Teaching Business Calculus: Methodologies, Techniques, Issues, and Prospects

Frederick A. Halcon
Business Management Department, De La Salle University-Manila

Calculus, as a subject matter, is an area in mathematics that deals with limits, rates of change, derivatives and integrals. University education often incorporates calculus in the curriculum with emphasis given to its applications in real-life situations. It is a common notion that calculus, as a course or subject, is intricate and complex. However, treating business calculus in the most elementary manner and making abstract concepts more concrete can prove to be effective teaching strategies inside the classroom. This study attempts to review certain literature so as to equip instructors of business calculus with the necessary teaching pedagogy intended to eventually help their students perform better in class. Aside from these objectives, this study also draws on the researcher's seven years of teaching business and mathematics, both in the secondary and tertiary levels.

Keywords: Business calculus, teaching methodologies, computer based mathematics education.

INTRODUCTION

The study of mathematics and its applications to business is a broad field by itself. In the College of Business and Economics of De La Salle University-Manila, for example, in order for students to successfully hurdle the demands of business calculus as a course, they need to be equipped with the rudiments of algebra and analytic geometry. Without this prerequisite knowledge and skills, students are more likely to fail their business calculus.

Failure in a math class may depend on several variables. First, there is the student's capacity to cope with the demands of the subject matter. In this case, too, it is assumed that the student is equipped with necessary prerequisite skills. And because most students have the preconceived notion that mathematics is difficult, this may greatly affect their study habits from the very beginning. Second, the instructor may fail to capture the student's interest because of his teaching methods. Instructors may need to resort to more student-centered activities than the usual "chalk-and-talk" or lecture type of teaching. The instructor also has to consider the time constraint and the scope of the subject in a usual three-unit course. Third, the learning environment may not be conducive to studying. Features of the student's physical environment (e.g. poor ventilation, crowded classrooms, noise) are factors to be considered, too.

What teaching strategies can an instructor of business calculus use to ensure maximum learning by his students? What modifications can be done on the usual "chalk-and-talk" or lecture for the teacher to be more effective in courses on business calculus? What are the different issues and prospects that may arise from incorporating improvements in classroom teaching? These are the questions that this study aims to answer.

THE BASICS OF EDUCATION

The Lesson Plan

The lesson plan is "the blueprint of the lecture." A well-constructed lesson plan is necessary to ensure the effective transmission of concepts from instructor to student. In the elementary and secondary levels of education, the construction of a well-written lesson plan is imperative for teachers; for which reason, these are checked regularly by subject coordinators. However, in the tertiary level, instructors are no longer required to construct a lesson plan, but they should still keep a "mental" lesson plan as a script for the day's lecture or lesson.

A typical lesson plan contains five parts namely (1) objectives, (2) subject matter, (3) procedure, (4) evaluation, and (5) assignments.

Objectives. Good objectives are usually written using the SMART principle, that is, objectives should be Specific, Measurable, Attainable, Realistic, and Time-bound. Also consider the three types of objectives: cognitive, psychomotor, and affective.

Subject Matter. This section includes the title of the lesson as well as the materials needed. This part of the plan also states the different references used in the lesson.

Procedure. The lesson usually starts with the motivation part. It is in this section where the instructor introduces the lesson through a short activity. It aims to gain the attention of the student as well as sustain it. After this comes the lesson proper.

Evaluation. This section of the lesson plan includes strategies that will check the student's understanding of the lesson. In math courses, this is usually in the form of seatwork, boardwork or recitation.

Assignments. To reinforce learning, the instructor may want to give exercises as homework for the students. These assignments serve to check up o whether or not the delivery of the lesson for the day was effective. Assignments are usually checked the next meeting before a new lesson is introduced; they may also serve as the motivation for the next lesson. Students find this practice beneficial since it refreshes their memory, given the fact that topics and skills required in mathematics build on previous lessons.

The format of the lesson plan is usually stipulated either by the school in general, or by the specific department/unit where a teacher is based. Or, if the teacher is given a free hand in constructing the lesson plan, its format may depend on the nature and demands of the subject matter.

Methodologies of Teaching

Traditionally, mathematics relies on the lecture type of teaching where the instructor presents and elaborates on definitions and concepts, and gives examples. Since most concepts in mathematics are best learned by practice (e.g. solving for derivatives which represent marginal functions), instructors assign seatwork, homework, and boardwork to check the progress of students. Boardwork is a decent way of giving recitation points to students, and is an appropriate venue for students to offer to and discuss alternative solutions with their classmates. It also allows for sharing of resources, since many math problems can be solved by using more than technique. Moreover, giving a chance to students to explain their answers enhances their oral communication skills and makes them familiar with mathematical and business jargon (e.g. "rationalize," "factor," "integrate," "differentiate," "marginal cost, "breakeven," "consumers' surplus," etc.). To reinforce social skills and values, instructors also use group activities to familiarize students with group dynamics.

PROSPECTS IN TEACHING BUSINESS CALCULUS

With the advent of new technology, instructors may now deviate from the usual "chalk-and-talk" method. There are certain freeware available online that the instructor may use to complement the classroom lecture. For example, the software *Graphmatica* can be used to draw graphs simply by typing an equation on its toolbar and by clicking "Enter". Graphing by point plotting is cumbersome, especially if the given equation is complex and of higher degrees. With the use of software, students will be able to relate concepts, technology, and business principles to each other.

As for teaching strategies, instructors may also opt to use games when introducing a new lesson as part of the lesson plan's motivation section. To make business calculus more interesting and appealing to students, instructors may use concepts in corporate social responsibility and link these to business practices.

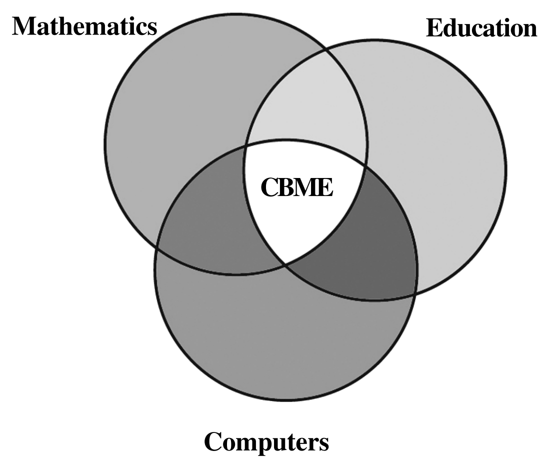
For the more efficient dissemination of information, instructors may opt to set up their own electronic groups (e-groups) hosted by websites such as Yahoo! and Google. These e-groups can serve as the repository of lecture notes, assignments, and exercises, which students can readily access. Furthermore, this saves the instructor a considerable amount of time writing lessons on the blackboard. The e-group can even be used as a message board on which instructors can post their announcements and reply to their students' queries.

Exploring Computer Technology in Optimization Problems

The use of computer technology in teaching mathematics is explored in an emerging field in education called *Computer Based Mathematics Education* (CBME). CBME pertains to the method of enriching mathematics education through the use of computers. The analysis of CBME in detail requires of the researcher a comprehensive understanding of the following fields: (1) mathematics, (2) education, and (3) computer technology. CBME lies in the intersection of these three fields. Computers are used in a number of ways in education, including tutorials, hypermedia, simulation, drill and practice, and more (*Computer Based Mathematics Education*, n.d.).

Figure 1 shows Computer Based Mathematics Education as an intersection of sets. The domain of CBME is simply the intersection of the spheres of computer technology, education, and mathematics. It is a new discipline by itself, and as such, is marked by innovations and developments. It is also influenced by innovations in education such as behaviorism, constructivism, and by rapid developments in computer technology.

***Figure 1*. Computer Based Mathematics Education.**



Source: http://en.wikipedia.org/wiki/Image:CBME11.gif

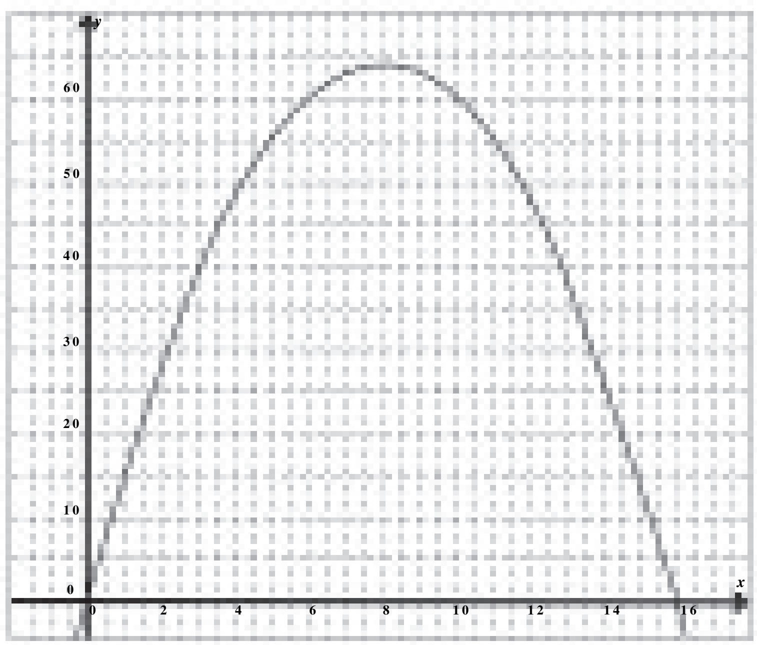
Businessmen and economists are primarily interested in optimization. Vasquez (1997) describes optimization problems as problems that involve the maximization of some benefit or the minimization of some cost. In economic analysis, firms always attempt to maximize revenue, profit, and output. They also attempt to minimize cost.

From the point of view of a rational consumer, he/she always aspires to maximize utility. Given the definition of economics as a branch of social science, it must be emphasized that human wants are unlimited while resources are limited.

A good example of a graph that could be best applied to business is the quadratic function. Quadratic functions take the form $y = a_2x^2 + a_1x + a_0$ where a_2, a_1, and a_0 are constants, and a_2 ‘“ 0 (Ramirez, Lial, & Miller, 1981). Should a_2 be positive, the graph will be a parabola opening upward. Otherwise, it opens downward. An example of the quadratic function is one given by $y = 16x - x^2$. In analytic geometry, this is a parabola opening downward with vertex at (8, 64), and with x-intercepts at the origin, (0, 0), and at (16, 0).

Parabolas concave downward serve as good examples of total revenue curves, total utility curves, and total product curves. First, firms wish to maximize their revenue and production, while consumers wish to maximize their utility. A parabola attains its maximum or minimum point at its vertex. At the vertex, revenue or production level, or utility is maximized. Parabolas concave downward also exhibit diminishing returns. Prior to reaching its vertex, the parabola concave downward is increasing at a decreasing rate. In the case of utility, each additional unit of x consumed contributes less and less to total utility, say y. Specifically, a consumer experiences diminishing marginal utility. A sample graph of a parabola opening downward as an output of *Graphmatica* is shown in Figure 2.

***Figure 2*. Graph of a Parabola Opening Downward**



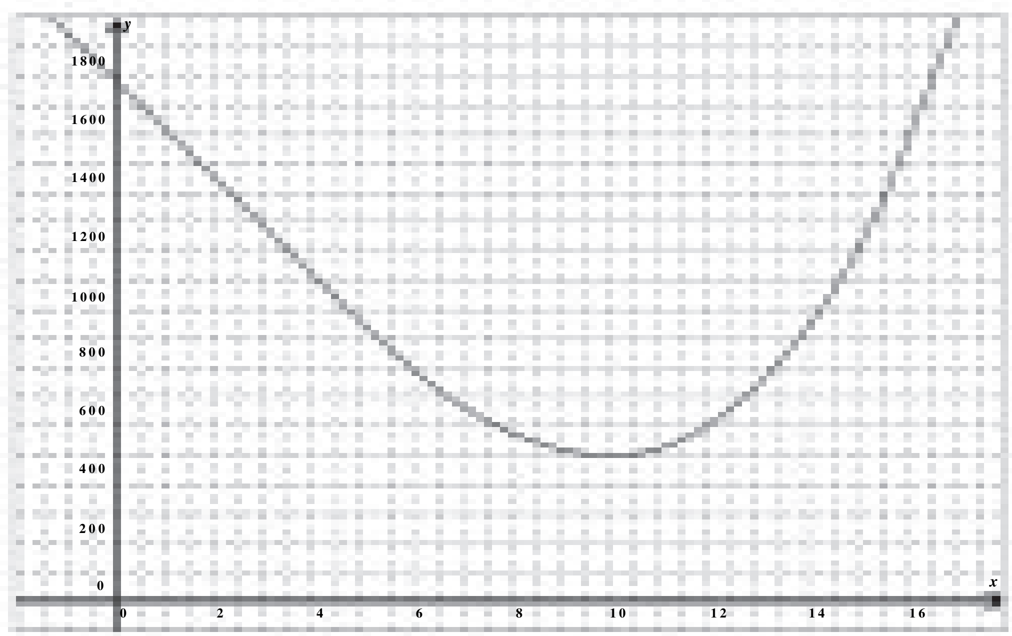
Most problems in business and economics deal with graphs falling on the first quadrant of the Cartesian plane since negative quantities and prices are meaningless. In most analyses, prices are plotted along the y-axis while quantity is plotted along the x-axis. It is important to emphasize to students that total revenue curves, total utility curves, and total product curves pass through the origin. No output or no production often implies no sales and no revenue. Similarly for production, no input is tantamount to no output. Analogously for utility, no units consumed amounts to no satisfaction. Developed by ksoft, *Graphmatica* is a shareware that enables its users to graph equations with ease. It is particularly helpful for students learning algebra and calculus.

It is also equally important to emphasize the profit maximizing behavior of most firms. Samuelson and Nordhaus (1992) stated that maximum profit will occur when output is at that level where the firm's marginal revenue is equal to its marginal cost. Firms increase their output as long as its marginal revenue is less than its marginal cost. This implies that each additional unit generates more revenue than it costs. When discussing profit maximization, students should have already mastered the different techniques of differentiation since the concepts of marginal revenue and marginal cost rely heavily on the concept of the derivative.

As for cost minimization, an ideal candidate would be the cubic function $y = x^3 - 7x^2 - 160x + 1800$, where x is expressed in quantity produced and y is in Pesos. Since firms wish to minimize their cost of production, it is best for them to operate at the relative minimum point of the cost curve. The relative minimum point is a relative extremum which is the lowest among the points in the neighborhood of that point (Vasquez, 1997). This is in contrast with the vertex of a parabola concave downward which is a relative maximum point.

With the aid of *Graphmatica*, it can be readily seen that the relative minimum point occurs at (10, 500). The firm should, therefore, produce 10 units for a minimum cost of P500. Prior to reaching its relative minimum point, it can be said that the firm's total cost is decreasing. Thus, the firm is enticed or encouraged to produce more. However, to the right of the relative minimum point, the firm's total cost is already increasing. Hence, the firm will be forced to produce less output. The cost curve as an output of *Graphmatica* is shown in Figure 3.

***Figure 3*. Graph of a Cost Curve**



However, great caution must be taken by the instructor since heavy reliance on computer technology may compromise the learning skills of the students. It should be made clear that students should understand the behavior of graphs and how graphs are manually plotted. Graphing software may, however, be used as a checking aid. In optimization, it is necessary that the students learn the basic rules of differentiation. It is also equally important to emphasize that students should learn the properties of a graph of a certain equation, such as whether it is increasing or decreasing in an interval, and concave or convex on an interval, rather than just relying on the graphing software.

The use of graphing software is also very advantageous in analytic geometry where students study how certain equations are graphed. Quadratic equations in two variables depict the different conic sections – circle, ellipse, parabola, and hyperbola – as graphs. In addition, students are also exposed to the different types of functions (including their graphs and their properties), and are expected to master the different general and standard forms of such quadratic equations. Instructors can interactively illustrate the different properties of their graphs through the use of graphing software. Functions of higher degrees, or polynomial functions, are even more tedious to graph manually. Thus, graphing software comes in handy.

Students must be informed that equations representing economic functions and the relationship between variables are estimated and analyzed using different statistical tools, more specifically regression and correlation. Regression and correlation are discussed in courses in business statistics and in econometrics.

Incorporating Corporate Social Responsibility in Business Calculus

Stoner, Freeman, and Gilbert (1995) stated that corporate social responsibility focuses on what an organization does that affects the society in which it exists. It is inevitable that the firm's pursuit of profit maximization is bound to affect groups of people called stakeholders, both internal and external. Internal stakeholders include those people that are not necessarily part of the firm's environment but for whom a manager in the firm is responsible. On the other hand, external stakeholders include the firm's suppliers and customers, labor unions, financial institutions, competitors, the government, special interest groups, etc. These groups of people influence the firm's STEP environment, which is composed of Social, Technological, Economic, and Political variables.

Firms may maximize profit and yet end up exploiting their natural and human resources. Consumers, meanwhile, may maximize utility but waste personal resources. Some firms even compromise the quality of their goods and services in the pursuit of profit maximization. To some extent, this even gives rise to negative diseconomies. Samuelson and Nordhaus (1992) describe external diseconomies as situations in which production or consumption imposes uncompensated costs on other parties. A classic example of an external diseconomy would be the pollution emitted by factories. Though such emission is a by-product of their operations, it causes adverse effects on human beings in general. These groups of people are "uncompensated" as they bear the effects of industrialization, mass production, and economic progress. In addition, Samuelson and Nordhaus reiterate that injured parties are not at all compensated.

It is expected of instructors of business calculus to make abstract concepts in business and economics, such as demand, supply, and utility functions, more concrete. The desire to gain profit at the expense of consumers may be explained by hoarding. The desire to gain more in the anticipation of a disaster may be explained by panic- buying, and is linked to the demand function. With regard to consumer behavior, the consumption of an additional unit of good beyond the saturation point (which represents the maximum utility) until the consumer experiences continuous disutility (where marginal utility is negative) may be attributed to greed. All these topics can be explored by future researchers, but instructors can already encourage students to examine the interplay of business, economics, morality, and public administration in light of social responsibility. It is evident that whatever a person does affects other groups of people. Corporate social responsibility can also be explored further in courses on business ethics, ethical consumerism, and corporate governance.

PRESENTATION AND ANALYSIS OF DATA

Utilizing the methods of delivering lectures in business calculus explained above, this writer gathered data on the final grades of students as a measure of teaching effectiveness. In the College of Business and Economics of De La Salle University-Manila, the passing raw score in mathematics courses is 60 percent, which is equivalent to a grade of 1.0. It is assumed that the instructor employed different teaching methods with some degree of consistency in all of his business calculus classes, and that all students were treated fairly. Quizzes and departmental final exams are, as a regular practice, administered to students in all classes, and these are standardized in terms of content and levels of difficulty, despite the use of different sets. It is also assumed that the instructor complied with the requirements stipulated in the course syllabus given to the students at the beginning of the term.

Commerce Calculus (COMCALC) is the third mathematics course taken by CBE students whose ID number is 105 and below. Its prerequisite courses are Commerce Mathematics 1 & 2 (COMATH1 & COMATH2), both of which are courses on basic and advanced algebra. A student's final grade is composed of 60 percent average quiz score, 30 percent score in the departmental final exam, and 10 percent class standing. Business Calculus (BUSCALC), meanwhile, is the third mathematics course taken by CBE students whose ID number is 106 and up. Its prerequisite courses are Commerce Algebra (COMALGE) and Business Mathematics (BUSMATH). A student's final grade is composed of 65 percent average quiz score, 30 percent score in the departmental final exam, and 5 percent class standing. The mathematics courses of the College of Business and Economics are offered by the Interdisciplinary Business Studies (IBS) Office. In these courses, the standard grading scale shown in Table 1 is used.

Table 1
Standard Grading Scale for Mathematics Courses in CBE

Numerical Grade	Percentage
4.0	96 – 100
3.5	90 – 95
3.0	84 – 89
2.5	78 – 83
2.0	72 – 77
1.5	66 – 71
1.0	60 – 65
0.0	Below 60

Tables 2 through 7 show the different calculus classes handled by the instructor, along with the term and the school year, the number of students enrolled in each class, the number of failures and its corresponding percentage equivalent, the average class final grade in percentage, and its corresponding numerical grade equivalent, and the standard deviation in the class. The average class final grade is simply the arithmetic mean of the final grade in percentage of the class. The standard deviation is used to show the consistency of the class performance. A lower standard deviation generally implies a more consistent performance while a higher standard deviation implies a dispersed performance. The instructor started teaching calculus in the 3rd Term of SY 2004-2005. No sampling procedure was utilized; all of the instructor's calculus classes from the beginning comprise the population of the study. Therefore, there is no need to perform any inferential statistical methods but simply descriptive statistics. The instructor used measures of central tendency and variation to explain the data gathered.

To verify the effectiveness of this writer/researcher's methodology, the results of the instructor's evaluation at the end of the term were also assessed. A score of 4.5000 and above is considered to be "Outstanding" and a spreading index of 3 indicates more or less a "grouped" or consistent response from students. A higher spreading index would mean that the ratings given by students to the instructor are diverse and inconsistent. The evaluation instrument is deemed to be indigenous as it was developed by the Institutional Testing and Evaluation Office (ITEO) of De La Salle University-Manila. The instrument covers the instructor's teaching skills (e.g. ability to maintain a harmonious teacher-student relationship), mastery of the subject matter, and ability to organize and manage the classroom well.

Table 2
Results from 3rd Term, SY 2004-2005

Class	No. of Students	No. of Failures	Average Class Final Grade	Standard Deviation
COMCALC C37	38	2 (5.26%)	82.70% (2.5)	11.77
COMCALC C38	40	0 (0.00%)	85.42% (3.0)	9.06
COMCALC C39	40	0 (0.00%)	82.00% (2.5)	8.56

Instructor's ITEO Evaluation: 4.5376 (Outstanding); Spreading Index: 3 (Grouped)

Table 3
Results from Summer Term, SY 2005-2006

Class	No. of Students	No. of Failures	Average Class Final Grade	Standard Deviation
COMCALC M71	22	0 (0.00%)	87.87 %(3.0)	7.14
COMCALC M73	18	0 (0.00%)	85.97% (3.0)	11.72

Instructor's ITEO Evaluation: None

Table 4
Results from 2nd Term, SY 2006-2007

Class	No. of Students	No. of Failures	Average Class Final Grade	Standard Deviation
COMCALC C31	40	5 (12.50%)	70.75% (1.5)	11.67
COMCALC C32	40	2 (5.00%)	74.00% (2.0)	11.49

Instructor's ITEO Evaluation: 4.7002 (Outstanding); Spreading Index: 3 (Grouped)

Table 5
Results from 3rd Term, SY 2006-2007

Class	No. of Students	No. of Failures	Average Class Final Grade	Standard Deviation
COMCALC C31	44	5 (11.36%)	68.46% (1.5)	8.49
COMCALC C32	40	4 (10.00%)	70.19% (1.5)	8.71

Instructor's ITEO Evaluation: None

Table 6
Results from Summer Term, SY 2006-2007

Class	No. of Students	No. of Failures	Average Class Final Grade	Standard Deviation
BUSCALC M71	27	1 (3.70%)	76.79% (2.0)	13.67
BUSCALC M72	20	1 (5.00%)	72.59% (2.0)	9.65

Instructor's ITEO Evaluation: None

Table 7
Results from 1st Term, SY 2007-2008

Class	No. of Students	No. of Failures	Average Class Final Grade	Standard Deviation
BUSCALC C31	39	1 (2.56%)	81.09% (2.5)	10.03
BUSCALC C32	36	4 (11.11%)	71.05% (1.5)	11.27
BUSCALC C33	26	3 (11.54%)	71.54% (2.0)	11.28

Instructor's ITEO Evaluation: 4.7449 (Outstanding); Overall Spreading Index: 3 (Grouped)

Table 8
Results from 2nd Term, SY 2007-2008

Class	No. of Students	No. of Failures	Average Class Final Grade	Standard Deviation
BUSCALC C31	42	1 (2.38%)	79.90% (2.5)	10.32
BUSCALC C32	41	5 (12.20%)	66.03% (1.5)	7.79
BUSCALC C33	39	7 (17.95%)	67.32% (1.5)	8.83

Instructor's ITEO Evaluation: 4.5795 (Outstanding); Overall Spreading Index: 4 (Somewhat Spread)

It can be seen that by incorporating improvements in the methodology of the instructor, few or no failures can be seen at the end of each grading period with the students earning satisfactory final grades in the course. The maximum number of failing students in a class was five. Excluding summer classes, the number of enrollees in the instructor's calculus classes is usually 40, the standard class size. (The maximum allowable class size is 45 students.) Therefore, it can be inferred that the teaching methodology of the instructor is effective. In addition to this, the instructor has been consistently rated "Outstanding" by the students in the terms he was evaluated. The instructor's evaluations also show a consistent spreading index of 3, which implies a grouped response from his students, indicating uniformity.

RECOMMENDATIONS AND AREAS FOR FUTURE STUDY

To empirically test the improvement of students' performance, future researchers and instructors may assess the examination results of one group undergoing traditional teaching methods and another group in which student-centered and technology-aided techniques are used. In addition, future researchers may also compare the results of a class taught by instructors using these methods with those taught by other instructors using other methods, traditional or otherwise. This will determine if there are any significant differences in terms of overall student performance. This will also further validate whether or not the methodologies studied above are effective on all students, assuming homogeneity in their learning abilities.

Using the data presented above, future researchers can also perform statistical tests on the average class final grade and standard deviations to see whether or not there are any significant differences among them. Through this, one can also make inferences from the students' ability to learn calculus and consequently, identify areas for improvement.

For future research purposes, instructors of the same subject (i.e. business calculus) may wish to explore the domain of Computer Based Mathematics Education (CBME) further to learn how this model can be used in the teaching of business courses.

Another area for further research is the tradeoff between profit maximization and the practice of corporate social responsibility since the latter runs counter to profits and is a potential cost driver. To what extent should firms practice corporate social responsibility at the expense of profit maximization?

REFERENCES

Computer based mathematics education. (n.d.) Retrieved October 30, 2007, from *http://en.wikipedia.org/wiki/Computer_Based_Mathematics_Education*

Ramirez, R. A., Lial, M. L., Miller, C. D. (1981). *Essential calculus: with applications*. San Juan: Academe Publishing House.

Samuelson, P. A., & Nordhaus, W. D. (1992). *Economics* (14th ed.). Singapore: McGraw-Hill Publish.

Stoner, J., Freeman, R. E., & Gilbert, D. R. (1995). *Management* (5th ed.). Englewood Cliffs, N.J.: Prentice Hall.

Vasquez, E. E. (1997). *Calculus, with analytic geometry for business and economics*. Manila: GIC Enterprises.