROASTERS
Offer: P135 per person

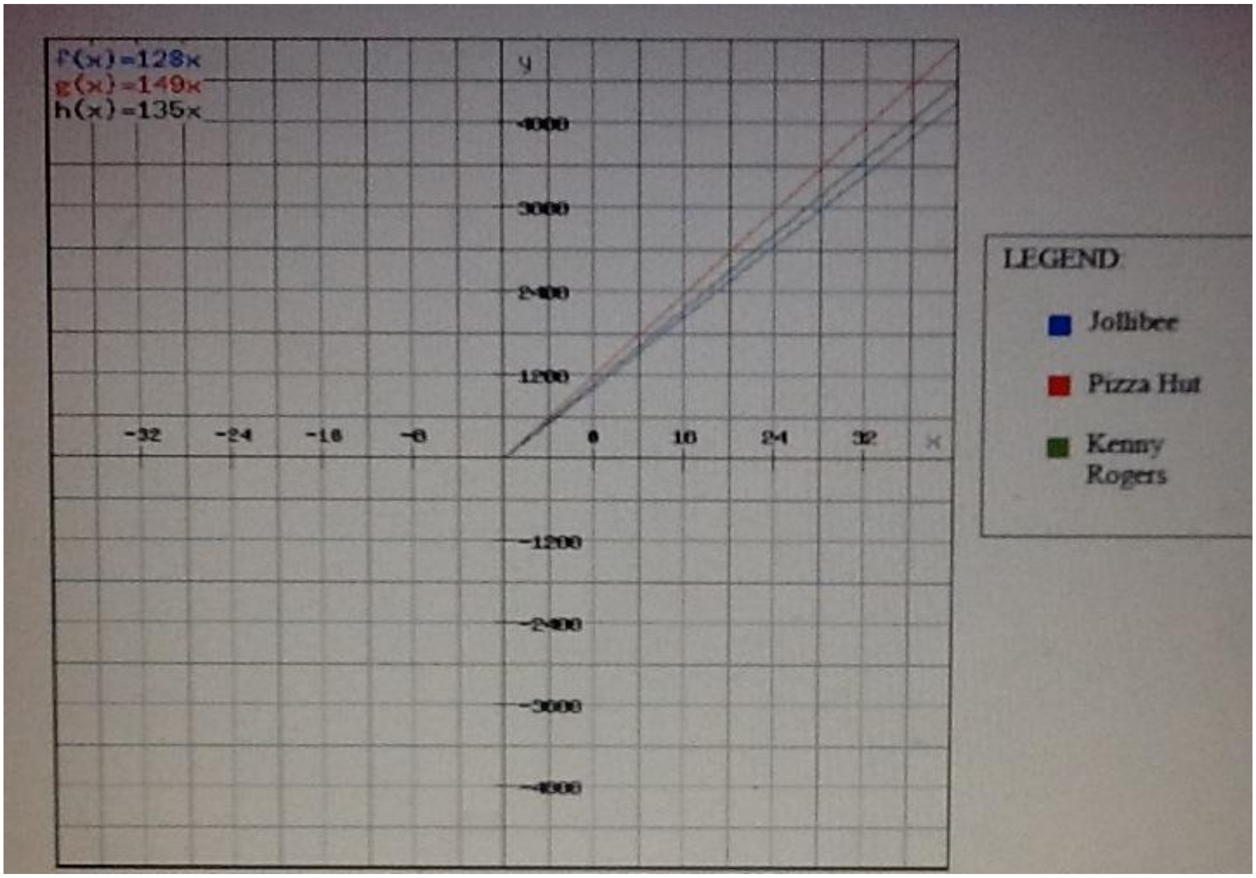
x	10	20	30	40
y	1,350	2,700	4,050	5,400

$y = 135x$
$y = 135(10)$
$y = 1{,}350$

$y = 135x$
$y = 135(20)$
$y = 2{,}700$

$y = 135x$
$y = 135(30)$
$y = 4{,}050$

$y = 135x$
$y = 135(40)$
$y = 5{,}400$



Group 4's conclusion for the performance task, food preparation:

J is more practical compared to PH and KR because first, of the reasonable prize it offers. J charges 128 per head, KR charges 135 per head and PH charges 149 per head. If 10 people are served, PHwill cost P1,490 already, Jwill only cost us P1,280 and KRwill cost us P1,350. Forty guests are invited to the party which will give us a total of P5,960for PH, P5,400 for KR and P5,120 for J. J's offer is also more practical because they offer us with more choices than PH and KR. Jollibee's offer includes spaghetti, regular fries, regular softdrink, and a sundae. In PH, however, offer includes a slice of family Hawaiian pan pizza, spaghetti with meat sauce, & a glass of Pand KR only offers us one main course with drink. J's offer is the cheapest and provides us with more choices of food.

Group 1 sample work on their performance task

For group 3, they may have committed errors in writing linear equations which lead to a wrong sketch of graphs but since they were able to connect their answers to address the problem in the performance task, it shows that they have acquired some conceptual knowledge in linear equations in two variables. They have the ideas that the packages may be modelled by mathematical equations, and that the best thing to identify which of the following packages is the most affordable is to sketch the graph of the equations and determine the least steep line. But since they failed to identify the correct linear equations for each of the following chosen packages, then their drawn conclusion might not be correct. This shows how minor piece-meal of learning, like the instrumental understanding and procedural knowledge, could affect the acquisition of the higher learning.

Two of the five groups seem that they did not have developed relational understanding (conceptual knowledge). Group 2 did not give any elaborated explanation on their work. They simply sketched the graphs after representing each of the following chosen packaged into mathematical equations and chose which among is the most affordable. They did not explicate the reason why they have chosen a certain package. The group was not able to give meaning on their answers. They group did not develop some conceptual knowledge on using mathematical equations to explain real world situations, recognize and connect mathematical ideas, and the like. The group's performance task shows that they have developed instrumental understanding and relational understanding (procedural knowledge). This is a good foundation to develop relational understanding (conceptual knowledge) and later on achieve the mathematical understanding.

The foregoing shows the sample conclusion drawn from group 5. This shows how failure to develop instrumental understanding and relational understanding (procedural knowledge) may also affect the acquisition of further knowledge and understanding which are more complex. In their work, since they committed mistakes from the very beginning, their drawn conclusions were also affected. Their problem was somehow similar to group 2 in terms of getting informative and correct conclusions due to undeveloped minor skills on instrumental understanding and procedural knowledge, but their case was more critical. Their group seem not to see how mathematics can address real world problems. The group's conclusions were very subjective – they did not refer to the results they got after sketching the graph as they drew their conclusions. This indicated the undeveloped conceptual knowledge on modelling real world situations using mathematics. This shows that the members of this group may seem to need reinforcements to address their flaws in mathematics.

Group 5 sample conclusion on their performance task

In this situation (invitation package) I would like to point out, that the most affordable price would be deal number two or the ticket style invitation. As you can imagine the design of the invitation you will not regret this nice offer, to top it all that, it has a very affordable offer. If you would compare the three deals, the second deal is the best choice. Why? Because I would think of the first offer, yes, it is affordable but it is too simple unlike the second offer, it also has the same price but when you consider the design, it is the best decision to choose deal two/ticket style invitation. And if you will compare the third once, it is still the best decision to choose deal two, because it is said the design is simple and if you look at the deal there would be an additional cost if you want the invitation to be presentable.

Students' Perceptions of Their Skills and Proficiency

The self-assessment checklist (see Appendix A) asking for the students' perceptions of their skills and proficiency on linear equations in two variables was administered to the students twice – before and after the implementation of the UbD unit plan. It was then checked if there were significant differences in students' perceptions of their skills and proficiency on linear equations in two variables after carrying out the unit plan.

Table 4 shows the students' perceptions of their skills and proficiency on linear equations in two variables. It shows the number of students who thought that they could carry out each of the following skills before and after the implementation of the unit plan, and if there are significant changes in their perceptions after the execution of the unit plan using z-test statistic for differences in two proportions.

Table 4: *Students' Perceptions of Their Skills and Proficiency on Linear Equations in Two Variables Before and After the Implementation of the UbD Unit Plan*

Key Standards	**Number of Students who Perceived They could Carry Out the Skills**	
	Before	**After**
1) Plot points on a coordinate plane*	40	40
2.a) Represent relations from tables	31	34
2.b) Represent relations from graphs	28	34
2.c) Represent relations from formulas*	25	32
3.a) Describe relations from tables	30	35
3.b) Describe relations from graphs*	33	38
3.c) Describe relations from formulas	24	29
4) Add, subtract, multiply, and divide positive and negative real numbers	40	40
5) Solve problems involving real numbers	38	39
6) Simplify and evaluate algebraic expressions, using commutative, associative, and distributive properties as appropriate*	31	37
7) Add and subtract linear expressions*	36	40
8.a) Define a variable	32	36
8.b) Write linear equations	34	34
8.c) Solve linear equations	34	37
8.d) Solve slopes	36	38
9) Use the addition and multiplication properties of equality to solve one- and two-step linear equations*	33	38
10) Build new mathematical knowledge through problem solving	25	28
11) Solve problems that arise in mathematics and in other contexts	28	32
12) Apply and adapt a variety of appropriate strategies to solve problems*	27	34
13) Monitor and reflect on the process of mathematical problem solving	29	34
14) Recognize reasoning and proof as fundamental aspects of mathematics	20	27
15) Make and investigate mathematical conjectures*	13	15
16) Develop and evaluate mathematical arguments and proofs*	20	22
17) Select and use various types of reasoning and methods of proof	22	27
18) Organize and consolidate their mathematical thinking through communication	29	32
19) Communicate their mathematical thinking coherently and clearly to peers, teachers and others	31	35
20) Analyze and evaluate the mathematical thinking and strategies of	27	34

others*		
21) Use the language of mathematics to express mathematical ideas precisely	29	32
22) Recognize and use connections among mathematical ideas	21	25
23) Understand how mathematical ideas interconnect and build on one another to produce a coherent whole	26	27
24) Recognize and apply mathematics in contexts outside of mathematics	17	24
25) Create and use representations to organize, record and communicate mathematical ideas	15	22
26) Select, apply, and translate among representations to solve problems*	24	34
27) Use representations to model and interpret physical, social, and mathematical phenomena*	21	28

Legend: (*) – significant at 0.05

The high P-values using z-test statistic for differences in two proportions indicate that there are no significant differences in most of the perceptions of the students of their skills and proficiency on linear equations in two variables. These signify that the students have, more or less, the same perceptions about their prior knowledge before and after the implementation of the UbD unit plan except for the other nine key standards. The percentages in terms of the changes on the pupils' perceptions are quite remarkable. The range on the positive changes of pupils' perceptions is from 0.00% to25.00% which shows that the frequencies have gone up.

The first key standard with the following skills: the plotting points on the Cartesian coordinate plane, representing and describing relations from tables, graphs and formulas, has an increased rate from 8.33% to 17.50%. There is an increase of 15.00% from the students' perceptions on the first skill, namely, plotting points on the rectangular coordinate plane. From 34 out of 40 (85%) students believing that they can carry out the skill before the implementation of the unit plan, it is increased to 40 out of 40 (100%) students believing that they can perform the skill after the execution of the unit plan.

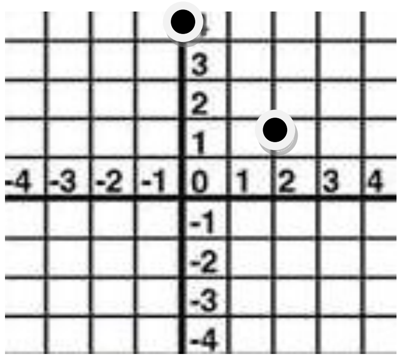
Pauline, one of the six out of 40 students who thought that they cannot carry out the skill of plotting points on a rectangular coordinate plane before the implementation of the UbD unit plan, has demonstrated misconception about the said skill when she answered an item on the board before discussing the lesson to check students' prior knowledge.

She solved the boardwork problem this way:

Plot the point on the cartesian coordinate plane:

1. (2, 3)

Pauline's solution:



and 3 is the y-coordinate/ordinate. But she did not know that ordered pair is only one point which lies on the quadrant I.

To address this, the first activity for the Cartesian coordinate system included helping the teacher to physically move the students around to illustrate an example. Pauline learned that (2, 3) signifies that from the origin, the teacher needs to walk 2 units to the right and 3 units upward.This demonstration may have helped Pauline understand the application of the points on the rectangular coordinate plane. This may also be true with the other five out of 40 students who thought that they cannot deal with the said skill. As what Quilter and Harper (cited in Johnston-Wilder, et al., 2011) emphasized, that the main reason for students' difficulty is explained not in terms of the conceptual complexity of the subject matter, but in terms of its apparent irrelevance and/or the teacher's inability to provide learning experiences that would present it in a coherent, meaningful way. The teacher's failure to lead students to the realization of the significance of what they are doing would possibly discourage them to do the mathematics. This may be one of the reasons why after the implementation of the unit plan, the students who believed that they can perform the skill of plotting points increased from 36 to 40 out of 40 (100%).

The students' perception on the skill about representing and describing relations from tables, graphs, and formulas increased from 0.00% to 17.50%. During the implementation of the UbD unit plan, the students were provided with lots of activities, boardwork, homework, and the like where construction of learning was given importance. These appropriate and meaningful learning experiences made the noticable increase in the change of students' perceptions possible.

Just as what Nebres and Intal (1988) stated in the literature, engaging students to meaningful learning experiences where construction of mearnings is involved would trigger students' interest and curiosity to do the mathematics.

The second key standard consisting of skills such as performing operations involving real numbers and solving problems involving real numbers increased from 0.00% and 2.50%, respectively. The students believed that they were able to master the skill of adding, subtracting, multiplying and dividing real numbers even before the implementation of the unit plan since the lesson pre-requisite to linear equations in two variables, which is linear equations in one variable, has been discussed during their sixth grade and during the first quarter of first year. Students were given enough training because they said that they were not allowed, at first, to use calculators when they were taking the topic during the first quarter. They were allowed after making sure that they were able to carry out the skill of performing operations involving real numbers. This supports what Eisenhower (1998) says that Mathematics is a field of study that build on previously established concepts. A student who does not know basic multiplicaiton and division will have a difficult time learning factoring, primes, simplification of fractions, and the like. Basic algorithms of arithmetic are a needed basis for understanding the corresponding operations with polynomials in algebra. Through restricting the use of calculator, one may develop number sense. But it is also important to use calculator for exploring new and more complex concepts. For this case, one of the reasons why students were confident that they could deal with the skill even before the implementation was because of the training that was provided to them. And at the same time, the implementation of the unit plan made the others further equipped with the skills.

In the third key standard which is about representing and evaluating quantities using algebraic expressions increased, ranging from 5.00% to 15.00%. There is a noticeable increase in the second skill which is about simplifying and evaluating algebraic expressions, using commutative, associative, and distributive properties as appropriate. One of the reasons for this was at first, students forgot the terms such as commutative, associative, and distributive properties. That is why there were only 31 out of 40 (77.50%) students who believed that they can carry out the skill. They knew that the terms were not new to them, but they forgot how to apply the said properties. But when they were provided with activities, the students realized that what they were doing was already the application of commutative, associative and distributive properties. One even reacted, "*Ay! 'Yun lang pala!*" (Ah! That's it?), which showed that the student already has the skill – he only forgot the concept behind it. In this key standard in general, students believed that they could perform the given skills even before the implementation of the unit plan.

In the fourth key standards, the increase of students' perceptions starts from 0.00% to 12.50%. The most noticeable in this standard is the skill about writing linear equations with a 34 out of 40 (85.00%) rate. This is considerably high – they may have believed that they could perform the skill even before the implementation of the unit plan, but the six students who believed that they could not carry out the skill remained uncertain even after the execution of the UbD unit plan. Claire, who according to the student teacher is really struggling in mathematics, wrote in the self-assessment checklist that she is still unsure if she can perform the skill even after the implementation of the unit plan. This signifies the need of more meaningful activities involving the said skill that the researcher should have provided to the students. The response of Claire shows how the instrument identifies not only the perception, but also the determines the lessons to be given more emphasis and students to be given more attention in order to achieve one of the goals of education – for the students to be provided with equal learning. No one should be left behind.

The next sets of key standards deal with higher order thinking skills. The results for the students' perceptions before the implementation were moderately alarming. The percentage of the students who did not believe that they could perform the skills even before the implementation of the UbD unit plan ranges from 32.50% to 77.50%. The skills which have minimal number of students who thought they could perform them before the implementation are about (1) making and investigating mathematical conjectures (32.50%), (2) creating and using representations to organize, record and communicate mathematical ideas (37.50%), (3) recognizing and applying mathematics in contexts outside of mathematics and recognize reasoning and proof as fundamental aspects of mathematics (37.50%), (4) developing and evaluating mathematical arguments and proofs (50.00%), (5) recognizing and using connections among mathematical ideas (52.50%), and (6) using representations to model and interpret physical, social, and mathematical phenomena (52.50%). All these skills require the ability that goes beyond mere solving. These require the ability to apply the learning in the real world scenarios and to see the connections of mathematical ideas. The poor results support what Nebres and Intal (1998) say about the lack of practice of science culture here in the Philippines, a culture which highlights empirically-based and systematic knowledge generation, research, attention to detail, precision, emphasis on measurement and quantification, accurate written records, problem solving approaches to things, persistence, aiming for excellence, following of systems and rules, an emphasis on facts rather than opinion, curiosity and observation, creativity, compassion and openness to new ideas, accuracy, discipline, honesty and objectivity. The responses of the students only show they were not provided with learning experiences that deal with exploration, discovery, construction of learning, and the like – learning which leads to enduring understanding. The results demonstrate evidence of the importance of providing real-world experiences to the students and showing the students the "Big Picture" of the lessons for them to obtain enduring understanding – a long-term understanding and is transferrable to other schools of learning.

On the other side of the coin, the results imply that the UbD unit plan is effective in exposing students to real world experiences which could lead to realization of the significance in doing the mathematics and attainment of higher order thinking skills. The increase of students' perceptions of their skills and proficiency after the implementation is noticeable: it ranges from 2.50% to 17.50%.

Table 5 shows the students' perceptions of their skills and proficiency on linear equations in two variables and the developed instrumental understanding (see Appendix D) of the students through written exercises and performance task. It shows the number of students who thought that they could carry out each of the following instrumental understanding after the implementation of the unit plan and the students who were able to exhibit development of instrumental understanding through written exercises and performance task, and if there are significant changes between the two factors using z-test statistic for differences in two proportions.

Table 5: *Students' Perceptions of their Skills and Proficiency After UbD Implementation and Their Developed Instrumental Understanding*

Instrumental Understanding	Number of Students who Perceived They could Carry Out the Skills After the Implementation of the UbD Plan	Number of Students who Attained Instrumental Understanding through Written Exercises	Number of Students who Attained Instrumental Understanding through Performance Task
1) Plot points on a coordinate plane	40	40	32*
2) Add, subtract, multiply, and divide positive and negative real numbers	40	32*	32*
3) Solve problems involving real numbers	39	30*	32*
4) Simplify and evaluate algebraic expressions, using commutative, associative, and distributive properties as appropriate	37	38	32
5) Add and subtract linear expressions	40	36*	32*
6.a) Define a variable	36	40	24*
6.b) Write linear equations	34	30	24*
6.c) Solve linear equations	37	30	32
6.d) Solve slopes	38	34	32*
7) Use the addition and multiplication properties of equality to solve one- and two-step linear equations	38	38	32*

Legend: (*) – significant at 0.05

The high P-values using z-test statistic for differences in two proportions indicate that majority of the key standards have no significant differences between the number of students who thought that they could perform each of the following instrumental understanding after the implementation of the UbD unit plan and the students who were able to exhibit evidence of development of the instrumental understanding through written exercises. The results show that seven out of 10 (70.00%) key standards have no significant differences between the students' perceptions of the their skills and proficiency and the students who were able to exhibit evidence in the development of instrumental understanding, while three out of 10 (30.00%) show significant differences. This is a good indication that students' beliefs on their capabilities to carry out instrumental understanding were materialized in the written exercises.

In the items where there are significant differences such as the (1) Adding, subtracting, multiplying, and dividing positive and negative real numbers, (2) Solving problems involving real numbers, and (3) Adding and subtracting linear expressions, the results show that the number of students who were able to developed each of the following said skills are significantly lower compare to the number of students who perceived that they could carry out the plan. Despite of the fact that the listed skills are already introduced and given fair strengthening during their 5^{th} and 6^{th} grades, students still find it very difficult to deal with performing basic operations between real numbers. This shows that the students should be given reinforcements because these skills are pre-requisite to other skills. Students may encounter difficulty to developed higher learning if they will not be able to master these skills.

The low P-values using z-test statistic for differences in two proportions indicate that there are significant differences between the number of students who thought that they could perform each of the following instrumental understanding after the implementation of the UbD unit plan and the students who were able to exhibit evidence of development of instrumental understanding through performance task. The results show that eight out of 10 (80.00%) key standards have significant differences between the students' perceptions of their skills and proficiency and the students who were able to exhibit evidence of development of instrumental understanding, while two out of 10 (20.00%) show no significant differences.

For the skills where there are significant differences, not to mention that there are 8 out of 10 skills with significant differences, the results indicates that the number students who were able to show evidence of the acquisition of the each of the following instrumental understanding are significantly lower compare to students who believed that they could carry out the following skills. This signifies that there is a need to focus more on engaging students on authentic learning experiences for them to see how the subject can be used to address real world situations. Students seemed to get so challenged when they were provided with non routine, innovative and authentic assessment. They should get used to the mathematics culture which focuses more on discovery, construction, experimentation and connection of learning to real world circumstances. Not merely on mathematics as algorithm.

Table 6 shows the students' perceptions of their skills and proficiency on linear equations in two variables and the developed relational understanding (procedural knowledge)(see Appendix E) of the students through written exercises and performance task. It shows the number of students who thought that they could carry out each of the following instrumental understanding after the implementation of the unit plan and the students who were able to exhibit development of relational understanding (procedural knowledge) through written exercises and performance task, and if there are significant changes between the two factors using z-test statistic for differences in two proportions.

Table 6: *Students' Perceptions of their Skills and Proficiency After UbD Implementation and Their Developed Relational Understanding (Procedural Knowledge)*

Relational Understanding (Procedural Knowledge)	**Number of Students who Perceived They could Carry Out the Skills After the Implementation of the UbD Plan**	**Number of Students who Attained Instrumental Understanding through Written Exercises**	**Number of Students who Attained Instrumental Understanding through Performance Task**
1) Plot points on a coordinate plane	40	34*	32*
2.a) Represent relations from tables	34	33	24*
2.b) Represent relations from graphs	34	33	24*
2.c) Represent relations from formulas	32	33	24*
3.a) Describe relations from tables	35	19*	24*
3.b) Describe relations from graphs	38	19*	24*
3.c) Describe relations from formulas	29	19	24
4) Add, subtract, multiply, and divide positive and negative real numbers	40	32*	32*
5) Solve problems involving real numbers	39	30*	32*
6) Simplify and evaluate algebraic expressions, using commutative, associative, and distributive properties as appropriate	37	38	32
7) Add and subtract linear expressions	40	38	32*
8.a) Define a variables	36	40	24*
8.b) Write linear equations	34	30*	24*
8.c) Solve linear equations	37	30*	32
8.d) Solve slopes	38	25*	32*
9) Use the addition and multiplication properties of equality to solve one- and two-step linear equations	38	38	32*

Legend: (*) – significant at 0.05

The low P-values using z-test statistic for differences in two proportions indicate that there are significant differences between the number of students who thought that they could perform each of the following relational understanding (procedural knowledge) after the implementation of the UbD unit plan and the students who were able to exhibit evidence of the development of relational understanding (procedural knowledge) through written exercises. The results show that 11 out of 16 (68.75%) key standards have significant differences between the students' perceptions of their skills and proficiency and the students who were able to exhibit evidence of development of relational understanding (procedural knowledge), while five out of 16(31.25%) show no significant differences. This shows that almost half of the students who believed that they could perform skills on procedural knowledge were not able to exhibit in the performance task.

Moreover, the low P-values using z-test statistic for differences in two proportions indicate that there are significant differences between the number of students who thought that they could perform each of the following relational understanding (procedural knowledge) after the implementation of the UbD unit plan and the students who were able to exhibit evidence of development of relational understanding (procedural knowledge) through the performance task. The results show that 14 out of 16 (87.50%) key standards have significant differences between the students' perceptions of their skills and proficiency and the students who were able to exhibit evidence of development of relational understanding (procedural knowledge), while two out of 16 (12.50%) show no significant differences. Majority of the students who believed that they could carry out the skills on procedural knowledge did not manifest in the performance task.

Table 7 shows the students' perceptions of their skills and proficiency on linear equations in two variables and the developed relational understanding (conceptual knowledge) (see Appendix F) of the students through written exercises and performance task. It shows the number of students who thought that they could carry out each of the following instrumental understanding after the implementation of the unit plan and the students who were able to exhibit development of relational understanding (conceptual knowledge) through written exercises and performance task, and if there are significant changes between the two factors using z-test statistic for differences in two proportions.

Table 7: *Students' Perceptions of their Skills and Proficiency After the UbD Implementation and Their Developed Relational Understanding (Conceptual Knowledge)*

Relational Understanding (Conceptual Knowledge)	**Number of Students who Perceived They could Carry Out the Skills After the Implementation of the UbD Plan**	**Number of Students who Attained Instrumental Understanding through Written Exercises**	**Number of Students who Attained Instrumental Understanding through Performance Task**
1) Build new mathematical knowledge through problem solving	28	9*	24
2) Solve problems that arise in mathematics and in other contexts	32	27	24*

3) Apply and adapt a variety of appropriate strategies to solve problems	32	27	24*
4) Monitor and reflect on the process of mathematical problem solving	34	9*	24*
5) Recognize reasoning and proof as fundamental aspects of mathematics	27	27	24
6) Make and investigate Mathematical conjectures	15	27	24
7) Develop and evaluate mathematical arguments and proofs	22	12*	24
8) Select and use various types of reasoning and methods of proof	27	9*	24
9) Organize and consolidate their mathematical thinking through communication	32	12*	24*
10) Communicate their mathematical thinking coherently and clearly to peers, teachers and others	35	9*	24*
11) Analyze and evaluate the mathematical thinking and strategies of others	34	9*	24*
12) Use the language of mathematics to express mathematical ideas precisely	31	9*	24*
13) Recognize and use connections among mathematical ideas	25	9*	24
14) Understand how mathematical ideas interconnect and build on one another to produce a coherent whole	27	9*	24
15) Recognize and apply mathematics in contexts outside of mathematics	24	9*	24
16) Create and use representations to organize, record and communicate	22	9*	24

mathematical ideas			
17) Select, apply, and translate among representations to solve problems	34	9*	24*
18) Use representations to model and interpret physical, social, and mathematical phenomena	28	12*	24

Legend: (*) – significant at 0.05

The results seem to be similar with Table 9 where the most of the P-values are low using z-test statistic for differences in two proportions which indicate that there are significant differences between the number of students who thought that they could perform each of the following relational understanding (conceptual knowledge) after the implementation of the UbD unit plan and the students who were able to exhibit evidence of development of relational understanding (procedural knowledge) through written exercises. The results show that 14 out of 18 (77.78%) key standards have significant differences between the students' perceptions of the their skills and proficiency and the students who were able to exhibit evidence of development of relational understanding (conceptual knowledge), while three out of 18 (16.67%) show no significant differences. This shows that majority of the students who believed that they could perform skills on procedural knowledge were not able to exhibit such in the written exercise. The results suggest that students might have found it difficult to express their developed conceptual knowledge in paper-and-pencil test. This confirms the results for the Philippines released by the Trends in the International Mathematics and Science Study (TIMSS) conducted by the International Association for the Evaluation of Educational Achievement (IEA) for 2003. According to the result of the Trend in the International Mathematics and Science Study (TIMSS), Philippines ranked the 41th out of 46 participating schools with an average scale score of 378 from the eighth grade students which was significantly below the international average of 467. Moreover, participating students appeared to be capable of performing relatively well in terms of knowledge and skills areas but exhibited lack of competency in the word problems and application areas. This may be an indication that our students should be engaged in more on trainings which will lead them to see the relevance and application of mathematics in real world. Our students, even up until now, are not getting used to problems which will challenge them to link learning to real-life situations. That is why students lose interest in learning the subject. They see mathematics as merely numbers and algorithms. Findings from the article: *Maths? Why Not?*, report commissioned by the Department of Education, Employment and Workplace Relations (McPhan, Morony, Pegg, Cooksey, & Lynch, 2008) identified five areas contributing to students decisions to not continue with higher level mathematics: (a) Self-perception of ability; (b) Interest and liking for higher-level mathematics; (c) Perception of the difficulty of higher–level mathematics subjects; (d) Previous achievement in mathematics; and (e) Perception of the usefulness of higher-level mathematics . One of the recommendations made in the McPhan et al. report was for the teachers of mathematics to implement pedagogical strategies that will lead students to experience the beauty of studying mathematics.

The results support the efforts of Wiggins and McTighe (2005) in providing another means of assessing the students which is the performance task; a tool which is very authentic and would test more complex learning for the students to be trained in transferring the learning in real world scenarios.

However, one skill (5.55%) got a remarkable result where it shows that the number of students who thought that they could not carry out the skill of making and investigating mathematical conjectures is less compared to the number of students who were able to exhibit the skill in the written exercises. It may be an indication that somehow the written exercises brings up the ability and potential of the students which could lead to a better acquisition of higher learning if will be provided with appropriate trainings and learning experiences.

For the results on checking for differences between the number of students who thought that they could perform each of the following relational understanding (conceptual knowledge) after the implementation of the UbD unit plan and the students who were able to exhibit evidence of development of relational understanding (conceptual knowledge) through performance were notable. The high P-values using z-test statistic for differences in two proportions indicate that majority of the key standards have no significant differences between the number of students who thought that they could perform each of the following relational understanding (conceptual knowledge) after the implementation of the UbD unit plan and the students who were able to exhibit evidence of development of the relational understanding (conceptual knowledge) through written exercises. The results show that 10 out of 18 (55.56%) key standards have no significant differences between the students' perceptions of their skills and proficiency and the students who were able to exhibit evidence of development of relational understanding (conceptual knowledge), while eight out of 18 (44.44%) show significant differences. This is a good indication that students' beliefs on their capabilities to carry out instrumental understanding were materialized in the performance task. Performance task may have triggered students' interest to exhaust everything they have because of its authenticity. The tasks gave them the Big Picture of the lessons that made them see the connection of what they have studies for the past few meetings in reality. Some groups may have failed to carry out parts of the performance tasks that tested their instrumental understanding and relational understanding (procedural knowledge), but the results show that few students were able to show strong evidence of the acquisition of relational understanding (conceptual knowledge); the ability of the students to relate learning to real word set-up.

4. CONCLUSIONS

The Students' Developed Instrumental Understanding and Relational Understanding (Procedural and Conceptual Knowledge) Through Written Exercises

The study revealed that the rate of the students who were able to develop the instrumental understanding ranged from 75.00 to 100.00%. Majority of the students have totally developed the instrumental understanding on linear equations in two variables. It was highlighted that Skemp (1989) mentioned instrumental understanding as may be not understanding at all because it is simply the ability on replicating the mentor's action but may serve as a good foundation for the acquisition of higher learning. Thus, this type of understanding should still be given attention.

The developed relational understanding (procedural knowledge) ranged from 47.50 to 100.00%. Most of the students were able to acquire the relational understanding (procedural knowledge) of the unit. It was given emphasis that there was one skill which should be given attention – the skill on the describing relations from tables, graphs and formulas.

Moreover, the number of students who developed relational understanding (conceptual knowledge) ranged from 30.00 to 67.50%. The result was quite notable for this indicates poor performance in terms of assessing the development of higher learning.

There were 27 students who were able to get the answers correctly in Test V which dealt with the application of linear equations in two variables. However, there were only 12 students who were able to see the context of their answers: nine of which were able to give complete solutions, answers and explanations while three were able to get the answers by simply sighting a concrete example which they linked to the given problem. The other 15 got the answers correctly but either they gave short and very vague explanations or totally did not give explanations to their answers. This supports the statement of one of the evaluators that there is a need to provide another authentic assessment tool which will gauge the developed conceptual knowledge of the students.

The results call for a need to focus on higher learning which are transferrable to other learning.

The Students' Developed Instrumental Understanding and Relational Understanding (Procedural and Conceptual Knowledge) Through Performance Task

It was given emphasis in this part that the students were graded as groups. Therefore, if the group had not manifested the development of a certain skill, then each member of the group was affected.

The rate of the students who developed instrumental understanding through performance task ranged from 60.00 to 80.00%. There were three groups whose works manifested weak evidences for the development of the instrumental understanding through a performance task. The work of the three groups showed how failure to develop minor skills may affect tasks which require more complex skills. The results support Skemp's (1989) suggestion on giving fair attention to the development of instrumental understanding because it may lead students on the acquisition of higher learning.

The rate of the students who developed relational understanding (procedural knowledge) ranged from 60.00 to 80.00%. The results verified how minor skills may affect carrying out other tasks. Since the three groups committed mistakes on the first part of the task, it followed that tasks which gauged procedural knowledge were not also accomplished correctly.

Lastly, the rate of students who developed each of the following relational understanding (conceptual knowledge) through performance task was 60.00%. It was given emphasis that assessing the conceptual knowledge through a performance task was easier compared to written exercises because students were given authentic assessment as it tested their ability to relate the discussed lessons to the given real world situation. Three groups were seen to have manifested the development of the relational understanding (conceptual knowledge): one of which had demonstrated evidences in the development of the conceptual knowledge while two have developed conceptual knowledge but needed improvement on the procedural knowledge and instrumental understanding.

Students' Perceptions of Their Skills and Proficiency

Using z-statistic for differences in two proportions, the high P-values indicate that there were insignificant differences in most of the students' perceptions regarding their skills and proficiency on linear equations in two variables. These signify that the students had, more or less, the same perceptions about their prior knowledge before and after the implementation of the UbD unit plan except for the other nine key standards. The percentages in terms of the changes on the pupils' perceptions were quite remarkable. The range on the positive changes of pupils' perceptions was from 0.00% to 25.00% which showed that the frequencies have gone up.

In the checking of differences between students' perceptions of their skills and proficiency and the developed instrumental understanding through written exercises, two altered results were obtained. Results showed that there were insignificant differences between the number of students who thought that they could perform each of the following instrumental understanding after the implementation of the UbD unit plan and the students who were able to exhibit evidence in the development of the instrumental understanding through written exercises. 60.00% of the key standards had insignificant differences between the students' perceptions of the their skills and proficiency and the students who were able to exhibit evidence of the development of instrumental understanding, while 40.00% showed significant differences. For the performance task, the results indicated that there were significant differences between the number of students who thought that they could perform each of the following instrumental understanding after the implementation of the UbD unit plan and the students who were able to exhibit evidence in the development of instrumental understanding through performance task. The results showed that 80.00% key standards had significant differences between the students' perceptions of the their skills and proficiency and the students who were able to exhibit evidence of development of instrumental understanding, while 20.00% showed insignificant differences.

For testing differences on the students' perception of their skills and proficiency and the students who were able to exhibit procedural knowledge in the written exercises, the low P-values using z-test statistic for differences in two proportions indicated that there were significant differences between the number of students who thought that they could perform each of the following relational understanding (procedural knowledge) after the implementation of the UbD unit plan and the students who were able to exhibit evidence in the development of relational understanding (procedural knowledge) through written exercise. Of the key standards, 68.75% had significant differences between the students' perceptions of their skills and proficiency and the students who were able to exhibit evidence of development of relational understanding (procedural knowledge) while 31.25% showed insignificant differences. Moreover, the results showed that 87.50% of the key standards had significant differences between the students' perceptions of their skills and proficiency and the students who were able to exhibit evidence in the development of relational understanding (procedural knowledge) through performance task while 12.50% showed insignificant differences.

Lastly, in checking for differences between the students' perceptions of their skills and proficiency after the implementation of the unit plan and the students who exhibited relational understanding (conceptual knowledge), the results showed that 77.78% of the key standards had significant differences between the students' perceptions of the their skills and proficiency and the students who were able to exhibit evidence in the development of relational understanding (conceptual knowledge) through written exercises while 16.67% showed insignificant differences. For performance task, 55.56% of the key standards had insignificant differences between the students' perceptions of their skills and proficiency and the students who were able to exhibit evidence in the development of relational understanding (conceptual knowledge) while 44.44% showed significant differences.

The researcher, therefore, claims that results in the study shows indications how a UbD unit plan or the backward curriculum can lead students to the development of mathematical understanding because of the following reasons: the students have developed almost all the instrumental understanding, the rate of the students who developed relational understanding (procedural knowledge) was remarkably high, the results on the students' developed relational understanding (conceptual knowledge) were promising, and the results on how UbD affects the pupils' perceptions of their skills and proficiency were a potential for further improvement.

The researcher would like to take not that in the implementation process of the UbD unit plan, the slopes as a rate of change was not given emphasis. The researcher suggests to consider this one for further researches.

Pedagogical Implications of the Results of the Study

In the light of the findings of this research study, the researcher has drawn the following considerations in the teaching of linear equations in two variables in particular and in the teaching of mathematics in general.

1. In teaching linear equations in two variables, mathematics teachers/educators must not limit themselves on traditional algorithmic instrumental and procedural techniques and focus more on innovative approaches which delve on the conceptual knowledge, i.e. focusing on how linear equations can model real world situations. This would improve the ability of the students to connect lessons in mathematics to real word scenarios.
2. To make the teaching of linear equations in two variables more meaningful to the students, the activities should be geared on real-life activities and authentic performance assessments. This will lead to the acquisition of the relational understanding (procedural and conceptual knowledge).
3. The employment of the Understanding by Design framework in teaching Mathematics in all levels of learning must be given consideration. First, students might experience anxiety on the sudden shift from a traditional teacher-centered culture of teaching to an innovative student-centered culture. In the study, the student teacher of the class stated that since the students came from different elementary schools, not all of the pupils experienced UbD framework. They still find some students encounter difficulty in dealing with the performance tasks for they got used to the traditional way of teaching. Second, UbD promotes deeper understanding of content rather than formulaic or recall

learning. To achieve deeper conceptual understanding of mathematical content the congested Philippine mathematics curriculum must be reviewed and updated.

4. The theory of constructivism must be applied in all levels of the basic education settings. Math teachers must move toward a constructivist approach in teaching mathematics where the teacher serves as the facilitator of learning and the student plays a significant role in constructing of his own understanding. Students will be given opportunities to be self-managers in the construction of knowledge. As facilitators of learning, the teachers must help the pupils discover and reflect on their own their prior knowledge of mathematical contents. Moreover, they must be guided to discover the correct knowledge about the concept and this new knowledge in a different situation.
5. The imposing collaborative or cooperative approach to learning must be given more attention in math instructions as it gives the students opportunity to share what they have learned to their peers and acquire learning from each other at the same time. Just like in the study, the number of students who acquired relational understanding (procedural and conceptual) through group performance task is remarkably higher compare to the number of students who acquired relational understanding (procedural and conceptual) through paper-and-pencil exercises. This shows how collaborative approach could bolster the acquisition of the students' understanding in mathematics.

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