RESEARCH ARTICLE

Assessing Equity Based Mutual Funds and Stock Market Indices in India Using the Engle-Granger Cointegration Technique

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Mutual funds are one of the most suitable investment alternatives for common investors. Mutual funds are affordable, professionally managed, transparent, and at the same time, offer a wide gamut of schemes to invest in. They can generate superior returns and, at the same time, reduce risk through the diversification of the portfolio. Out of various schemes offered by mutual funds, equity funds are quite popular among investors because of their ability to generate higher returns. They not only generate higher returns but also sometimes beat the market. As the portfolio of equity-based mutual funds consists of equity shares, the movement of NAVs of these funds and stock market indices' movement are often found to be highly correlated. This study attempts to determine the long-term relationship between returns generated by prominent equity-based mutual funds across various categories and the returns of stock market indices in India by Engle-Granger test of cointegration and Johansen's cointegration test. For this study, 10 selected prominent funds under each equity-based mutual fund category operating in India and the two most diversified stock market indices of India (i.e., BSE 200 and Nifty 500) have been taken. The study reveals a high long-term equilibrium between equity mutual funds and stock market indices, which ultimately helps investors make investment decisions.

Keywords: equity mutual funds; BSE 200; Nifty500; decision-making; Engle-Granger cointegration test; Johansen's cointegration test

Classification Code: G11

Steady source of income is always desirable irrespective whether a person is working or retired. Investors investment behaviour has been migrating from investing in traditional avenues like FD, PPF, Gold, etc towards modern investment options like mutual funds, exchange traded funds, index funds & so on.

Indian capital market always considered as one of the dynamic market in terms of investment avenues in financial assets. Every investor is different in terms of savings, return expectation and risk endurance. It is imparetive to have a wide variety of investment opportunities to substantiate the investors need. Mutual funds in todays era is considered as one of the most promising investment instrument that provides a wide variety of options to suits ever investors motive.

One prominent category of mutual funds is equitybased mutual funds, which invest in equity shares. The main objective of these schemes is to generate higher returns for investors. However, we should also remember that these schemes carry a higher risk as well. Equity funds are further categorized into subcategories, for example, large-cap funds, mid-cap funds, small-cap funds, multi-cap funds, equity-linked savings schemes (ELSS), and sectoral funds.

As the portfolio of equity-based mutual funds consists of equity shares, the movement of NAVs of these funds and stock market indices' movement are often found to be highly correlated. This study attempts to determine the relationship between returns generated by prominent equity-based mutual funds across various categories and the returns of Indian stock market indices. For the study, we have taken 10 prominent funds under each equity-based mutual fund category and two most diversified stock market indices, that is, BSE 200 and Nifty 500.

Literature Review

The main objective of Mutual fund schemes is to generate higher returns for investors; however, these schemes carry a higher risk as well. In this regard number of studies have examined the return and risk relationship with respect to benchmark indices. The study conducted by Philips et al. (2010), Kumar and Kumar (2012), Husain and Sharma (2014), Damayanti and Cintyawati (2015), Toyo and Damayanti (2015), and Agarwal and Nawazish (2017) measured the performance of different equity mutual funds schemes ranging from multi-cap to small-cap based on risk and return. They compared these scheme's performance with a benchmark index to envision whether the scheme is outperforming or underperforming the benchmark.

As the portfolio of equity-based mutual funds consists of equity shares, the movement of NAVs of these funds and stock market indices and movements are often found to be highly correlated. To assess the performances of the mutual funds and the relationship between behavioral biases and investment decisionmaking, various researchers like Nimalathasan and Gandhi (2012), Narayanasamy and Rathnamani (2013), Aydogan et al. (2014), Choudhary and Chawla (2014), Rangasamy and Sathiyapriya (2017), Nandhini and Rathnamani (2017), and Raheja and Dhiman (2019) utilized many analytical tools like alpha, beta, standard deviation, multiple regression, Treynor's, Sharpe's, and Jensen's measure, among others. Furthermore, the cointegration framework and standard Engle-Granger tools are utilized to analyze investment opportunities in the stock market and mutual funds by Matallin and Nieto (2002), Allen and Macdonald (2006), and Pojanavatee (2014) in their respective studies.

From the literature review, no study has taken into consideration various equity-based mutual funds. Thus, this study attempts to determine the long-term relationship between returns generated by prominent equity-based mutual funds across various categories and the returns of stock market indices in India by the Engle-Granger test of cointegration and Johansen's cointegration test.

Study Objective

The study's objectives are to study the long-term cointegration between returns generated by prominent equity-based mutual funds across various categories and the returns of Indian stock market indices, that is, BSE 200 and Nifty 500.

Research Methodology

This research is based upon an analysis of historical data. Monthly data related to NAV of selected mutual funds and the closing value of BSE 200 and Nifty 500 have been collected from official websites of NSE, BSE, AMFI, and other relevant sources for five years from January 2015 to December 2019. Monthly returns have been calculated from these NAVs and closing

prices. These returns have been used to measure the cointegration between them.

The two most broad-based indices, that is, BSE 200 and Nifty 500, have been taken as representatives of Indian stock market indices. At the same time, 10

prominent mutual fund schemes under various subcategories of equity funds, that is, multi-cap funds, large-cap funds, mid-cap funds, small-cap funds, ELSS, and sectoral funds have been selected for the study (Table 1).

Table 1.	Details	oj	^f Mutual	Fund	Categories
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Mutual Fund Category	Mutual Funds					
Multi-Cap Funds	Canara Robeco Equity Diversified Fund					
	UTI Equity Fund					
	HDFC Equity Fund - Growth Option					
	Mirae Asset India Equity Fund					
	Aditya Birla Sun Life Equity Fund					
	ICICI Prudential Multicap Fund					
	JM Multicap Fund					
	Kotak Standard Multicap Fund					
	Aditya Birla Sun Life Equity Fund					
	Franklin India Equity Fund					
Large- Cap Funds	Canara Robeco Blue Chip Equity Fund					
	Edelweiss Large Cap Fund					
	HDFC Top 100 Fund					
	Axis Bluechip Fund - Growth					
	Invesco India Largecap Fund					
	IDFC Large Cap Fund					
	ICICI Prudential Bluechip Fund					
	Reliance Large Cap Fund					
	IDFC Large Cap Fund					
	JM Core 11 Fund					
Mid-Cap Funds	ICICI Prudential MidCap Fund - Growth					
	Invesco India Mid Cap Fund - Growth					
	DSP Midcap Fund - Regular Plan-Growth					
	HDFC Mid-Cap Opportunities Fund-Growth Option					
	L&T Midcap Fund - Growth					
	Tata Mid Cap Growth Fund Regular Plan-Growth					
	Taurus Discovery (Midcap) Fund - Growth					
	Edelweiss Mid Cap Fund - Regular Plan-Growth Option					
	Franklin India Prima Fund					
	Kotak Emerging Equity					

Small-Cap Funds	HDFC Small Cap Fund					
	Reliance Small Cap Fund - Growth					
	Kotak Small Cap Fund - Growth					
	Aditya Birla Sun Life Small Cap Fund					
	DSP Small Cap Fund - Regular Plan-Growth HSBC Small Cap Equity Fund					
	Sundaram Small Cap Fund - Growth					
	Franklin India Smaller Companies Fund					
	SBI Small Cap Fund					
	CICI Prudential Smallcap Fund - Retail					
Equity Linked Savings	Aditya Birla Sun Life Tax Relief '96					
Schemes (ELSS)	Axis Long Term Equity Fund					
	DSP Tax Saver Fund					
	HDFC Long Term Advantage Fund					
	Taurus Tax Shield					
	IDFC Tax Advantage (ELSS) Fund - Regular Plan					
	Invesco India Tax Plan					
	JM Tax Gain Fund					
	Kotak Tax Saver Regular Plan					
	L&T Tax Advantage Fund					
Sectoral/Thematic Funds	ICICI Prudential Banking and Financial Services Fund					
	Franklin Build India Fund					
	Aditya Birla Sun Life Banking & Financial Services Fund-Regular Pla					
	Reliance Banking Fund					
	UTI Banking and Financial Services Fund - Regular Plan					
	Baroda Banking and Financial Services Fund					
	SBI Infrastructure Fund					
	L&T Infrastructure Fund					
	Invesco India Infrastructure Fund					
	LIC MF Infrastructure Fund					

Time Period of the Study

Relevant monthly data has been collected through authorized websites for five years, from 2015 to 2019.

Calculation of Return

For this research work, the monthly return of each mutual fund scheme understudy has been calculated. To calculate monthly return following two NAVs of the fund have been considered:

- The opening NAV of the month
- The closing NAV of the month

The formula for calculating monthly return:

(Closing NAV of the month – Opening NAV of the month)

The opening NAV of the month

For stock market indices, opening and closing values have been taken instead of NAVs.

Statistical Tools for Analysis

For analysis, some of the statistical tools are used in the study are descriptive statistics, ADF test, Engle-Granger cointegration test, and Johansen cointegration test. Excel and R 3.5.2 software has been used for all the calculation purposes.

Hypotheses

Following are the hypotheses for the study:

- H_{01} : Returns of Nifty 500 and multi-cap funds are not cointegrated.
- H_{02} : Returns of Nifty 500 and large-cap funds are not cointegrated.
- H_{03} : Returns of Nifty 500 and mid-cap funds are not cointegrated.
- H_{04} : Returns of Nifty 500 and small-cap funds are not cointegrated.
- H_{05} : Returns of Nifty 500 and equity-linked savings schemes are not cointegrated.
- H_{06} : Returns of Nifty 500 and sectoral funds are not cointegrated.
- H_{07} : Returns of BSE 200 and multi-cap funds are not cointegrated.
- H_{08} : Returns of BSE 200 and large-cap funds are not cointegrated.
- H_{09} : Returns of BSE 200 and mid-cap funds are not cointegrated.
- H₁₀: Returns of BSE 200 and small-cap funds are not cointegrated.
- H_{11} : Returns of BSE 200 and equity-linked savings schemes are not cointegrated.
- H₁₂: Returns of BSE 200 and sectoral funds are not cointegrated.

Engle-Granger Test of Cointegration

Cointegration is a moderately new idea presented by Granger (1983) and Engle and Granger (1987). Two factors are supposed to be cointegrated when a straight blend of the two factors is fixed, suggesting a drawn out connection between them. Absence of cointegration recommends that no such relationship exists. For a time series, residuals testing from ordinary least square (OLS) regression is required for cointegration testing. Then, the residuals are obtained as:

$$Y_t = \beta_0 + \beta_1 x_t + \beta_2 z_t + (1)$$

The above equation represents the linear relationship between y on x and z. The residuals are obtained from the OLS, and a Dicky fuller unit root test is carried out to check for unit root. If a unit root is not present, the residuals are stationary, and the variables are cointegrated (Subha and Nambi, 2013).

Johansen Cointegration Test

On the name of Søren Johansen, a method for testing cointegration of multiple time series, say k, I(1) imerged as the Johansen test. Through this test more than one cointegrating relationship can be determined, so it is more commonly pertinent than the Engle–Granger test based on the Dickey-Fuller (or the augmented) test for unit roots in the residuals from a single (estimated) cointegrating relationship.

Johansen test are of two types that is with trace or with eigenvalue. The inferences of both the types might be a slightly vary. For the trace test the null and alternative hypothesis are:

H0: The number of cointegration vectors is $r=r^* < k$. H1: The number of cointegration vectors is r=k.

Testing progresses in order for $r^{*}=1,2$, etc. and the first acceptance of the null hypothesis is taken as an estimate of r. The null hypothesis for the "maximum eigenvalue" test is the same as trace test, but the alternative is $r=r^{*}+1$ and, again, testing progresses in order for $r^{*}=1,2$, etc., with the first acceptance used as an estimator for r.

Data Analysis and Interpretation

Cointegration of different equity mutual funds concerning NIFTY 500 and BSE 200 is carried out for a sample period of five years, from January 2015 to December 2019. Secondary data of respective mutual fund returns and return on indices are retrieved from Jan 2016 to Dec 2020 (https://www.advisorkhoj.com/ mutual-funds-research). Figures 1 and 2 depicts the same.

Time

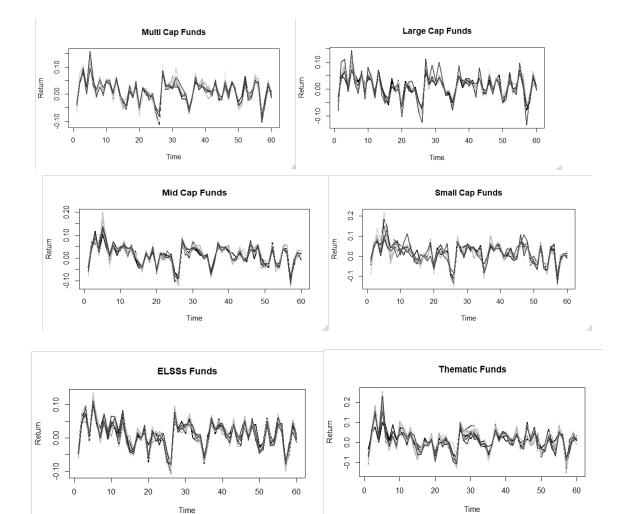


Figure 1. Graphical Representation of Equity Mutual Funds

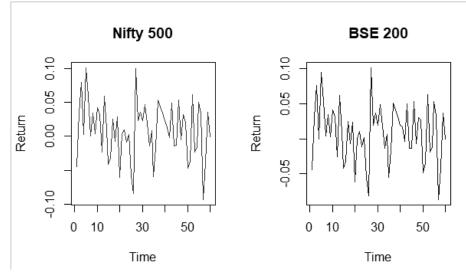


Figure 2. Graphical Representation of Indices

Particulars	Min.	1st Qu	Median	Mean	3rd Qu	Max.
NIFTY 500	-0.092922	-0.014455	0.010107	0.009161	0.034969	0.100891
BSE 200	-0.086495	-0.013504	0.009641	0.008904	0.034774	0.101224
Multi Cap Fund	-0.082995	-0.008269	0.012238	0.011199	0.04175	0.124713
Large Cap Fund	-0.082314	-0.01099	0.013132	0.009712	0.040074	0.100412
Mid Cap Fund	-0.1053	-0.01248	0.01984	0.01317	0.04436	0.14785
Small Cap Fund	-0.12084	-0.01405	0.02098	0.01388	0.04425	0.13697
ELSSs Fund	-0.086521	-0.007362	0.013036	0.011383	0.042787	0.106874
Thematic Fund	-0.09427	-0.01221	0.012	0.01291	0.04184	0.18491

 Table 2. Descriptive Statistics of Indices and Six Categories of Mutual funds

 Table 3. Augmented Dickey-Fuller (ADF) Unit Root Test

Time Series	ADF Statistics	p-value
Nifty 500	-3.7582	0.02746
BSE 200	-3.7266	0.03013
Multi Cap MF	-3.7885	0.02495
Large Cap MF	-3.8578	0.02192
Mid Cap MF	-3.6998	0.03239
Small Cap MF	-3.6483	0.03673
ELSSs	-3.623	0.03886
Thematic MF	-4.2111	0.01

Table 2 shows the descriptive statistics of the mutual funds taken under the research. Values that describe the characteristics of a sample or population are termed descriptive statistics. Mean and median are different for each of the selected mutual funds, which shows that data are not normally distributed.

To test the cointegration, we will check the time series data's stationarity using the augmented Dickey-Fuller (ADF) test. The ADF test contains the unit root in the time series and is a negative number. The more negative it is, the stronger the rejection of the hypothesis that there are unit roots at some level of confidence. To test the cointegration between the two series, the time series data must be stationary.

From Table 3, all the indices and returns by all the equity mutual funds are stationary.

Now, to check the cointegration between:

1. NIFTY 500 and equity mutual funds

2. BSE 200 and equity mutual funds

The Engle and Granger cointegration test has been employed, and the results are summarized in Tables 4 and 5.

The Engle and Granger test is applied to test the cointegrating relationship between two variables in the long run. Based on the results shown in Tables 4 and 5, the null hypothesis is rejected at a 1% significance level, and it can be concluded that there is cointegration in case of lag 3 between NIFTY 500 and six categories of equity mutual funds as well as between BSE 200 and six categories of equity mutual funds. Also, from results for type 2 and 3 time series are double smoothed and have quadratic trends. This implies that six categories of equity mutual funds under study are cointegrated with the long-run benchmark indices.

NIFTY 50	0 and Mu	lti-Cap Fund		
	Lag	EG	p-value	
Type 1 (no trend)	3	-3.4810	0.0139	
Type 2 (linear trend)	3	0.282	0.100	
Type 3 (quadratic trend)	3	1.12	0.10	
NIFTY 50	0 and Lar	ge Cap Fund		
	Lag	EG	p-value	
Type 1 (no trend)	3	-4.23	0.01	
Type 2 (linear trend)	3	-0.527	0.100	
Type 3 (quadratic trend)	3	2.33	0.10	
NIFTY 5	00 and Mi	d Cap Fund		
	Lag	EG	p-value	
Type 1 (no trend)	3	-3.71	0.01	
Type 2 (linear trend)	3	0.757	0.100	
Type 3 (quadratic trend)	3 0.00742		0.100	
NIFTY 50	0 and Sm	all Cap Fund		
	Lag	EG	p-value	
Type 1 (no trend)	3	-3.71	0.01	
Type 2 (linear trend)	3	0.787	0.100	
Type 3 (quadratic trend)	3	-0.875	0.100	
NIFTY	500 and E	LSS Fund		
	Lag	EG	p-value	
Type 1 (no trend)	3	-3.68	0.01	
Type 2 (linear trend)	3	0.402	0.100	
Type 3 (quadratic trend)	3	0.781	0.100	
NIFTY 5	00 and Th	ematic Fund		
	Lag	EG	p-value	
Type 1 (no trend)	3	-5.37	0.01	
Type 2 (linear trend)	3	0.22	0.10	
Type 3 (quadratic trend)	3	0.823	0.100	

 Table 4. Engle and Granger Cointegration Test Between NIFTY 500 and Equity Mutual Fund Returns

BSE 200 and Multi-Cap Fund							
	Lag	EG	p-value				
Type 1 (no trend)	3	-3.481	0.014				
Type 2 (linear trend)	3	0.539	0.100				
Type 3 (quadratic trend)	3	0.795	0.100				
BSE 200 and I	Large Caj	o Fund					
	Lag	EG	p-value				
Type 1 (no trend)	3	-4.37	0.01				
Type 2 (linear trend)	3	-0.165	0.100				
Type 3 (quadratic trend)	3	1.86	0.10				
BSE 200 and Mid Cap Fund							
	Lag	EG	p-value				
Type 1 (no trend)	3	-3.76	0.01				
Type 2 (linear trend)	3	0.861	0.100				
Type 3 (quadratic trend)	3	-0.228	0.100				
BSE 200 and S	Small Caj	o Fund					
	Lag	EG	p-value				
Type 1 (no trend)	3	-3.69	0.01				
Type 2 (linear trend)	3	0.825	0.100				
Type 3 (quadratic trend)	3	-0.96	0.10				
BSE 200 and	d ELSS F	und					
	Lag	EG	p-value				
Type 1 (no trend)	3	-3.465	0.015				
Type 2 (linear trend)	3	0.661	0.100				
Type 3 (quadratic trend)	3	0.415	0.100				
BSE 200 and 7	Thematic	Fund					
	Lag	EG	p-value				
Type 1 (no trend)	3	-5.15	0.01				
Type 2 (linear trend)	3	0.482	0.100				
Type 3 (quadratic trend)	3	0.394	0.100				

 Table 5. Engle and Granger Cointegration Test Between BSE 200 and Equity Mutual Fund Returns

Johansen Cointegration Test Results

From the trace and eigenvalue statistics generated by Johansen's cointegration test (see Appendix-I), the null hypothesis is rejected. This implies that the variables are cointegrated, and they have a long-run relationship. The maximum eigenvalue test also shows two cointegrating equations at the 5% level. Therefore, these two tests confirm a cointegrating relationship over the sample period. Hence, the results obtained by the Engle and Granger test are supported by statistical evidence obtained by Johansen's test. In other words, all the variables move together in the long-run means that they have a long-run association, so now the ECM and VECM model can be run to check the short-run relationship.

Conclusion and Future Implications

This study selected six prominent equity mutual funds (multi-cap, large-cap, mid-cap, small-cap, ELSSs, and sectorial) to check the long-term association between them and benchmark indices, NIFTY 500 and BSE 200. For this purpose, the series' stationarity has been reviewed by the ADF test and found that all the time series are stationary. Engle-Granger cointegration test was used to find the long-run relationship between the equity mutual fund categories and benchmark indices. The results were found to be statistically significant and show long-run cointegration between them. The Johansen cointegration test provides statistically significant evidence of long-run cointegration between all the categories of equity mutual funds and benchmark indices. Though returns of all the six categories are cointegrated with stock indices returns, in the long run, based on the degree of cointegration for no trend (given in Tables 4 & 5), the categories of equity mutual funds have ranked as multi-cap, ELSSs, small-cap, mid-cap, large-cap and sectorial equity mutual funds.

Although the study provides valid results to help investment makers in decision-making, they can be further extended by considering some other investment alternatives. Also, the same study can be done to study the long-term relationship between the Indian stock indices and other countries' indices. This study utilized the data till December 2019; in the future, the impact of COVID-19 can also be studied by taking the data of post pandemic.

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Appendix-I

Johansen Cointegration Test

Test type: trace statistic, with a linear trend **Eigenvalues (lambda):**

NIFTY 500	BSE 200	Multi-Cap	Large Cap	Mid Cap	Small-Cap	ELSS	Thematic
0.64853	0.566217	0.492305	0.432885	0.376313	0.302959	0.221288	0.152035

Values of the test statistic and critical values of test:

	test	10pct	5pct	1pct
r <= 7	9.57	6.50	8.18	11.65
r <= 6	24.07	15.66	17.95	23.52
r <= 5	45.00	28.71	31.52	37.22
r <= 4	72.39	45.23	48.28	55.43
r <= 3	105.28	66.49	70.60	78.87
$r \le 2$	144.60	85.18	90.39	104.20
r <= 1	193.04	118.99	124.25	136.06
r=0	253.69	151.38	157.11	168.92

Test type: maximal eigenvalue statistic (lambda max), with a linear trend **Eigenvalues (lambda):**

NIFTY 500	BSE 200	Multi-Cap	Large Cap	Mid Cap	Small-Cap	ELSS	Thematic
0.64853	0.566217	0.492305	0.432885	0.376313	0.302959	0.221288	0.152035

Values of the test statistic and critical values of test:

	test	10pct	5pct	1pct
r <= 7	9.57	6.50	8.18	11.65
$r \le 6$	14.51	12.91	14.90	19.19
$r \leq 5$	20.93	18.90	21.07	25.75
$r <= 4 \mid$	27.38	24.78	27.14	32.14
$r \le 3$	32.90	30.84	33.32	38.78
$r <= 2 \mid$	39.32	36.25	39.43	44.59
r <= 1	48.44	42.06	44.91	51.30
r=0	60.65	48.43	51.07	57.07