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RESEARCH ARTICLE

# A Test of the Generalized Yunus Equation and its Implications for Microcredit in the Philippines

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Microcredit is one of the most important financial services offered by microfinance institutions (MFIs) in the Philippines. Loan repayment by frequent installments faces the challenge of possible random delays, which leads to random interest rates. Such randomness affects both microlenders and borrowers. This research utilizes a mathematical model called the generalized Yunus equation (GYE) to study the microloan repayment process in the Philippine setting. Using this model, an explicit formula was formulated expressing the effective interest rate involved in repayment as a function of the time when the delay takes place, which closely approximates the actual values of the rate that microlenders receive. The model can potentially help lenders identify how much interest they gain or lose, which is a viable part of their operation's sustainability.

***Keywords:*** generalized Yunus equation, loan repayment, microcredit, random interest rate

***JEL Classifications:*** C69, E43, G21

Microcredit, from the term itself, means very small loans. It provides credit access to poor or low-income people, without collateral. One of the pioneers of microcredit practice is Muhammad Yunus, an economics professor at Chittagong University, Bangladesh. He spearheaded the Grameen Bank Project that lent money to people in small villages in Bangladesh. He founded what was then called the Grameen Bank in 1983. Yunus and the Grameen Bank were awarded the Nobel Peace Prize in 2006 for their efforts for the overall development of both the social and economic conditions of the poor (Nobel Media AB, 2006).

Brau and Woller (2004) presented a comprehensive review of existing literature in microcredit and microfinance in general, mostly dealing with economics or social dimension and very few on mathematics. The authors believed that mathematical research could help improve microfinance theory and practice worldwide. Tedeschi (2006) came up with a repayment model of individual microlending, which takes into consideration the dynamic incentives to prevent borrowers' strategic default. F. Diener, Diener, Khodr, & Protter (2009) introduced a Markov chain representation of Tedeschi's model, which features credit exclusion as a penalty for non-repayment. The study showed that in terms of expected return, a borrower is better off paying a loan than choosing to default. Results paved the way for a probabilistic approach to microcredit. Mauk (2013) formulated the deterministic Yunus equation that describes microloan repayment, as described by Yunus and Jolis (1999), which considers partial installments. Mauk (2013) also formulated the random Yunus equation, which takes into account random delays in installments. The Yunus equations were modified and generalized by Diener and Santos (2016) in what they called the generalized Yunus equation (GYE). Results showed that actual values of the rates when there is a single random delay in microcredit repayment could be theoretically approximated using the equation.

Several studies focused on discovering the relationship between the interest rate charged by microcredit institutions and loan repayment. Abbas and Zhang (2016) used descriptive assessment techniques to identify the impact of interest rate on loan repayment in MFIs in Tanzania. Results show that there were high levels of loan default when clients are given higher loan amounts, which comes with high interest rates. A project conducted in Nigeria ("The effect of interest rate", n.d.) appraised the effect of MFI interest rates on loan repayment through systematic random sampling. This study suggests that a low interest rate encourages loan repayment, thus leads away from the risk of missing scheduled installments, even worse, default. It also identified other factors and their effects on loan repayment. As these studies focused on the impact of interest rates in loan repayment, very few looked at the possible impact of delays in repayment—considered as one of the major risks in microcredit—on the interest received by lenders. Rosenberg (2002) came up with an explicit pricing formula for the annualized effective interest rate charged on microloans in terms of different factors, which include the loan loss rate. This rate is attributed to loans that are not paid on time, which may eventually lead to write-offs.

Recent studies on microcredit and microfinance, particularly in the Philippines, were published. Daley and Sautet (2005) discussed recent policies in microcredit, its barriers, and reforms. It was noted that making the microcredit market open to competition from other countries will ensure low interest rates. Karlan and Zinman (2011) used randomized credit scoring to evaluate borrowers from a microlender operating in the National Capital Region and few provinces in Luzon. The method turned out to be a practical and reasonable tool in measuring the impacts of expanding access to microcredit. Habaradas and Umali (2013) studied the contribution of several factors, such as the adoption of a sound business model, government and donor interventions, technology-based innovations, to the growth of the MFI industry in the country. Conchada and Tiongco (2017) researched how micro-savings works on the country's informal sector.

This study focuses on the possible effects of delays in the repayment of Filipino borrowers on the microcredit market. We aim at identifying the effect of delayed installments, specifically on microcredit interest rates. We check if GYE works in the Philippine setting, as we partner with one of the country's top MFIs, ASA Philippines. We hypothesize that our model works in identifying the effect of a single random delay on the effective interest rate that the ASA receives.

The paper will proceed as follows. The next two sections will discuss the development of a stochastic model of microcredit repayment to GYE and the case where there is one delay in repayment. A background of microcredit and microfinance in general in the Philippine setting will then be discussed. Borrower and repayment information will be studied as we verify if GYE reflects the real practice of microcredit in the Philippines. We study and compare two situations, considering the repayment of ASA borrowers. The last section gives the conclusion and future directions of the study.

***Microcredit Repayment Models***

The development of the Yunus equations for microcredit was discussed by Mauk (2013) and Santos (2017). The deterministic Yunus equation for microcredit models repayment of 1000 Bangladesh Taka (BDT) by weekly installments of 22 BDT, based on a 10% flat rate, in a year. It is given by

$$1000 = 22\sum_{j=1}^{50} q^j, \quad q = e^{-\frac{r}{52}}, \tag{1}$$

where $r$ is the actual interest rate. Notice that if we denote by N the number of installments and by $r_f$ the flat rate, (1) can be written as

$$N = \left(1 + r_f\right)\sum_{j=1}^{N} q^j, \qquad q = e^{-\frac{r}{N}}. \tag{2}$$

A stochastic model was developed to consider the possibility of the borrower having delays in repayment. As the loan is small, a borrower will exert utmost efforts to avoid delaying or even defaulting, so as not to lose the advantage of continued access to microcredit. But as the number of installments in a year is considerably large, a borrower may indeed face some burden leading to difficulties in paying installments on time. This would postpone by one or more time periods all the subsequent settlements. So, we now treat the installment date as a random variable. The $j^{th}$ installment takes place at the stopping-time

$$T_j = \min\{t \mid B_1 + B_2 + \cdots + B_t = j\},\ j = 1, 2, \ldots N,$$

where the $B_i s$ are independent and identically distributed Bernoulli random variables such that $B_i = 1$ if the borrower is able to pay at week $i$, and 0 otherwise. In this model, the actual interest rate, $r$, in (2) now becomes random and will be denoted by $R$. This rate is implicitly defined by the GYE (Diener & Santos, 2016):

$$N = (1 + r_f) \sum_{j=1}^{N} Q^{T_j}, \quad Q = e^{-\frac{R}{N}} . \qquad (3)$$

***Asymptotics of the Interest Rate in Microcredit Repayment with One Delay***

Using a nonstandard approach, through concepts of infinitesimals and infinitely large numbers (F. Diener & Diener, 1995; Keisler, 1986; Nelson, 1977), the random interest rate involved in GYE for the case when there is a single unique delay was studied by Diener and Santos (2016).

Define $R_1$ to be the implicit interest rate assuming a single delay. Clearly, the rate can take $N$ different values, denoted by $r(k) = -N \log q(k)$, $k \in \{1, 2, \ldots, N\}$. The GYE given one delay, at week $k$ is given by

$$N = (1 + r_f) \left( \sum_{j=1}^{N+1} q^j - q^k \right), \quad q = e^{-\frac{R_1}{N}} . \qquad (4)$$

Diener and Santos (2016) obtained an asymptotic expansion of both the discount factor and interest rate when there is one delay in repayment, given in the following theorems.

**Theorem 1:** For $k = 1, 2, \ldots, N$, the unique real solution $q(k)$ to GYE (4) given delay at week $k$ has an order three expansion given by

$$q(k) = 1 - \frac{\beta_1}{N} + \frac{\beta_2}{N^2} + \frac{\beta_3(k)}{N^3} + \frac{\varepsilon_k(N)}{N^3}, \qquad (5)$$

with $\lim_{N \to \infty} \varepsilon_k(N) = 0$, where

$\beta_1$ is the real nonzero solution of $1 - e^{-\beta_1} = \frac{\beta_1}{1 + r_f}$,

$$\beta_2 = \frac{\beta_1^2 (3 + \beta_1 - r_f)}{2(\beta_1 - r_f)},$$

$\beta_3(k) = \vartheta k + \mu$, where $\vartheta$ and $\mu$ are constants given by $\vartheta = -\frac{\beta_1^2 (1 + r_f)}{\beta_1 - r_f}$ and

$$\mu = -\frac{\beta_1 (1 + r_f)}{\beta_1 - r_f} \left\{ \frac{\beta_2^2}{\beta_1^2 (1 + r_f)} \cdot \left(1 - \frac{\beta_1}{1 + r_f}\right) \left[ \beta_2 \left( \frac{3}{2} - \frac{\beta_2}{\beta_1^2} - \frac{\beta_2}{2\beta_1} \right) - \beta_1 \left( 1 + \frac{2\beta_1}{3} - \frac{\beta_2}{2} + \frac{\beta_1^2}{8} \right) \right] \right\}.$$

**Theorem 2:** The actual interest rate $r(k)$ associated with $q(k)$ in (5), for one delay in repayment at time $k$, has the following order two asymptotic expansion given by

$$r(k) = \alpha_0 + \frac{\alpha_1}{N} + \frac{\alpha_2(k)}{N^2} + \frac{\epsilon_k(N)}{N^2}, \qquad (6)$$

with $\lim_{N \to \infty} \epsilon_k(N) = 0$, where $\alpha_0 = \beta_1, \alpha_1 = \frac{1}{2}\beta_1^2 - \beta_2$, and $\alpha_2(k) = \frac{1}{3}\beta_1^3 - \frac{1}{2}\beta_1\beta_2 - \beta_3(k)$.

Theorems 1 and 2 imply that up to terms small with respect to $\frac{1}{N^3}$ and $\frac{1}{N^2}$, respectively, the discount factor $q$ and the actual interest rate $R_1$ defined in (4) are quasi-linear functions of the time $k$ when the delay occurs, given by (5) and (6). Derivations of the asymptotic expansions were discussed in detail by Santos (2016). Figure 1 displays the actual values of the interest in case of a single delay, at week $k$, and the corresponding values using the asymptotic expansion for $N = 50$ and $r_f = 10\%$. The horizontal line represents the interest rate when there is no delay.

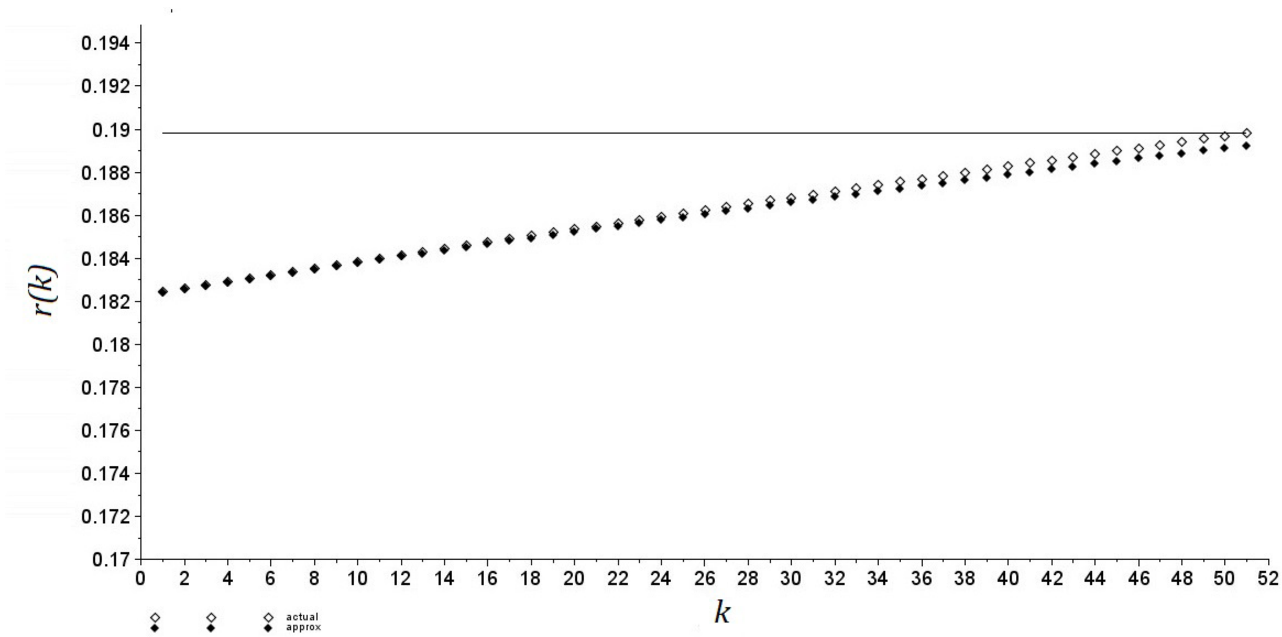


***Figure 1.*** Actual versus approximate values of the interest rate.

Results show that the asymptotic expansion for the interest rate when there is one random delay out of the 50 installments captures the behavior of the actual values of the rate. Also, as the delay takes place at a later date, the corresponding rate differs less, almost negligible, from the rate when there is no delay. This means that the effective interest rate that the lender receives becomes smaller than the expected one once there is a delay by a borrower and even decreases as it occurs in an earlier period (Diener & Santos, 2016). This shows that GYE can provide good insight to the lender in terms of the effective rate it receives from the borrower who misses one installment at a random week.

## GYE in the Philippine Setting

Microfinance is one effective tool in providing aid to Filipinos living below the poverty line. An increase in income was evident due to small to medium scale entrepreneurial activities funded by microloans, and improvement was observed in the way of living of families. Microcredit remains the most important service among all financial products offered by MFIs, not just in the Philippines but worldwide (Guntz, 2011). In this section, we discuss microcredit and microfinance, in general, in the Philippine setting. We then check if GYE can be utilized to model the process of microloan repayment of borrowers of ASA Philippines.

From the 1960s up to the early 1980s, the Philippine government has mostly been supporting the poor by providing financial credit at subsidized interest rates. However, despite the government and donor resources allotted for these financial programs, a lot of people living under the poverty line, especially those in the rural areas, still lack access to it. From 1985 to 2000, poverty count in the country decreased by 10.5%. However, the actual number of Filipinos, who live below the poverty line, increased by more than four million over the same period (Schelzig, 2005). Poverty incidence among Filipino families registered at 21.6% as of the year 2015 (Bersales, 2017).

In 1989, the Philippines was among the first group of countries to replicate Grameen banking through the Grameen Bank Approach Replication Project, run by local non-governmental organizations (NGOs). Despite the high repayment rate and positive impact on the standard of living of the borrowers, it was deemed unsustainable due to inadequate interest rates and high operating costs (Hossain & Diaz, 1997). In 1995, the People's Credit and Finance Corporation (PCFC) was established by the Philippine government to execute a nationwide Grameen replication project, called the Rural Micro-enterprise Finance Project (RMFP), which was jointly funded by the Asian Development Bank

(ADB) and the International Fund for Agricultural Development (IFAD) (Felloni, Seibel, & Cornejo, 2005). All these led to the creation of the National Credit Council in 1993, which established the National Strategy for Microfinance in 1997. This paved the way for the development of microfinance in the country (Geron, 2011).

Microfinance in the Philippines is provided by three types of institutions—NGOs, credit cooperatives, and banks. NGOs focus mainly on the social development of their borrowers in various geographical locations. Credit cooperatives are financial organizations owned and controlled by its members. They provide loans only to their members who usually belong to the middle income and poor sector, at low interest rates. Rural and thrift banks offer a wide array of credit products, usually with simple requirements and no collateral.

The Microfinance Council of the Philippines, Inc. (www.microfinancecouncil.org) is the national network of MFIs in the Philippines. It was established in 1999 and is currently composed of 57 institutions that account for at least 75% of the total active outreach of the microfinance sector in the Philippines. In 2012, a private credit bureau was formed by seven of the biggest MFIs in the country. The Microfinance Data Sharing System (MiDAS) is the first credit bureau that allows participating MFIs to submit reports and retrieve credit and borrower information. The seven MFIs responsible for the creation of MiDAS are Taytay sa Kausagan Inc. (TSKI), OK Bank, CARD Bank, CARD NGO, Negros Women, Ahon sa Hirap, and ASA Philippines (Snelling, 2012).

The Economist Intelligence Unit Limited (www.eiu.com) named the Philippines as the top country in East and South Asia and third in the world with the most conducive environment for financial inclusion in 2014. Delivery of financial services at affordable costs to low-income members of the population was made possible by the efforts of different MFIs in the country. The US-based NGO, Microfinance Information Exchange (www.themix.org), reported data of around 140 MFIs in the country with a total gross loan portfolio of US$1.27B and 5.4 million borrowers as of the year 2017.

### *ASA Philippines*

We consider borrower and repayment information in one of the biggest microcredit institutions in the Philippines. ASA Philippines Foundation, Inc. (www.asaphil.org) is a non-profit and non-stock corporation specializing in microfinance. The word *asa*, in both Filipino and Bengali languages, means “hope”. ASA was established in 2004 through the efforts of its current CEO and president, Mr. Kamrul F. Tarafder, and Ambassador Howard Q. Dee of the Assisi Development Foundation.

ASA Philippines was named the leading institution in the Philippines microfinance industry in pricing transparency and client protection. It is the first MFI to be registered in BSP as an “entity with credit granting facility” in 2013. It is also one of the leading MFIs in the country in terms of the number of active borrowers and gross loan portfolio. As of 2017, ASA has recorded 1,669,650 borrowers all over 1,150 branches in all 82 provinces of the country. It reports a loan portfolio of PhP16,233,167,714.

Information on microloan repayment of ASA borrowers was obtained through a one-on-one interview with an ASA branch administrator. ASA’s microloan caters to those who live below the poverty line and of 18 to 65 years of age. Because the aim of the institution, just like any other NGO, is mainly to help and improve the lives of its clients, borrowers who are considered for microloans are those who do not have stable jobs and regular income. Each applicant is evaluated and is then supervised by a loan officer once a loan is approved. The borrowers are then organized in groups of 30 with one loan officer in charge of them. One cycle of a microloan runs for 23 weeks or six months, and the interest, which they call the service charge, is based on a 15% flat rate. ASA has been charging the same rate since the start of its operations, which is relatively lower and is not like that of other MFIs that tend to change from time to time. Since the time that ASA came up with rebates for accelerated payments, almost 30% of the borrowers aim to pay in less than six months. Borrowers with accelerated payments have the privilege to pay lower rates and the opportunity to renew their loans at an earlier period, which could be very beneficial to their small businesses.

The repayment or recovery rates published by ASA Philippines for the past 15 years of its operation are higher than 97%, the value which is considered as the internationally accepted rate, defined simply as the complement of the default rate equal to 3% (Addae-Korankye, 2014). The rates were computed as the ratio of actual over target collections on microloans, computed at the end of each year. Different ratios are used to measure repayment, as enumerated and analyzed by Rosenberg (1999). The ratio ASA used is consistent with the definition of what is called the cumulative collection rate. This rate considers all principal payments received since the start of the program over all repayments of principal that have fallen due as of the date of measurement, or in short, all loan disbursements within the considered period.

Such high recovery rates could be justified by the institution's lenient approach to loan repayment. Group meetings are held at the start of the week, where loan officers collect installments. Borrowers who miss paying during the meeting have until the end of the week to pay. This approach is opposite that of loan sharks, which forces borrowers to pay, even imposing punishments. If a borrower delays for consecutive weeks without a valid excuse, such as illness or calamity, then the loan will be declared as a past due account. This is different from saying that the loan is a default. Contrary to other MFIs, especially in other countries, ASA has not set the maximum number of consecutive weeks of delay in declaring a loan default. So, borrowers are still encouraged to pay even after weeks of missing repayment. Moreover, if a client delays, say once, they would allow payment of twice the installment amount during the following week to compensate for the missed one. Hence, the loan can still be fully paid when due. Loan officers will explore different possibilities to be able to collect all installments of past due accounts. At the end of each year, if the loan is still not repaid, or if in cases of declaring bankruptcy or fraud to some extent, problematic loans are written off, that is, declared uncollectible.

It is deemed important that interest rates be studied as these are used by non-profit MFIs, such as ASA Philippines, in funding different costs in their operation. These include administration and transaction costs, cost of funds such as salaries, benefits, utilities, and rent. High recovery rates of MFIs positively contribute to their sustainability and growth.

***Microloan Repayment Scheme***

We study the repayment process of a basic microloan offered by ASA Philippines. We utilize GYE and come up with the model that reflects microloan repayment of ASA's clients.

On average, ASA provides loans amounting to PhP5,000 in a cycle of six months. Based on a 15% flat rate, the loan is repaid by 23 weekly installments of PhP250. The total of payments that equals 5,000 is represented by the following equation:

$$5000 = \sum_{j=1}^{23} 250\, e^{-\frac{jr}{52}} \,. \tag{7}$$

Solving (7) yields $r \approx 62\%$. Note that we wish to study microloan in a cycle of half a year. Without loss of generality, we consider studying the semi-annual interest rate compounded continuously. Note that the rate obtained in (7) corresponds to approximately 31% semi-annual effective interest rate. Utilizing GYE, we consider dividing half of a year into $\frac{N}{2}$ intervals. For the case when $N$ is odd, we use $\left[\!\left[\frac{N}{2}\right]\!\right]$. So for any loan amount $L$ and flat rate $r_f$, we have

$$L = \sum_{j=1}^{\frac{N}{2}} \frac{L(1+r_f)}{N/2}\, e^{-\frac{jr}{N/2}}. \tag{8}$$

Hence, for the case that $N = 46$ and $r_f = 15\%$, the semi-annual rate that solves (8) is $r \approx 27.4\%$.

***Repayment With and Without Compensation***

We study two cases wherein there is one delay in repayment based on ASA Philippines' process, that is, when the delay occurs at time $k, k \in \left(1, 2, \ldots \frac{N}{2}\right)$.

The first case is like what was discussed in the previous section, where a borrower misses to make one installment and is able to continue paying the week after. This means that the repayment of the loan will be extended for one more week. Hence, the loan will be fully repaid at time $T_{N/2} = \frac{N}{2} + 1$. We call this *repayment without compensation*. Utilizing GYE, we model repayment without compensation by the following equation.

$$\frac{N}{2} = \left(1 + r_f\right)\left(\sum_{j=1}^{k-1} e^{-\frac{jr}{N/2}} + \sum_{j=k+1}^{\frac{N}{2}+1} e^{-\frac{jr}{N/2}}\right), \tag{9}$$

where $r$ is the semi-annual interest rate. Denoting $r = r_k, k \in \left\{1, 2, \ldots \frac{N}{2}\right\}$, with $N = 46$ and $r_f = 15\%$ in (9), we obtain $r_k \in (0.25, 0.28)$.

The second case considers the following situation where ASA allows its borrowers to pay twice the installment the week after missing an installment. Hence, the loan will still be fully repaid on time. That is, the last installment would still be at time $T_{N/2} = \frac{N}{2}$. We call this *repayment with compensation*. Utilizing GYE, we suppose that a borrower delays just once, say at week $k$, and pays twice the installment amount, denoted $I$, at week $k + 1$. So, given the loan amount $L$ and flat rate $r_f$, we write

$$L = \frac{L(1+r_f)}{N/2} \left( \sum_{j=1}^{k-1} e^{-\frac{jr}{N/2}} + 2\left(e^{-\frac{r}{N/2}}\right)^{k+1} + \sum_{j=k+2}^{\frac{N}{2}} e^{-\frac{jr}{N/2}} \right). \quad (10)$$

Rewriting (10), we have

$$\frac{N}{2} = \left(1+r_f\right) \left( \sum_{j=1}^{k-1} e^{-\frac{jr}{N/2}} + 2\left(e^{-\frac{r}{N/2}}\right)^{k+1} + \sum_{j=k+2}^{\frac{N}{2}} e^{-\frac{jr}{N/2}} \right). \quad (11)$$

Solving (11) for each $k, k \in \left\{1,2, \ldots \frac{N}{2} - 1\right\}$, we obtain $r_k \in (0.2726, 0.2729)$.

Figure 2 plots the interest rates $r_k$ for repayment processes with and without compensation. Observe that for delay without compensation, the earlier the delay takes place, the more significant the difference of the rate is with that when a delay occurs at the last weeks of repayment. On the other hand, if there is compensation, the rate is maintained at approximately 27.3%. So clearly, there is almost no change in the rate that ASA receives when repayment with compensation is considered. This justifies ASA's decision to allow borrowers to compensate for delays in repayment. This approach in collecting installments clearly benefits not just ASA but also its borrowers. This process will incur no loss in interest for ASA. Borrowers, on the other hand, will be able to finish repayment on time and continue to be granted a loan. This is plausible as borrowers tend to return repeatedly for new loans to support their businesses, despite the seemingly exorbitant interest rate of 27% for six months. All these would suggest that ASA Philippines would indeed prefer repayment with compensation, as the interest payments they receive are maintained at a level where there is no delay in repayment.

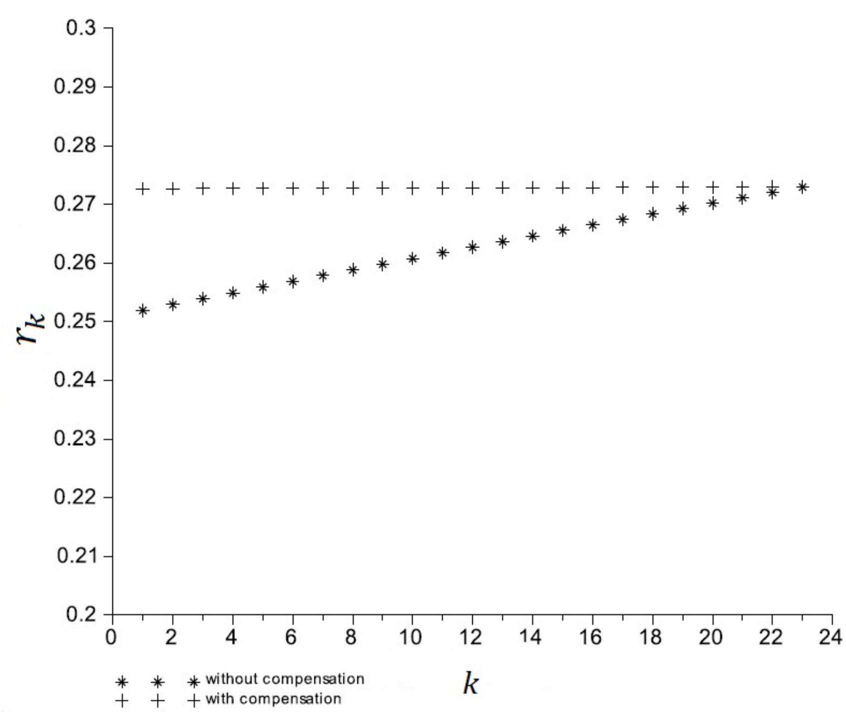


***Figure 2.*** Interest rates with and without compensation.

***Asymptotics of the Interest Rate***

Working on the hypothesis that GYE can be utilized to model microloan repayment, with and without compensation, of ASA borrowers, we obtain an approximation of the interest rates involved in both scenarios. Utilizing equations (9) and (11), we obtain the following results.

**Theorem 3:** For $k = 1,2, \dots \frac{N}{2}$, the implicit interest rate associated with a single delay in repayment, at week $k$, without compensation, has an order two expansion given by

$$r_k = \alpha_0 + \frac{\alpha_1}{N} + \frac{\alpha_2(k)}{N^2} + \frac{\epsilon_k(N)}{N^2}, \qquad (12)$$

with $\lim_{N \to \infty} \epsilon_k(N) = 0$, where $\alpha_0 = \beta_1, \alpha_1 = \beta_1^2 - 2\beta_2$ and $\alpha_2(k) = \frac{4}{3}\beta_1^3 - 2\beta_1\beta_2 - 4\beta_3(k)$. Expressions for $\beta_1$, $\beta_2$, and $\beta_3(k)$, and are as given in Theorem 3.1. The proof using a nonstandard approach is detailed by Santos (2017).

Figure 3 plots the actual and approximate values of the interest rate involved when there is one delay in repayment, without compensation, for the case of ASA Philippines. It shows that the values obtained using equation (12) closely approximate the actual values of the interest rate given repayment without compensation. This result is consistent with that obtained by Diener and Santos (2016). That is, if a borrower delays, ASA will receive interest that is lower than expected. But because ASA allows delays, the approximations will guide them on how much interest they lose and help them prepare for possible future losses.

**Theorem 4:** For $k = 1,2, \dots \frac{N}{2}$, the implicit interest rate associated with a single delay in repayment, at time $k$, with compensation, has an order three expansion given by

$$r_k = \alpha_0 + \frac{\alpha_1}{N} + \frac{\alpha_2}{N^2} + \frac{\alpha_3(k)}{N^3} + \frac{\epsilon(N)}{N^3}, \qquad (13)$$

with $\lim_{N \to \infty} \epsilon(N) = 0$, where $\alpha_0 = \beta_1, \alpha_1 = \beta_1^2 - 2\beta_2$ and $\alpha_2 = \frac{4}{3}\beta_1^3 - 2\beta_1\beta_2 - 4\beta_3$, and $\alpha_2 = 2\beta_1^4 + 4\beta_2^2 - 4\beta_1\beta_2 + 4\beta_1\beta_3 - 8\beta_4(k)$ where $\beta_1$ is the real nonzero solution of $1 - e^{-\beta_1} = \frac{\beta_1}{1+r_f}$,

$\beta_2 = \frac{\beta_1^2(3+\beta_1-r_f)}{2(\beta_1-r_f)}$,

$$\beta_3 = \frac{\beta_1(1+r_f)}{\beta_1 - r_f}\left\{\beta_1 - \frac{\beta_2^2}{\beta_1^2(1+r_f)} + \left(1 - \frac{\beta_1}{1+r_f}\right)\left(\frac{\beta_1^2}{6} + \frac{\beta_2^2}{2\beta_1} - \frac{\beta_1\beta_2}{2} + \frac{\beta_1^3}{4} + \frac{\beta_2^2}{\beta_1^2} - \frac{\beta_2}{2}\right)\right\}, \quad \beta_4(k) = \sigma k + \tau$$

where σ and t are constants given by $\sigma = -\frac{\beta_1^3(1+r_f)}{\beta_1 - r_f}$, and

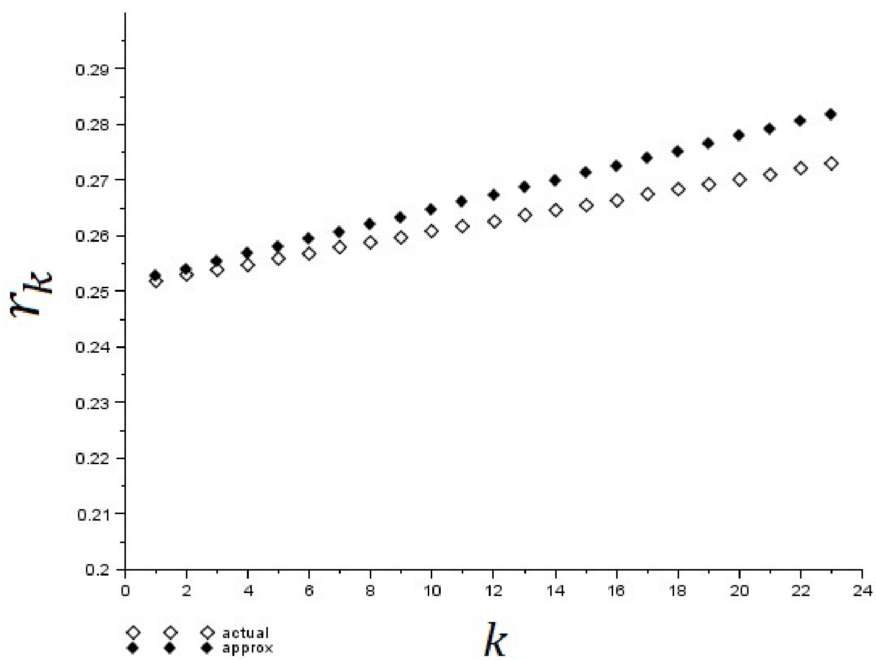


***Figure 3.*** Interest rates $r_k$, $k = 1,2, \dots, \frac{N}{2}$ given repayment delay without compensation, with $N = 46$ and $r_f = 15\%$.

$$\tau = \frac{\beta_1(1+r_f)}{\beta_1 - r_f}\left\{-\beta_2 - \frac{2\beta_2\beta_3}{\beta_1^2\,(1+r_f)} - \left(\frac{1-\beta_1+r_f}{\beta_1(1+r_f)}\right)\right.$$
$$\left(\beta_3 - \frac{\beta_1^2\,\beta_3}{2} + \frac{2\beta_3}{\beta_1} - \frac{\beta_1\beta_3}{2}\right) - \frac{2\beta_1\beta_2^3}{\beta_1^4\,(1+r_f)} + \left(1 - \frac{\beta_1}{1+r_f}\right)$$
$$\left(\frac{\beta_2^3}{\beta_1^3} + \frac{\beta_2^2}{2\beta_1} - \frac{\beta_1\beta_2}{3} + \frac{\beta_2^3}{2\beta_1^2} + \frac{\beta_2^2}{2} - \frac{\beta_1^2}{8} + \frac{\beta_1^3}{12} + \frac{\beta_2^1\,\beta_2}{12} - \frac{\beta_1^4}{24} -\right.$$
$$\left.\left.\frac{\beta_2}{2\beta_1} + \frac{\beta_2^3}{6\beta_1} - \frac{\beta_2^2\,\beta_1}{4} + \frac{\beta_2\beta_1^3}{8} - \frac{\beta_1^5}{48}\right)\right\}.$$

See Santos (2017) for the proof of the theorem.

Figure 4 plots the actual and approximate values of the interest rate involved when there is one delay in repayment, with compensation, for the case of ASA Philippines. Observe that each of both the actual and approximate values is approximately 27.3%. The mean of the differences of the approximate from the actual ones is approximately equal to 0.00167. This shows that the values obtained using equation (13) closely approximate the actual values of the interest rate given repayment with compensation, verifying that GYE can possibly predict values of the actual interest rates even for this setup.

To summarize the repayment process for the case of ASA Philippines, we have the following remark.

**Remark**: Consider microloan repayment with a single, unique delay.

A loan is to be repaid in 23 weekly installments based on a 15% flat rate.

2.a. When repayment without compensation is considered, a miss in installment would mean a delay in the subsequent installments, causing a week of delay in the full repayment of the loan. The actual interest rate, which ranges approximately from 25.19% to 27.29%, is less than the effective interest rate, which is approximately 27.4% when there is no delay in repayment.

2.b. When repayment with compensation is considered, a borrower can make up for a miss in installment by paying twice the installment the week after, repaying the loan in full when due. The real interest rate is maintained at the level of the effective interest rate because the values obtained range approximately from 27.26% to 27.29%.

3. Theorems 3 and 4 imply that GYE provides a good approximation of the actual interest rate values that ASA receives, given the time of delay.

## Conclusion

We have shown that GYE can be utilized to model microloan repayment in the Philippine setting. Using borrower repayment information obtained from ASA Philippines, we were able to study the case when there is a unique, single delay in repayment. We have shown

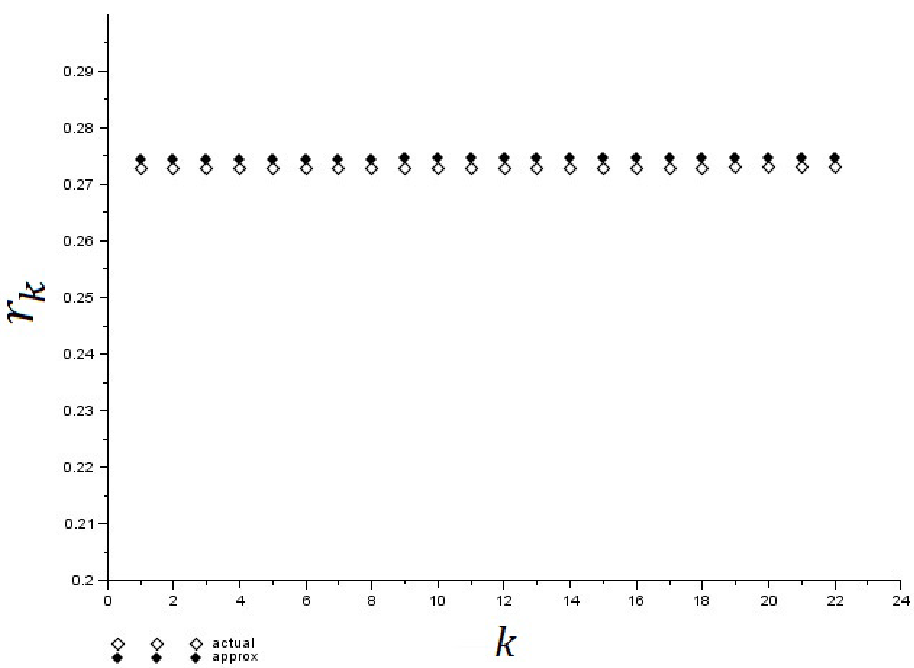


***Figure 4.*** Interest rates involved given repayment delay with compensation, with $N = 46$ and $r_f = 15\%$.

that actual interest rate values may be predicted through asymptotic expansions dependent on the time when the borrower misses to make an installment. It was shown that there exists a quasi-linear relationship between the rate $r_k$ and the time delay $k$.

These results suggest that in general, MFIs are better off having borrowers who delay in paying installments near the end and not at the start of their loan cycles. Borrowers who delay at the start of their repayment process could possibly burden microlenders as the effect on the interest rate is worse. Moreover, having a significant number of these types of borrowers would have a significant effect on ASA's source of funds for its operations.

Furthermore, we were able to study and compare situations involving a delay in repayment with and without compensation. This study gives us the insight that repayment with compensation, despite the delay, does not change the value of the interest rate benefiting both the microlender and the borrower.

As for future studies, one can explore other situations regarding the random repayment of microloans offered by ASA Philippines. It might be a challenge though, as the repayment rate ASA published is almost 100%, which means that its clients always tend to pay their loans on time. One can consider studying the behavior of the interest rate in cases such as compensation in random delays, which are not successive, accelerated payments, and of restructured loans. One can also gather and study data or information from other MFIs in the country to be able to compare loans and repayment processes.

Finally, as this study is limited to the case where interest rate is studied based on a single delay in repayment of an individual borrower, one can venture into studying other possible factors such as when there are two or more delays, when borrowers' savings are used to pay for the installments, joint liability in group lending, and other.

## References


Abbas, A. O., & Zhang, H. (2016). Empirical evidence impact of interest rate on loan repayment of microfinancial institution in Tanzania. *International Journal of Business and Economics Research*, *5*(5), 143–148. doi: 10.11648/j.ijber.20160505.12

Addae-Korankye, A. (2014). Causes and control of loan default/delinquency in microfinance institutions in Ghana. *American International Journal of contemporary Research, 4,* 36–45.

Bersales, L. (2017, July 4). Farmers, fishermen, and children consistently posted the highest poverty incidence among basic sectors – PSA. Retrieved from http://psa.gov.ph/poverty-press-releases

Brau, J., & Woller, G. (2004). Microfinance: A comprehensive review of the existing literature. *The Journal of Entrepreneurial Finance and Business Ventures, 9,* 1–28.

Conchada, M. P., & Tiongco, M. (2017). Making social health insurance and micro-savings programs work for the informal sector in the Philippines. *DLSU Business & Economics Review, 27(1),* 107–123.

Daley, S., & Sautet, F. (2005). *Microfinance in action: The Philippine experience* (Mercatus Policy Comment No. 1). Accessed from https://ssrn.com/abstract=1264013

Diener, F., & Diener, M. (Eds.). (1995). *Nonstandard analysis in practice*. Germany: Springer, Verlag/Berlin/Heidelberg.

Diener, F., Diener, M., Khodr, O., & Protter, P. (2009). *Mathematical Models for Microlending*. Accessed from https://www.researchgate.net/publication/228457730_MATHEMATICAL_MODELS_FOR_MICROLENDING

Diener, M., & Santos, J. (2016). Randomness of interest rates in microcredit. *Matimyás Matematika, 39,* 1–16.

Felloni, F., Seibel, H. D., & Cornejo, A. (2005). Mainstreaming Grameen banking. *Small customers, big market: Commercial banks in microfinance.* 21–25. New Delhi, India: TERI Press

Geron, M. (2011). *Microfinance industry report – Philippines – 2010*. Philippines: Microfinance Council of the Philippines, Inc.

Guntz, S. (2011). *Sustainability and profitability of microfinance Institutions* (Tech. Report Research Paper No. 4). Germany: Center for Applied International Finance and Development (CAIFD), University of Applied Sciences in Nuremberg.

Habaradas, R., & Umali, M. (2013). *The microfinance industry in the Philippines: Striving for financial inclusion in the midst of growth* (Working Paper No. 2013-05). Manila: DLSU Center for Business Research and Development. doi: 10.2139/ssrn.2568021

Hossain, M., & Diaz, C. (1997). Reaching the poor with effective microcredit: Evaluation of Grameen Bank replication in the Philippines. *Philippine Sociological Review, 45,* 89–121.

Karlan, D., & Zinman, J. (2011). Microcredit in theory and practice: Using randomized credit scoring for impact evaluation. *Science, 332*(6035), 1278–1284. doi: 10.1126/science.1200138

Keisler, H. (1986). *Elementary calculus: An infinitesimal approach*. California: Prindle, Weber & Scmidt, Inc.

Mauk. P. (2013). Modélisation mathémathique du microcredit. [Mathematical Modeling of Microcredit] (Unpublished doctoral dissertation). Université de Nice-Sophia Antipolis, France.

Nobel Media AB. (2006). Grameen Bank facts. Retrieved from https://www.nobelprize.org/prizes/peace/2006/grameen/facts/

Nelson, E. (1977). Internal set theory: A new approach to nonstandard analysis. *Bulletin of the American Mathematical Society, 83,* 1165–1193.

Rosenberg, R. (1999). *Measuring microcredit delinquency: Ratios can be harmful to your health* (Tech. Report Occasional Paper No. 3). Washington DC: The Consultative Group to Assist the Poorest (CGAP).

Rosenberg, R. (2002). *Microcredit interest rates* (Occasional Paper No. 1). Washington, DC: The Consultative Group to Assist the Poorest (CGAP). Retrieved from https://www.cgap.org/research/publication/microcredit-interest-rates

Santos, J. (2017). *On the generalized Yunus equation for microcredit* (Unpublished doctoral dissertation). Institute of Mathematics, University of the Philippines-Diliman.

Schelzig, K. (2005). *Poverty in the Philippines: Income, assets, and access.* Retrieved from https://www.adb.org/sites/default/files/publication/29763/poverty-philippines.pdf

Snelling, C. (2012, February 9). Microcapital brief: Microfinance Credit Bureau, MiDAS, launched by Microfinance Institutions in Philippines. Retrieved from https://www.microcapital.org/microcapital-brief-microfinance-credit-bureau-midas-launched-by-microfinance-institutions-in-philippines/

Tedeschi, G. A. (2006). Here today, gone tomorrow: Can dynamic incentives make microfinance more flexible? *Journal of Development Economics, 80,* 84–85.

*The effect of interest rate on loan repayment in microfinance institutions – A case study of Lapo Micro Finance Bank*. (n.d.). Retrieved from https://www.uniprojectmaterials.com/banking-and-finance/the-effect-of-interest-rate-on-loan-repayment-in-microfinance-institutions-a-case-study-of-lapo-micro-finance-bank/project-topics-materials-for-final-year-students

Yunus, M., & Jolis, A. (1999). *Banker to the poor: Micro-lending and the battle against world poverty*. New York: Public Affairs.