



HIGH SCHOOL STUDENTS' CONCEPTUAL UNDERSTANDING OF VARIABLES AND EQUALITY

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Abstract: This study focused on High School Students' conceptual understanding of variables and equality. Variables and equality, the use of equal sign, are two basic concepts in Mathematics. In Mathematics, we use letters which is called "variables", instead of numbers. Also, they are now introduced to properties of equality. This study is a qualitative data which particularly aimed to find out the level of understanding of high school students. The researchers used two instruments in this study; test and interview. After gathering the data, the researchers then looked at the answers of the participants then based on their answers, the researchers categorized them in their proper level of understanding. The levels of understanding are as follows: no knowledge, other alternative/s, operational and relational. Less than 50% of the participants were able to reach the highest level of understanding, relational level. The researchers can say that there are still a lot of students who lack the conceptual understanding of variables and equality. The students defined variables and equality based on their current lessons. This means that there should be a deliberate effort on the part of the teacher to connect concepts learned in their previous study to the current study of the subject matter.

Keywords: level of conceptual understanding; variables; equality

1. INTRODUCTION

Many students fear the word "algebra" because they find it complicated since it uses variables instead of numbers and equal sign to show equality between quantities. It is important for students to understand the concept of variables and equality as early as high school so when the time comes for them to use these concepts, they will not have any difficulty.

The use of a variable is more flexible than using a number or term/s in an equation because variable is a symbol or representation which may represent any number in a set of values or term/s. However, according to Asquith, Stephens, Knuth and Alibali (2007), some students interpret variable as labels of assigned values to its place in the alphabet when using letters. Teachers' concept of variable is not the same as to the understanding of the students (Asquith et al., 2007).

The use of an equal sign shows the equality of a value, quantity, or term/s to another value, quantity, or term/s (Mazur, 2007). The equal sign is the most prevalent symbol in mathematics (Knuth, Alibali, Hattikudur, McNeil & Stephens, 2008) and according to Knuth, Alibali, McNeil, Weinberg, and Stephens (2005), many students look at equal sign as an indicator of solving the equation and looking for an answer. Equal sign is introduced early on in lower grade levels as used in equations, but is not fully defined and believed no other concept or review needed to be given (Knuth et al., 2005).

In 2005, there was a study made by Knuth, Alibali, McNeil, Weinberg, and Stephens in the United States about the students' understanding on the concept of variables and equality and



they targeted middle school students. They found out that all three grade levels had almost the same understanding on equality. In the case of the variables, the researchers used n as their variable; some students felt the need to replace n with x (Knuth et al., 2005). Though, all three grade levels have the same understanding on the concept of variables. The percentage of improvement across the three grade levels on the concept of both variable and equality were significantly increasing.

In 2007, another study was made by Asquith, Stephens, Knuth, and Alibali in the United States, focused on the teachers' knowledge of students' understanding of the concept of variables and equality. The teachers expected that the students fully understand the concept of variables and equality by the time they reach middle school, sixth to eighth grade level. Unfortunately, the result of the study was very different from the expectations of the teachers.

In 2010, a study was made by Rivera, Sinfuego, and Tudayan in the Philippines focusing on the 2nd year high school students' understanding of equality and linear equations. Most of the time, students neglect the role of equal sign in an equation. Some students view equal sign as an operation to balance the left side and the right side of the equation. Some students view equal sign as a symbol that separates the equation from the answer (Rivera, Sinfuego & Tudayan, 2010).

This study aims to find out the level of conceptual understanding of high school students (1st year to 4th year) on the concept of variables and equality. There were only twenty participants, five per year level because of the nature of the study. The participants, who were categorized as average in their class/among the batch in mathematics, were selected by their mathematics teacher at random. The study does not aim to generalize high school students' conceptual understanding of variables and equality in all schools in the Philippines.

2. METHODOLOGY

There were only two instruments used in this study; test and interview. The test had two parts and consisted of a total of twenty two items. After taking the test, the students were interviewed individually to confirm their answers in the test and they can further explain their answers. The interview consisted of seven questions.

The researchers had validated and pilot tested the questionnaire to ensure the ambiguity of the instructions and it was clear to all students across four year levels.

The students' conceptual understanding of variables and equality were analyzed through their answers in the test and during the interview. The researchers first looked at their initial answer on the questionnaire, and then considered their answers on the follow up questions on the questionnaire and in the interview. The levels of conceptual understanding of the students were categorized as Relational, Operational, Other Alternative/s, or No Knowledge for variables and equality/equal sign. The levels of conceptual understanding are hierarchal, but Other Alternative/s is not better than No Knowledge. If a student has an answer, and it is a wrong idea/concept, it will be categorized as Other Alternative/s.

Table 1. The Corresponding Answer to Each Level of Conceptual Understanding of Variables and Equality/Equal Sign

| Level | Variable | Equality/Equal Sign |
|---------------------|--------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------|
| Relational | A variable represents any value, quantity, or term/s. | Equal sign means “the same as” |
| Operational | A variable represents a specific value, quantity, or term/s. | Equal sign means “the answer” |
| Other alternative/s | A variable represents the answer to an equation or a variable is simply a symbol/representation. | Equal sign is simply a symbol or separates the left and the right side of an equation. |
| No knowledge | No answer. | No answer. |

3. RESULTS AND DISCUSSION

The students in this study had varying degrees of understanding. With regard to variable, some students viewed variable as a representation of values, quantity, or term/s; while for others, a variable only represented a specific number. For some, a variable means something else in an equation. Moreover, when asked about the meaning of equality and equal sign, mostly answered that equality shows that two quantities have the same value and the equal sign means “to look for an answer”, while for some, equal sign indicates equality.

Students’ Level of Conceptual Understanding of Variables

Table 2. The Tally of Students on Each Level of Conceptual Understanding of Variables

| Year Level | Relational Level | Operational Level | Other Alternative/s Level | No Knowledge |
|----------------------|------------------|-------------------|---------------------------|--------------|
| 1 st year | 1 | 0 | 4 | 0 |
| 2 nd year | 0 | 3 | 1 | 1 |
| 3 rd year | 0 | 1 | 4 | 0 |
| 4 th year | 2 | 2 | 1 | 0 |
| Total | 3 | 6 | 10 | 1 |

The single 1st year high school student who was on the relational level was the only one who stated, “a variable is used to replace the missing number”. The other four 1st year high school students who were on the other alternative/s level commonly answered, “a variable is simply a symbol” and/or “a variable is a symbol that needs to be solved in order to find the value”.

The three 2nd year high school students who were on the operational level commonly answered, “a variable that represents ‘the’ number” which they said had only one value. The



single 2nd year high school student who was on the other alternative/s level answered, “a variable represents a number that is not yet solved in the equation”. The one on the no knowledge level was not able to define variable.

The single 3rd year high school student who was on the operational level was the only one who stated, “variable has or can be substituted by a number”. The four 3rd year high school students who were on the other alternative/s level, commonly answered, “variable is a number that needs to be solved”.

Two 4th year high school students were on the relational level because they commonly answered, “variables can be substituted by any number”. The other two 4th year high school students who were on the operational level commonly stated, “variables can be substituted by a number or a single value”. The single 4th year high school student who was on the other alternative/s level had an unusual answer.

Table 3. Comparison Between The Common Answers Per Year Level on Variables

| 1 st year | 2 nd year | 3 rd year | 4 th year |
|---------------------------------------------------------------------------------------------------------------------|--------------------------------------------|-------------------------------------------------|-----------------------------------------------------------------------------------------------------------------|
| “A variable is simply a symbol.” And/or “A variable is a symbol that needs to be solved in order to find the value. | “A variable that represents ‘the’ number.” | “Variable is a number that needs to be solved.” | “Variables can be substituted by any number.” And “Variables can be substituted by a number or a single value.” |

Students’ Level of Conceptual Understanding of Equality/Equal Sign

Table 4. Students’ Conceptual Understanding of Equality/Equal Sign per Year Level

| Year Level | Relational Level | Operational Level | Other Alternative’s Level | No Knowledge |
|----------------------|------------------|-------------------|---------------------------|--------------|
| 1 st year | 0 | 5 | 0 | 0 |
| 2 nd year | 2 | 3 | 0 | 0 |
| 3 rd year | 4 | 0 | 1 | 0 |
| 4 th year | 3 | 2 | 0 | 0 |
| Total | 9 | 10 | 1 | 0 |

All five 1st year high school students who were on the operational level answered, “equal sign is a symbol used to indicate the final answer”.

The two 2nd year high school students who were on the relational level stated, “equal sign means the two equations are equal” and “the equation is equal to the given number”. The



other three who were on the operational level said, “equal sign is a symbol used to indicate the answer”.

For the 3rd year high school students, four were on the relational level. They commonly answered, “the values on both sides of the equal sign have the same value”. There was only one 3rd year student who was on the other alternative/s level. The student answered, “the equal sign is a symbol used to separate numbers”.

There were three 4th year high school students who were on the relational level. They commonly answered, “the equal sign is a symbol used to indicate that both sides of the equation have the same value”. The other two 4th year high school students who were on the operational level answered, “the equal sign is a symbol used to indicate the answer on the given equation”.

Table 5. Comparison Between the Common Answers per Year Level on Equality/Equal Sign

| 1 st year | 2 nd year | 3 rd year | 4 th year |
|-------------------------------------------------------------|-------------------------------------------------------|-------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| “Equal sign is a symbol used to indicate the final answer.” | “Equal sign is a symbol used to indicate the answer.” | “The values on both sides of the equal sign have the same value.” | “The equal sign is a symbol used to indicate that both sides of the equation have the same value.” |

Looking at the results of this study, the subject matter taken by the students of each year level may had affected their answers on the questionnaire and in the interview. The subject matter for the 1st year students is basic algebra on variables, equality and the properties of equality; for the 2nd year student is algebra for they solve for the value of the variable and always use the equal sign as an indicator that the value of the variable is the answer to the problem/s; for the 3rd year students is geometry for they use variables and the concept of equality and equal sign in the process of proving and showing the equality of the values of both sides of the equation with the use of the property of equality, transitive property, and substitution; for the 4th year students is trigonometry and the basic lesson is the equations of functions with the use of Cartesian coordinate system. In a function, the use of variables is for the points in the Cartesian coordinate system and the use of the equal sign presents the value on one side of the equation is equal to the value of the term/s on the other side of the equation in order to know the value of the variables.

4. CONCLUSIONS

Many students still lack the conceptual understanding of variables and equality because less than 50% of the students reached the highest level of understanding on both variables and equality. The students defined variables and equality base on their current lessons, yet the students understand the applications of variables and equality. However, only one student who was on the relational level on variables because he was able to give a different answer, a smiley face and a sad face, which shows that this student really understood and applied the definition of a variable.

The findings in this study on variables is that majority of the students were on the other alternative/s level and had a different result compared to the study of Knuth et al. (2005), which majority were on the relational level. The concept of variables may still be vague to the students because majority answered that a variable needed to have a value and the equation must to be



solved. Moreover, the students did not take the representation of the variables literally as to its place value in the alphabet when using letters, unlike in the studies of Asquith et al. (2007) and Knuth et al. (2005). In addition, no student had the tendency to replace any variable to x , y , or z , which are the common representations of variables.

The finding in this study on equality is that majority of the students were still on the operational level and with similar results as of the study of Knuth et al. (2005), and some other previous studies, Kieran (1981) and Alibali (n.d.). However, this study's results showed that the students' view of equality and equal sign were progressing from each year level because all five 1st year high school students were on the relational level and when it comes to the 2nd year to 4th year high school students, there were some of them who were on the relational level. Though, the students understood the role of the equal sign in an equation and it will play a big role in their success in solving equations and word problems (Knuth et al., 2005). The concept of equality and equal sign were introduced in lower grade levels and not given much review and opportunity to develop the concept of equality of the students (Knuth et al., 2005).

There is a tendency for most students to conceptualize variables and equality, and the use of equal sign, based on their current lessons and discussions. This requires that there should be a deliberate effort on the part of the teacher to show and explain the connection of the concepts learned in their previous study to the current study of the subject matter.

It is important for teachers to know and be aware of their students' level of conceptual understanding so that they would be able to correct the misconceptions and improve the students' level of conceptual understanding among these two core mathematical concepts.

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