ADDRESSING STUDENTS' MISCONCEPTIONS AND DEVELOPING THEIR CONCEPTUAL UNDERSTANDING AND PROCEDURAL SKILLS ON FRACTIONS USING MANIPULATIVE MATERIALS

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Abstract: Research has shown that many students still have difficulty understanding fractions even when they already have reached high school. Consequently, this study aimed to determine the levels of conceptual understanding and identify the misconceptions on the basic concepts and operations on fractions that low-performing first year high school students in the Philippines have. The students' misconceptions and levels of conceptual understanding about fractions were identified through a pre-test consisting of six word problems on the basic concepts and operations on fractions. To address these misconceptions, the students underwent a learning intervention where manipulative tools were used to relearn about the basic concepts and operations on fractions. A parallel post-test was utilized to determine whether the misconceptions of the students were addressed and whether the levels of conceptual understanding of the students changed after the intervention. The pre-test results showed that the students held many misconceptions, which fall under all Domingo's (2004) categories and that the students were in the low levels of conceptual understanding based on the OGAP Fraction Framework (2008). The post-test results showed that the misconceptions of the students were significantly reduced and their levels of conceptual understanding were raised. The lessening of misconceptions of students after the intervention showed that the manipulative materials helped in addressing the students' misconceptions. Moreover, the results of the tests showed that with the aid of the manipulative tools, the decrease in the students' misconceptions resulted to the increase in the students' levels of conceptual understanding.

Keywords: fractions; misconceptions; conceptual understanding; manipulative materials

1. INTRODUCTION

Upon reaching high school, students should already have learned the concept of fractions since this serves as foundation for advanced mathematics (Clarke, Roche, & Mitchell, 2007). Yet research shows that students find fractions difficult to understand, (Behr, Lesh, Post, Silver, Kieren and Streefland as cited by Clarke et. al. 2007). According to Kilpatrick, Swafford, & Findell, fractions are hard to understand due to their many representations (Clarke, et. al., 2007).

Booker (1997) and Nickson (2000) in Domingo (2004) stressed that students have a tough time coping with the abstract idea of fractions because fractions do not occur in ordinary situations.

Mcguire (2004) deemed that students need to understand the concepts behind abstract representations before they can make use of these abstractions. He claimed that students need to experience physical models of fractions at the very start of learning fractions so that the mathematical representations of these quantities would be meaningful to the students and their schemas of fractions could be widened, which would enable them to be better with fractions.

Sadi (2007) identified that the four fundamental operations and fraction equivalence are the areas wherein elementary and high school students commonly exhibit misconceptions on fractions. Frederiksen, Mislevy, and Bejar (1993) believed that ―misconceptions affect knowledge application‖ (p. 191). Moreover, misconceptions influence students' conceptual understanding (Michael, n.d. as cited by Booth, 2011). Lucariello (n.d.) deemed that conceptual change must first happen for learning to take place; otherwise, the teachers will just build information on a faulty foundation of misconceptions (Booth, n.d.).

According to Wong & Evans (n.d.) in Domingo (2004), conceptual understanding of fractions involves the declarative and procedural knowledge and has taken place when students see the connections between the concepts and procedures and can give logical reasons for these concepts and procedures.

Although there have been several studies about misconceptions on fractions in foreign countries, there are limited studies about the topic in the Philippines. One is Domingo's (2004) study that identified first year education majors' alternative conceptions on fraction operations and concepts, which she categorized under procedural or conceptual understanding and further classified under theoretical, functional, or semantic. The study found that most of the alternative conceptions the students had fallen under conceptual understanding.

Consequently, this study aimed to identify the misconceptions on the basic concepts (i.e. definition of fractions; mixed numbers; equality of fractions; comparing and ordering fractions including similar and dissimilar fractions) and operations on fractions that Filipino first year high school students have, and categorize them under Domingo's (2004) classifications. In addition, this study intended to determine the students' levels of conceptual understanding based on the OGAP Fraction Framework (2008). An intervention program with the use of manipulative materials was developed to address the gathered students' misconceptions. Through the instruments used, the study intended to find out what the changes in the students' misconceptions were after having gone through the intervention with the aid of manipulative materials, what the students' levels of conceptual understanding were before and after the intervention, and how the decrease in the students' misconceptions would affect the students' conceptual understanding.

2. METHODOLOGY

The sampling used in the study was nonrandom and purposive. The participants were first year high school students from Ramon Magsaysay High School, which has a homogeneous sectioning. The participants were five low-achieving students from one of the lowest sections.

Firstly, the researchers determined the students' initial misconceptions and levels of conceptual understanding through the pretest and the interview where the participants explained their answers. The study qualitatively described in detail the students' errors and level of conceptual understanding using the two assessment tools: criteria in identifying and categorizing students' misconceptions modified from Domingo's (2004) rubric; and the OGAP Fraction Framework. Then the gathered misconceptions were categorized under conceptual or procedural and classified further under theoretical, functional or semantic.

Next, a learning intervention with the use of manipulative materials was implemented to correct the misconceptions of the students.

The *fraction strips*, *fraction tiles,* and *vertical and horizontal bars* were the manipulative tools utilized during the intervention. The *fraction strips* were used for the definition, equivalence (includes expressing fractions as lowest terms) and comparing of fractions. The *fraction tiles* were used for the topics of mixed numbers, addition of fractions and subtraction of fractions. The *vertical and horizontal bars* were used for multiplication and division of fractions. Alongside the concrete materials, pictorial representations were used to model fractions and operations on fractions in the intervention.

The learning intervention began with a review of basic number theory concepts and skills in order to help the students easily relearn about fractions and perform algorithms. The topics covered in the intervention were on the basic concepts (i.e. definition of fractions; mixed numbers; equality of fractions; comparing and ordering fractions including similar and dissimilar fractions) and operations on fractions. In every meeting, there were review of multiplication of whole numbers and review of the previous topic. In every topic, there were two worksheets: Worksheet (1) was aided by manipulative materials while the Worksheet (2) was purely questions developing number sense of students.

After the intervention, a post-test was given to the students to assess their new learning. Moreover, the pre-test and post-test were descriptively compared to evaluate the improvement on the students' conceptual understanding and the effectiveness of the teaching methodology on learning fractions. The two tests were compared also to determine whether the misconceptions of the students were reduced. An interview was also conducted to supplement the results of the posttest and to know the perceptions of the students regarding the intervention with the use of manipulative materials.

To analyze the answers in the two tests, similar errors were grouped together under conceptual (theoretical or semantic) or procedural (functional or semantic). From these groupings, the misconceptions were determined. The recurrence of the misconceptions was also considered. The levels of conceptual understanding of the students before and after the intervention were determined using the OGAP Fraction Framework. The pretest and posttest analysis was utilized to check the significance of the intervention.

3. RESULTS AND DISCUSSION

The pre-test results showed that the students held many misconceptions, which fall under all the categories.

The *theoretical* misconceptions under *conceptual understanding* were *inability to identify fractional part of a given figure or a set of objects*; *failure to determine what fractional part given a number is of another number*; *inability to compare and order fractions*; *inability to perform and use appropriate operations*; and *failure to see fraction as a quantity*.

The *semantic* misconceptions under *conceptual understanding* were *inability to comprehend a given problem* (e.g. the participant thought ―how much longer‖ means to compare or to choose which fraction is longer); *analyze given statements in a problem* (e.g. not recognizing a subset of a group and separating a subset from its group); and *misinterpretation of words or phrases in a problem* (e.g. confused given word with another word with similar letters).

The *functional* misconception under *procedural knowledge* was *incorrect algorithm used in performing the operations*. This category had the most number of misconceptions, such as: wrong multiplication of whole numbers; not converting mixed number to improper fraction correctly; adding the given fractions right away without expressing them as similar fractions; subtracting also the denominators to subtract fractions; multiplying fractions by getting the reciprocal of the multiplier; and dividing the fractions right away without getting the reciprocal of the divisor and multiplying it to the dividend.

The *semantic* misconception under *procedural knowledge* was *failure to visualize and understand the given information.*

The post-test results showed that most of the students' misconceptions before the intervention had been addressed. The only misconceptions left were *functional* misconceptions under *procedural knowledge*, which were *inability to convert mixed number to improper fraction* and *incorrect algorithm used in performing the operations*. From this, it could be observed that the smooth transition from concrete and pictorial models to abstract representations of fractions present in the intervention helped in correcting the students' misconceptions, as supported by Piaget's theory (Ojose, 2008) and Mcguire's (2004) study. Therefore, it could be seen that the misconceptions of the students were significantly lessened through the intervention.

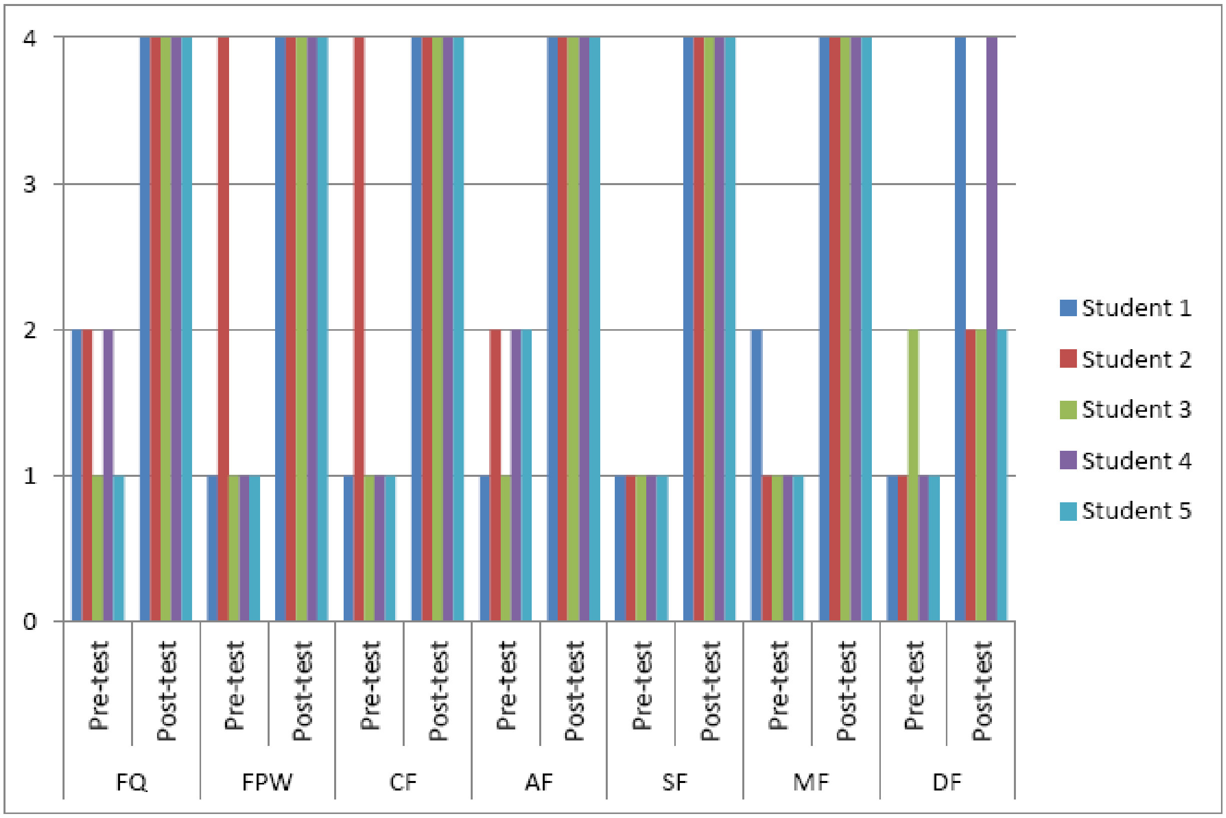

Figure 1. Students' Levels of Conceptual Understanding before and after Intervention
Legend: The vertical axis shows the levels of conceptual understanding: 1-Non-Fractional Reasoning, 2-Fractional Strategy with an Error or Misconception, 3-Transitional Fractional Strategy and 4-Fractional Strategy
The horizontal axis shows the topics: FQ-Fraction as a Quotient, FPW-Fraction as Part-to-Whole, CF-Comparing Fractions, AF-Addition of Fractions, SF-Subtraction of Fractions, MF-Multiplication of Fractions and DF-Division of Fractions

Figure 1 shows the levels of conceptual understanding of all the students before and after the intervention. It could be seen that in the pre-test, only one student reached the level of Fractional Strategy while in the post-test, each student was able to reach the Fractional Strategy level in almost all the topics. This shows that there had been a raise in the students' levels of conceptual understanding.

The results showed that the misconceptions had decreased while the levels of the students' conceptual understanding had increased. Thus, it could be observed that there had been an inverse relationship between the misconceptions and level of conceptual understanding. From these results, it could be seen that the intervention helped in improving the students' conceptual understanding through addressing the misconceptions of the students. When the students' misconceptions had been corrected, the students were able to understand the concepts in the fraction topics; for examples, they were able to understand how a fraction operation is done correctly, to comprehend the problems, and to use the appropriate strategies and operations in the problems. This was in line with the research of Frederiksen, Mislevy, and Bejar (1993) and Booth and Koedinger (2008) in Booth (2011). Thus, it could be observed that the by addressing the students' misconceptions through the intervention, conceptual change occurred, which enabled the improvement of the students' levels of conceptual understanding, as supported by (Lucariello, n.d.).

4. CONCLUSIONS

The lessening of misconceptions of students after the intervention showed that the manipulative materials helped in addressing the students' misconceptions. The significant raise in the students' levels of conceptual understanding after the intervention implied that the addressing the misconceptions of the students considerably helped in developing their conceptual understanding on the basic concepts and operations on fractions.

Further studies can be done with bigger samples to gather more misconceptions of students on fractions and look for other ways to address the gathered misconceptions and develop the conceptual understanding of students.

5. ACKNOWLEDGEMENTS

We extend our deepest gratitude to Ramon Magsaysay High School for allowing us to implement our research in their school. Moreover, we would like to thank our five participants, Aldine, Angelica, Eddielyn, Emmanuel and Loreto, for their active participation in our study and their parents for allowing them to participate. Last but not the least we would like to thank our thesis adviser Dr. Minie Rose C. Lapinid and our professors Mr. Jasper Q. Alontaga, Mr. Francis Joseph Campeña, Mr. John Vincent S. Morales and Dr. Auxencia A. Limjap for their constant advice, suggestions, support and understanding.

6. REFERENCES

Booth, J. (2011). Why can't students get the concept of math? Retrieved from http://www.resourceroom.net/math/ida_math_spring2011_booth.asp

Clarke, D. M., Roche, A., & Mitchell, A. (2007). Year six fraction understanding: A part of the whole story. In J. Watson & K. Beswick (Eds.), Mathematics: Essential research essential practice —Volume 1: Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia (pp. 207-216). Adelaide: MERGA.

Domingo, A. C. (2004). Problem-based learning and change in students' conceptual understanding of fractions. Philippines: De La Salle University, Manila.

Filep, L. (2001). The development, and of developing of, the concept of the fraction. Retrieved on October 19, 2011 from http://www.cimt.plymouth.ac.uk/journal/default.htm

Frederiksen, N. Mislevy, R. & Bejar, I. (1993). Test theory for a new generation of tests [NetLibrary version]. Retrieved from http://www.google.com.ph/books

Jaramillo, N., Chua, S., Gue, A. Y. & Lacandazo, V. (1995). 21st century mathematics. Quezon City, Metro Manila: Phoenix Publishing House. pp. 189-265

Lucariello, J. (n.d.). How do I get my students over their alternative conceptions (misconceptions) for learning? [Web log post]. Retrieved from http://www.apa.org/education/k12/misconceptions.aspx#

Michael, J. (n.d.). The significance of misconceptions for learning and teaching [PowerPoint slides]. Retrieved from http://www.google.com.ph

Mcguire, K. (2004). Exploring an interdisciplinary strategy for teaching fractions to second graders. International Journal for Mathematics Teaching and Learning. Retrieved from http://www.cimt.plymouth.ac.uk/journal/mcguire.pdf

Ojose, B. (2008). Applying Piaget's theory of cognitive development to mathematics instruction. The Mathematics Educator. 18, 26–30. Retrieved from http://math.coe.uga.edu/tme/issues/v18n1/v18n1_Ojose.pdf

Pearn, C. & Stephens, M. (2007). Whole number knowledge and number lines help to develop fraction concepts. In J. Watson & K. Beswick (Eds.), Mathematics: Essential research,essential practice —Volume 2: Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia (pp. 601-610). Adelaide: MERGA.

Reedal, K. (2010). Jean Piaget's cognitive development theory in mathematics education. Retrieved from http://ripon.edu/macs/summation

Rucker, W., Dilley, C. & Lowry, D. (1988). Heath mathematics. Massachusetts: D.C. Heath & Company. pp.127-186

Sadi, A. (2007). Misconceptions in numbers. UGRU Journal, Vol. 5. Retrieved July 7, from http://www.ugru.uaeu.ac.ae/UGRUJournal/UGRUJourna l_files/SR5/MIN.pdf

Seng, T. K. & Yee, L. C. (2007). Mathematics 1. Singapore: KHL Printing Co Pte Ltd. pp. 53-61

Tirosh, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: the case of division of fractions. Journal for Research in Mathematics Education, 31, 5-25. Retrieved July 14, 2011, from JSTOR database.

Vermont Institutes. (2008). OGAP fraction framework. Retrieved from www.vermontinstitutes.org/.../9-ogap-fractions-framework-may-200

Wong, M. & Evans, D. (n.a.). Students' conceptual understanding of equivalent fractions. In J. Watson & K. Beswick (Eds.), Mathematics: Essential research, essential practice Volume 2: Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia (pp. 601-610). Adelaide: MERGA.