RESEARCH ARTICLES

Forecasting the Term Structure of Philippine Interest Rates Using the Dynamic Nelson-Siegel Model

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Abstract: The three-factor Nelson-Siegel model is a widely used model for forecasting the term structure of interest rates. Several extensions have recently been proposed. Even for the original model, different methods of treating the parameters have been shown. Ultimately, what works best depends on the data used to estimate the parameters. In this paper, the original three-factor model with fixed shape parameter was applied to forecast the term structure using market data from the Philippines. Instead of giving a pre-determined model for the latent factors, the best time series model for them was searched using standard statistical tools. Based on the historical data, the best model for each latent factor is of the form ARMA(p,q)+eGARCH(1,1). The dependence structure of these parameters was considered in generating their future values. This was carried out by finding the joint distribution of the residuals via appropriate copula. Results show that forecast of interest rates for different tenors is reliable up to the near future. For an active market, this is good enough since the models for the parameters can be adjusted as new information comes in.

***Keywords*:** Yield curve, Nelson-Siegel, forecasting, copula, time series

JEL Classification: G17, E47, C53

In recent years, the influx of new financial products in the Philippines, including financial derivatives, has become inevitable. Some local banks have been granted limited authority by the Bangko Sentral ng Pilipinas (BSP) to engage in specified derivatives transactions, while some others are still in the stage of preparing to apply for a license for such transactions. The bond market is also on the rise with an increasing participation of corporate bond issuers. Amidst these developments is the need for sound risk management structure and reliable models for the pricing and valuation of financial products.

Financial risk management and valuation of financial securities require sound and reliable mathematical models. Loss estimates and pricing are based on current and forecast of underlying economic variables such as interest rates and currency exchange rates. Currently, several financial institutions in the Philippines are either using vendor-developed systems, which are essentially "black box," or have developed their internal models which are mostly based on popular or classical models. There is indeed a challenge for financial institutions, particularly in the banking industry, to develop their own models which are more reliable, relevant, and suited to the market data.

Public debt management also requires sound mathematical models for forecasting the evolution of macroeconomic factors such as interest rates, exchange rates, and primary budget deficit/surplus. Finding an optimal debt strategy means determining a mix of debt instruments that minimizes cost to the government, in terms of interest payment, subject to a prudent risk level. It may then be used to guide the policy makers in creating a short-term or medium-term borrowing plan, which will hopefully improve the domestic debt market due to enhanced predictability and transparency related to public debt.

Forecasting the term structure of interest rates has created a huge literature spanning several decades. Some of these make use of mean-reverting stochastic models, such as the models of Vasicek (1977) and Cox, Ingersoll, and Ross (1985). Another class which has gained popularity among government policy makers due to its good fit to observed term structure includes Nelson and Siegel's (1987) model and its variations.

This paper aims to present a method for forecasting the term structure of interest rates that is applicable to the Philippine market data. It is based on the framework developed by Diebold and Li (2006), which is a reinterpretation of the model introduced by Nelson and Siegel (1987). Although several extensions are available, the original three-factor model will be used. The procedure, however, will be modified and the results of the implementation will be highly dependent on the data used.

The Nelson-Siegel model is a popular model for the term structure of interest rates. It was introduced by Nelson and Siegel (1987) as a class of parametric functions to capture the range of shapes typically associated with yield curves. Diebold and Li (2006) reformulated it as a dynamic factor model and used it to forecast the yield curve. Several extensions and variations of the model have been introduced with the goal of improving the out-of-sample forecasts (see for example Christensen, Diebold, & Rudebuscha, 2011; Exterkate, Dijk, Heij, & Groenen, 2013; Koopman, Mallee, & Van der Wel, 2010).

This paper will focus on the original three-factor Nelson-Siegel model. Following the method in Diebold and Li (2006), the latent factors, which will be referred to throughout this paper as *beta parameters*, will be estimated from historical data by fitting the Nelson-Siegel equation to the yield curves over time and assumed to follow a time series model. However, instead of assuming independent univariate AR(1) processes for all the beta parameters, the best time series model will be searched using standard statistical tools. The model that will be used is of the form ARMA(p,q)+eGARCH(1,1) or ARMA for the mean equation and Exponential GARCH for the variance equation. Moreover, for forecasting, the dependence structure of the beta parameters will be considered by using an appropriate copula to obtain their joint distribution.

The remainder of this paper proceeds as follows: The next section discusses the Nelson-Siegel equation for the yield. We then describe the data and the general procedure that will be implemented, to be followed with the results of the implementation including the time series found for the beta parameters, the copula used to obtain their joint parameters, and the forecast performance of the model. Finally, we provide the conclusion.

The Nelson-Siegel Model

Let $p_t(\tau)$ be the price at time t of a zero-coupon bond that pays 1 peso at time $t+\tau$, that is, τ is the time to maturity of the bond in years. Let $y_t(\tau)$ be the corresponding continuously compounded nominal yield. This yield is also referred to as a zero rate. Then

$$p_t(\tau) = e^{-\tau y_t(\tau)} \tag{1}$$

If $f_t(\tau)$ is the instantaneous forward rate with maturity at time $t + \tau$ contracted at time t, then

$$f_t(\tau) = -\frac{\partial \ln p_t(\tau)}{\partial \tau} \tag{2}$$

It follows that

$$y_t(\tau) = \frac{1}{\tau}\int_0^{\tau} f_t(u)du \tag{3}$$

Yield curve at a particular point in time t is a curve that describes the spot interest rates $y_t(\tau)$ for different maturities τ. These curves are typically monotonic, humped or S-shaped (Nelson & Siegel, 1987). To generate this range of shapes, a parsimonious model introduced by Nelson and Siegel assumes that the forward rate follows the equation

$$f_t(\tau) = \beta_{0t} + \beta_{1t} e^{-\tau/\lambda_t} + \beta_{2t} \frac{\tau}{\lambda_t} e^{-\tau/\lambda_t} \quad (4)$$

where β_{0t}, β_{1t}, β_{2t}, and λ_t are constants and $\lambda_t \neq 0$. Note that $x(\tau) = f_t(\tau)$ is a solution to the second-order linear ordinary differential equation

$$x'' + \frac{2}{\lambda_t} x' + \frac{1}{\lambda_t{}^2} x = \frac{\beta_{0t}}{\lambda_t{}^2} \quad (5)$$

From Equation (3),

$$y_t(\tau) = \frac{1}{\tau} \int_0^\tau \left[\beta_{0t} + \beta_{1t} e^{-u/\lambda_t} + \beta_{2t} \frac{u}{\lambda_t} e^{-u/\lambda_t} \right] du \quad (6)$$

Evaluating the integral at the right, we obtain the Nelson-Siegel model for the yield curve

$$y_t(\tau) = \beta_{0t} + \beta_{1t} \left(\frac{1 - e^{-\tau/\lambda_t}}{\tau/\lambda_t} \right) + \beta_{2t} \left(\frac{1 - e^{-\tau/\lambda_t}}{\tau/\lambda_t} - e^{-\tau/\lambda_t} \right) \quad (7)$$

The parameters β_{0t}, β_{1t}, and β_{2t} are called **factors** and their coefficients are called **factor loadings**. The factor loading of β_{0t} is 1, a constant that does not decay to zero even as $\tau \to +\infty$. Thus, it has significant contribution to the yield for any maturity; hence, it is referred to as the long-term factor. The factor β_{1t} is called the short-term factor because its factor loading, $\frac{1-e^{-\tau/\lambda_t}}{\tau/\lambda_t}$, has significant contribution to the value of $y_t(\tau)$ at shorter maturities (smaller values of τ). Moreover, it decreases to 0 as $\tau \to +\infty$. Lastly, β_{2t} is referred to as the medium-term factor because its factor loading, $\frac{1-e^{-\tau/\lambda_t}}{\tau/\lambda_t} - e^{-\tau/\lambda_t}$, starts at 0, increases, then decays to 0. More precisely, it approaches 0 as $\tau \to 0^+$ and as $\tau \to +\infty$. Its significant contribution to the yield is in the medium term to maturity.

The three factors are also interpreted in terms of level, slope, and curvature of the yield curve (Diebold & Li, 2006). The factor β_{0t} governs the level of the curve since $\lim_{\tau \to +\infty} y_t(\tau) = \beta_{0t}$ and an increase in β_{0t} increases all yields equally. Thus, this factor is responsible for parallel yield curve shifts. The factor β_{1t} is related to the yield curve slope. If the slope of the yield curve is defined as $\lim_{\tau \to +\infty} y_t(\tau) - \lim_{\tau \to 0^+} y_t(\tau)$, then it is equal to $-\beta_{1t}$. Lastly, β_{2t} is related to the curvature, defined by Diebold and Li as $2y_t(24) - y_t(3) - y_t(120)$, where maturity is given in months.

Estimation of Parameters

In Equation (7), the parameter λ_t is referred to as the **shape parameter**. For each t, when λ_t is specified, the model becomes linear in the parameters β_{0t}, β_{1t} and β_{2t}. Let $Y_\tau^t = y_t(\tau)$, $X_{1\tau}^t = \frac{1-e^{-\tau/\lambda_t}}{\tau/\lambda_t}$, and $X_{2\tau}^t = \frac{1-e^{-\tau/\lambda_t}}{\tau/\lambda_t} - e^{-\tau/\lambda_t}$. Then Equation (7) becomes

$$Y_\tau^t = \beta_{0t} + \beta_{1t} X_{1\tau}^t + \beta_{2t} X_{2\tau}^t \quad (8)$$

Specific value of τ gives specific values of $X_{1\tau}^t$ and $X_{2\tau}^t$.

In practice, a record of daily datasets is collected, each dataset indexed by a particular day number t and indicating the interest rates $y_t(\tau)$ for different maturities τ. For each t, the parameters β_{0t}, β_{1t}, and β_{2t} in Equation (8) are estimated using, for example, the ordinary least squares (OLS) method for multiple linear regression.

Diebold and Li (2006) used the same value of λ_t for all t. Thus, a series of estimates $\{\beta_{0t}, \beta_{1t}, \beta_{2t}\}$ was obtained based on this single value of λ_t. Others (Nelson & Siegel, 1987; Annaert, Claes, De Ceuster, & Zhang, 2012) considered a range of values of λ_t for each t. The method was referred to as **grid search**. In this case, the value of λ_t and the set of estimates $\{\beta_{0t}, \beta_{1t}, \beta_{2t}\}$ were chosen based on the highest R^2. Thus, different datasets could have different shape parameters λ_t. Furthermore, the issue of multicollinearity was considered and ridge regression was implemented as a remedy (Annaert et al., 2012).

Data and Procedure

Data

Historical data of interest rates were initially obtained from the website of Philippine Dealing and Exchange Corporation or PDEx (originally http://www.pdex.com.ph, but data now available thru the website of the parent company Philippine Dealing Systems, http://www.pds.com.ph/). The rates for tenors at most one year were zero rates so these were used. The data for tenors longer than one year, however, were not zero rates so the corresponding zero rates were obtained from Bloomberg (2013). Note that the zero rates can also be computed using a method called bootstrapping.

PDEx is one of the two major exchanges in the Philippines, the other being the Philippine Stock Exchange (PSE). PDEx is a venue for trading fixed-income and other securities, most of which are government securities. From daily trading, PDEx calculates and publishes Philippine Dealing System Treasury Reference Rates such as PDS Treasury Reference Rate AM (PDST-R1) and PDS Treasury Reference Rate PM (PDST-R2). Both PDST-R1 and PDST-R2 benchmarks are intended to become the source of reference rates for the repricing of loans, securities, derivative transactions, and other interest rate sensitive instruments to be issued. They are also intended to become the bases for market valuation of Government Securities and other Philippine-peso-denominated fixed income securities.

In this work, the PDST-R2 rates from January 2, 2008 to December 25, 2013 were chosen. The PDST-R2 rate is the weighted average of the yields from done transactions of the set of benchmark securities for each tenor up to 4:15 p.m. For simplicity, weekly data were extracted by taking the rates Wednesday of each week. Index t is attached to each week.

Procedure

In using the Nelson-Siegel model to forecast the term structure, three major steps were implemented in this work. The first step was to run multiple linear regression in the form of Equation (8) for each dataset t to obtain the beta parameters β_{0t}, β_{1t}, and β_{2t} . This step required a prior choice of value or values of the shape parameter λ_t. The next step was to assume a model for the time series of beta parameters obtained. This model was then used to forecast future values of these parameters. The last step was to plug in the future values of the beta parameters and the same shape parameter in Equation (7) to obtain forecast of interest rates $y_t(\tau)$.

For the first step, regression using a fixed shape parameter across all datasets and using a grid search were both considered initially. For the grid search, the values of λ_t ranged from 2.000000 to 40.000000 with increments of 0.000001 for each t. The λ_t together with the set of beta parameters β_{0t}, β_{1t}, and β_{2t} that produced the highest R^2 for each t were taken. It was noted that quite a number of datasets had very high R^2 but the beta parameters obtained were not realistic, that is, the values were very far from the trend in the majority of datasets. This problem did not appear in the other method that used a fixed shape parameter across all datasets, which will be referred to for the rest of the paper as **fixed lambda method**. To determine this fixed shape parameter, first, for each lambda in the same range used in grid search, beta parameters β_{0t}, β_{1t}, and β_{2t} were estimated and corresponding R^2 were taken for each t. The average over all t was then calculated. Finally, the shape parameter that produced the highest average was chosen. Because the results of the fixed lambda method were more stable, the succeeding steps proceeded from these.

It is important to note that the results in either method, grid search or fixed lambda, were dependent on the choice of range of values of λ_t.

Having chosen the fixed shape parameter and the resulting beta parameters for each t, the second step was to fit a univariate time series model for each of $\{\beta_{0t}\}$, $\{\beta_{1t}\}$ and $\{\beta_{2t}\}$. First, each series was tested for stationarity and was found to have a unit root. Hence, each series was transformed by differencing to remove the unit root. Each series also showed strong presence of ARCH effect. The best model obtained for each differenced series followed ARMA(p,q)+eGARCH(1,1). Then the error terms of the estimated models for the betas were considered as a random vector whose joint probability distribution function (pdf) was to be determined by the copula method. Finally, random numbers from the joint pdf of the error terms were generated. These were then used in the final step, forecasting the term structure of interest rates.

Implementation and Results

Fitting the Yield Curve

A total of 313 datasets of zero rates from January 2, 2008 to December 25, 2013 were considered. Each dataset gave the interest rates in hundred basis points for maturities 1 month (Mo), 3 Mo, 6 Mo, 1 year (Yr), 2Yr, 3Yr, 4Yr, 5Yr, 6Yr, 7Yr, 8Yr, 9Yr, 10Yr, 15Yr, 20Yr, and 30Yr. Figure 1 shows sample yield curves.

Before performing linear regression, all rates were first converted to continuously compounding rates. For the processing of data, from estimation to time series modelling of the beta parameters, only the rates from January 2, 2008 to December 26, 2012 were used. Thus, 261 datasets were considered. The remaining

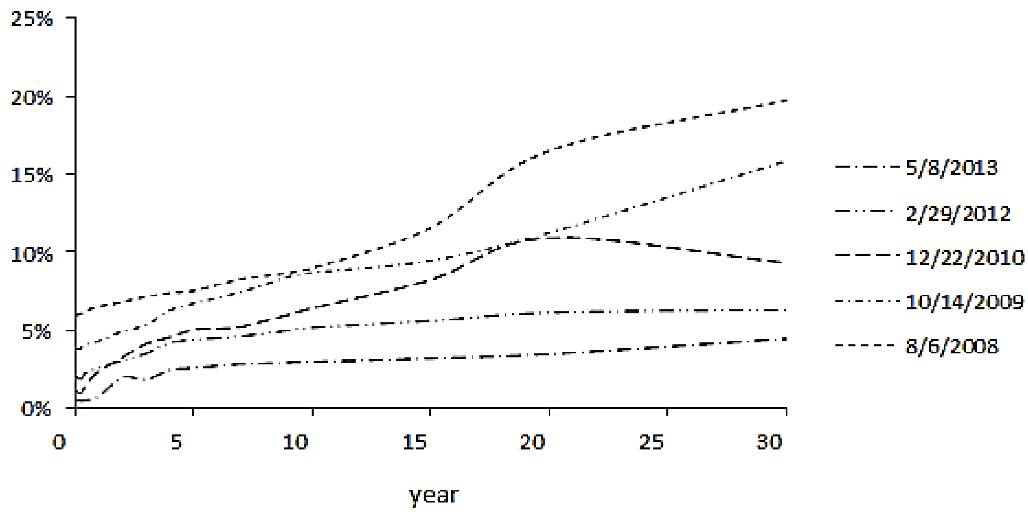

Figure 1. Actual yield curves for selected dates.

Table 1. *Minimum, Maximum, Mean Values, and Standard Deviations of R2 and Beta Parameters*

	Min	max	mean	standard deviation
R^2	80.26%	99.72%	96.22%	3.04%
β_0	0.02242	1.49527	0.17384	0.18941
β_1	-1.41810	0.01044	-0.13947	0.17833
β_2	-1.80766	0.24361	-0.04411	0.25549

historical data were to be used to check the reliability of the forecast.

The fixed lambda method described earlier produced $\lambda_t = 9.807527$. To summarize the results of linear regression, the values of R^2 and beta parameters of Equation (8) for the different datasets are described in Table 1.

Forecasting of interest rates requires forecast of the beta parameters. Thus, the next step was to find appropriate time series models for them.

Time Series Models for the Beta Parameters

All statistical procedures were implemented using the software R. First, univariate time series models were fitted on the estimated betas. As seen in Figures 2 and 3, presence of non-stationarity and of conditional heteroscedasticity were apparent. These observations were justified by KPSS test (for the actual series) and ARCH-LM test (for the differenced series), giving all significant *p*-values. These are standard procedures in detecting these features of most financial time series.

The best models for all three differenced series $d\beta_i, i = 0,1,2$, were ARMA for the mean equation and Exponential GARCH for the variance equation. In particular, based on the Bayesian Information Criterion (BIC), both $\{d\beta_0\}$ and $\{d\beta_1\}$ followed MA(1)+eGARCH(1,1) models, while $\{d\beta_2\}$ followed ARMA(1,1)+eGARCH(1,1).

The estimated models were given by the following equations. Let $d\beta_{i,t} = \beta_{i,t} - \beta_{i,t-1}$, $i = 0,1,2, t = 1,2,\dots,T$, where T is the sample size. Then

$$d\beta_{i,t} = \mu_i + \varphi_i d\beta_{i,t-1} + a_{i,t} + \theta_i a_{i,t-1} \quad (9)$$

$$a_{i,t} = \sigma_{i,t}\varepsilon_{i,t} \quad (10)$$

$$\ln(\sigma_{i,t}^2) = \omega_i + \alpha_i a_{i,t-1}^2 + \gamma_i\left(\frac{|a_{i,t-1}|}{\sigma_{i,t-1}} - E\left(\left|\frac{a_{i,t-1}}{\sigma_{i,t-1}}\right|\right)\right) + \kappa_i \ln(\sigma_{i,t}^2) \quad (11)$$

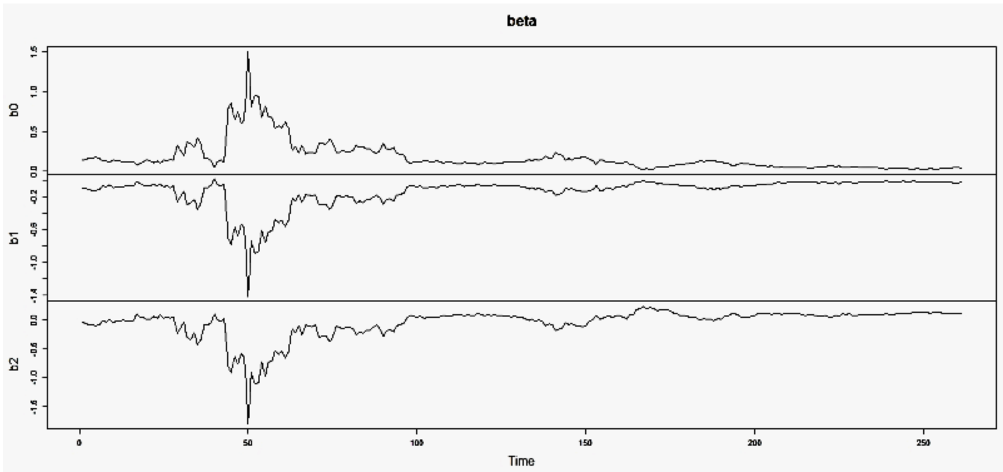

Figure 2. Time series plots of the estimated betas.

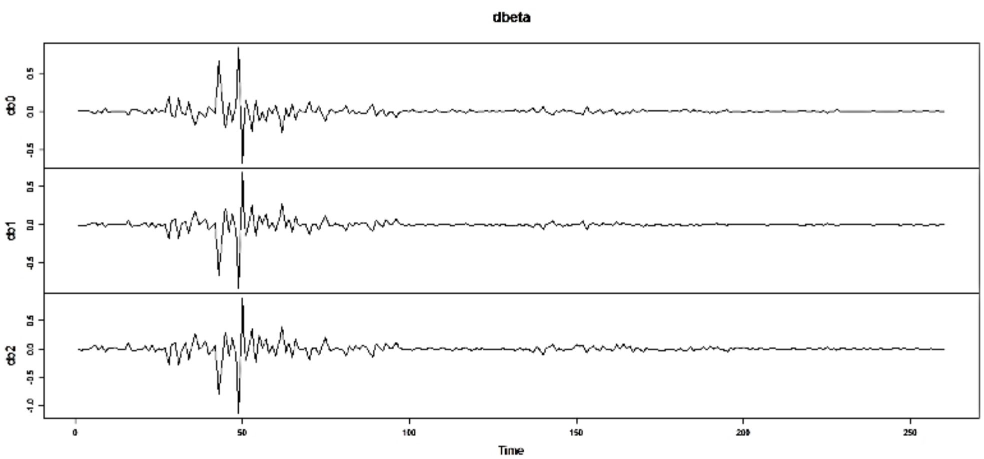

Figure 3. Time series plots of the differenced betas.

where $E\left(\left|\frac{a_{i,t-1}}{\sigma_{i,t-1}}\right|\right) = E\left(\left|\varepsilon_{i,t}\right|\right)$ when $\sqrt{\frac{\nu}{\nu-2}}\varepsilon_{i,t} \sim t_2$. The coefficients and corresponding standard errors are given in Table 2.

With T as the sample size, the estimated initial values to be used in forecasting are given in Table 3.

The assumed distribution of the error $\varepsilon_{i,t}$ was the t distribution with 2 degrees of freedom or t_2. Such distribution captures the heavy-tailedness of the series since it has infinite variance.

In generating future values of the beta parameters using the formulas in Equations (9) to (11), their underlying dependence structure based on their observed values were considered. This required estimating the distribution of the random vector $(\varepsilon_0, \varepsilon_1, \varepsilon_2)$ consisting of the error variables. As assumed in the time series models, the distribution of each error variable was t_2. The dependence structure of $(\varepsilon_0, \varepsilon_1, \varepsilon_2)$ was obtained using a copula function.

Using Copula for the Joint Density of the Error Variables

First, pseudo-observations defined by

$$U_i = \frac{R_i}{T+1} \tag{12}$$

were created, where T is the sample size and R_i is the rank of the i^{th} observation. These pseudo-observations were used to check the association among the error variables and in estimating the parameters of the

Table 2. *Estimated Parameters of ARMA + eGARCH Models and Corresponding Standard Errors*

	i=0	i=1	i=2
μ_i	-0.000385	0.000386	-0.000247
	(0.000480)	(0.000426)	(0.000409)
φ_i	0	0	-0.152738
			(0.043782)
θ_i	-0.215126	-0.199544	-0.099143
	(0.053564)	(0.050340)	(0.039429)
ω_i	-0.095836	-0.105876	-0.079393
	(0.036828)	(0.049583)	(0.051335)
α_i	0.390405	-0.276128	-0.316096
	(0.207581)	(0.188705)	(0.206423)
γ_i	0.537282	0.626099	0.558905
	(0.277760)	(0.343980)	(0.360170)
κ_i	0.984873	0.983536	0.983487
	(0.006315)	(0.008814)	(0.008566)

Table 3. *Estimated Initial Values Based on the Fitted ARMA + eGARCH Models*

	$a_{i,T}$	$\sigma_{i,T}$
i=0	-0.00492075	0.01729929
i=1	0.00121345	0.01571196
i=2	0.00144197	0.02802909

copula model. The pairwise scatterplots of the pseudo-observations are shown in Figure 4.

Based on the scatterplots of the pseudo-observations, the first error variable had strong negative association to the second and third error variables, while the second and the third had strong positive correlation. Notice that the associations among the error variables showed a symmetric-type dependence. That is, the dependence on the tails was almost the same as the distributions on the middle part of the distributions. One model that captures such symmetry in association between two uniform random variables, as depicted by the scatterplots, is the elliptical family. This family contains the Gaussian and *t* copulas.

Estimation and goodness of fit test were performed for the Gaussian copulas. The estimation procedure was based on maximum pseudo-likelihood estimation proposed by Genest, Ghoudi, and Rivest (1995) and the goodness of fit test was based on the empirical copula proposed by Genest, Rémillard, and Beaudoin (2009). The results are shown in Table 4. Note that $\rho_{i,j}$ is the Pearson's correlation coefficient of the i^{th} and j^{th} pseudo-observations.

Table 4. *Results of the Goodness-of-Fit Test Under the Gaussian Copula Assumption*

Estimated parameter	S_n	p-value
$\rho_{0,1}$= -0.981	0.0047	0.9296
$\rho_{0,2}$= -0.959		
$\rho_{1,2}$= 0.909		

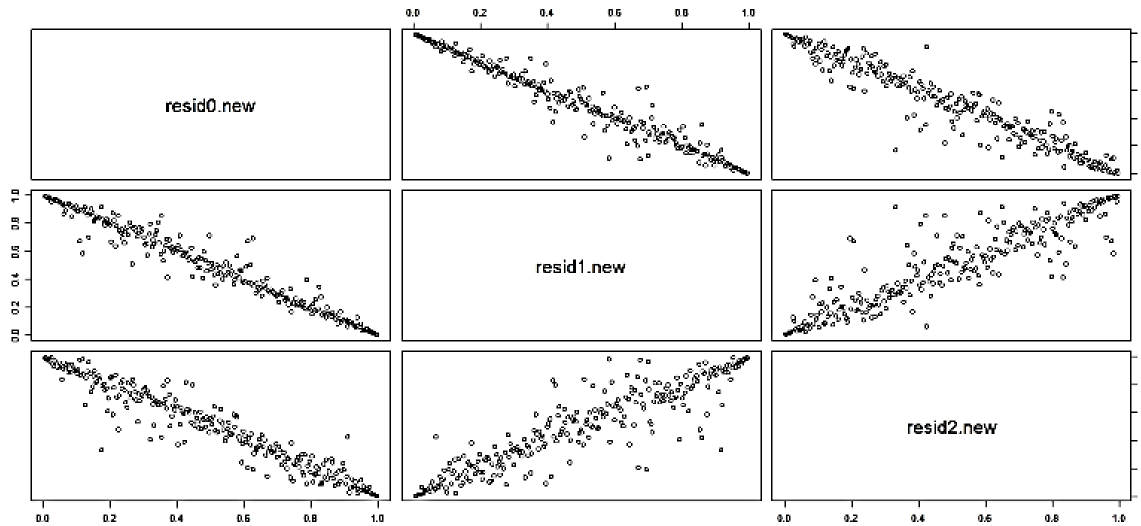

Figure 4 . Pairwise scatterplots of the pseudo-observations.

With p-value of 0.9296, it seemed that the Gaussian copula was adequate to capture the dependence of the pseudo-observations. The estimated correlation matrix was given by

$$\begin{pmatrix} 1 & -0.981 & -0.959 \\ -0.981 & 1 & 0.909 \\ -0.959 & 0.909 & 1 \end{pmatrix}$$

These estimated parameters were then used to simulate trivariate data with t_2 margin and normal copula to capture the dependence.

From Equations (9) to (11) and the random numbers generated from the distribution of the random vector (ε_0, ε_1, ε_2), future values of the beta parameters were computed and used in the Nelson-Siegel equation

$$y_t(\tau) = \beta_{0t} + \beta_{1t}\left(\frac{1-e^{-\tau/\lambda_t}}{\tau/\lambda_t}\right) + \beta_{2t}\left(\frac{1-e^{-\tau/\lambda_t}}{\tau/\lambda_t} - e^{-\tau/\lambda_t}\right) \tag{13}$$

with λ_t = 9.807527. The initial values were those obtained for the December 26, 2012 dataset: $\beta_{0,0}$ = 0.042670232, $\beta_{0,1}$ = 0.035735575, and $\beta_{0,2}$ = 0.0107586694. The randomness of the beta parameters were due to the residuals $\varepsilon_{i,t}$., i = 0,1,2.

Forecasting

To validate the model, a fraction of the available data, from January 2 to December 25, 2013 (52 Wednesdays), was taken out. The goal was to compare the forecasts and the actual data taken out.

First, 10,000 triples of random numbers ($\varepsilon_{0,1}$, $\varepsilon_{1,2}$, $\varepsilon_{2,1}$) from the joint distribution of the random vector (ε_0, ε_1, ε_2) of the residuals were produced. Each triple was substituted into the time series model to obtain the values of β_{01}, β_{11}, β_{21}. Thus, 10,000 triples (β_{01}, β_{11}, β_{21}) were obtained. Then the median of each beta was taken to obtain one triple (β_{01}, β_{11}, β_{21}). This median triple was then used in Equation (7) to get the yield curve of week 1 (January 2, 2013), y_1 (τ), where τ is the maturity.

For the second week (t = 2),10,000 triples of random numbers ($\varepsilon_{0,2}$, $\varepsilon_{1,2}$, $\varepsilon_{2,2}$) from the joint distribution of the random vector (ε_0, ε_1, ε_2) were again produced. These and the betas of previous weeks were plugged in the time series model to obtain 10,000 triples (β_{02}, β_{12}, β_{22}). Again, the median of each beta was taken to get one triple (β_{02}, β_{12}, β_{22}). This median triple was used in the Nelson-Siegel equation to get the yield curve y_2 (τ) for week 2 (January 9, 2013). This process was continued to find the yield curve for week t, $t > 2$. Notice that medians of beta parameters were taken first before computing the yield y_t (τ). An alternative approach is to compute for each t the yield y_t (τ) using each of the 10,000 triples ($\varepsilon_{0,t}$, $\varepsilon_{1,t}$, $\varepsilon_{2,t}$) then take

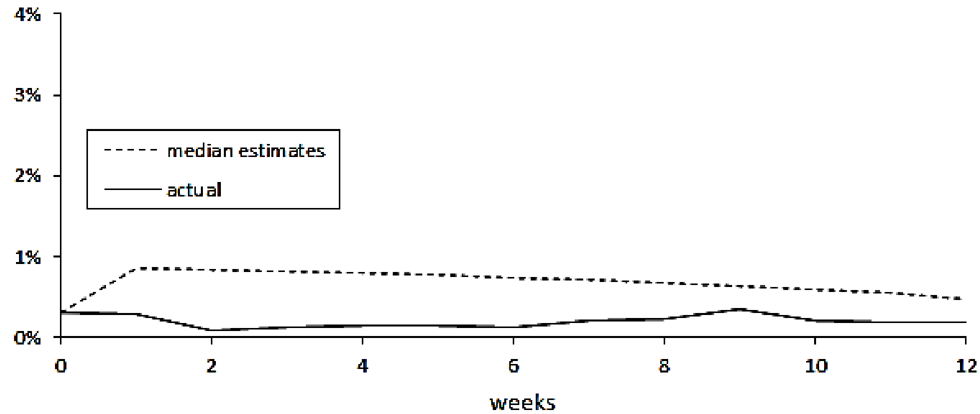

Figure 5. Forecast for 3-month tenor.

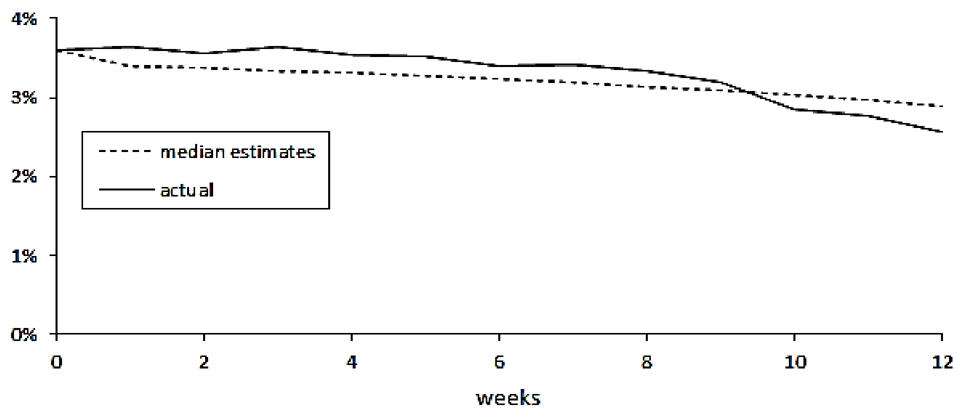

Figure 6. Forecast for 5-year tenor.

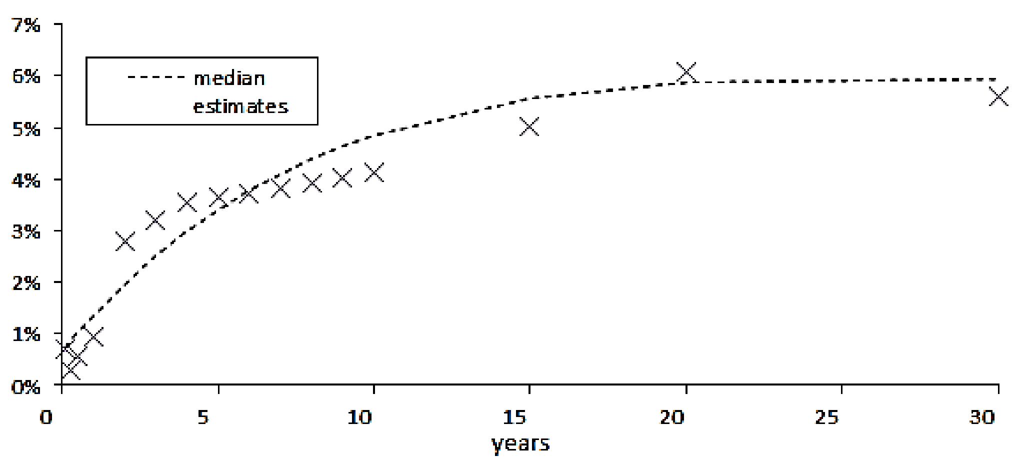

Figure 7. Yield curve forecast for week 1.

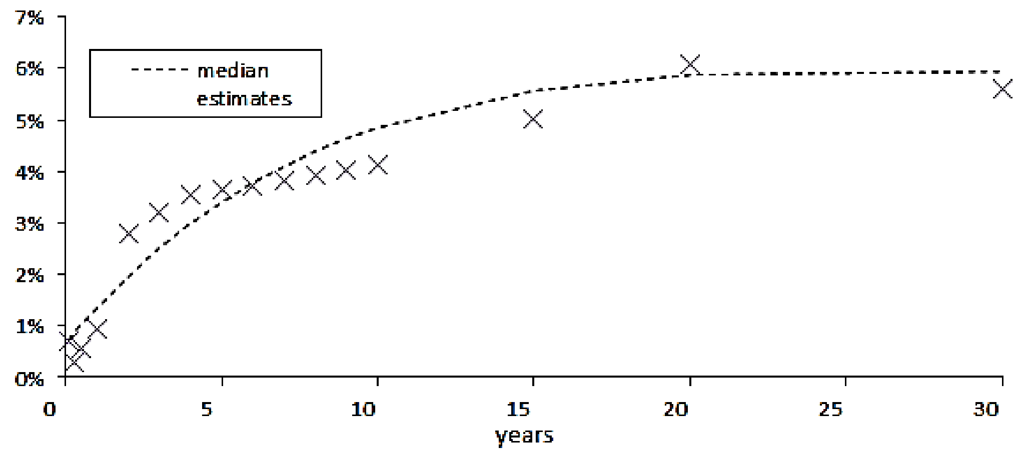

Figure 8. Yield curve forecast for week 9.

the median $y_t(\tau)$ for each τ. This, however, will still resort to taking a single triple of beta parameters from previous weeks; otherwise, the process will be computationally expensive.

Figures 5 to 8 present sample forecast of interest rates per tenor and per week along with the actual data for comparison.

It was observed that the forecasting accuracy is weakened as one goes farther into the future. This is a limitation of any time series models. Thus, instead of 1-year forecast, only 12 weeks were considered.

The root mean square errors (RMSE) per tenor and per week are shown in Tables 5 and 6. In can be seen that the best forecast is for 5-year yield, with only 6 bps RMSE, and the least accurate is for 10-year yield with 27 bps RMSE.

Conclusion and Recommendation

In this paper, the three-factor Nelson-Siegel model was applied to forecast the term structure of interest rates using Philippine market data. Using a fixed shape parameter, the equation for the yield became linear in the beta parameters. Such equation was fitted to each historical dataset of zero rates thereby producing a series of beta parameters. An appropriate time series model was then obtained for each beta parameter. Based on the historical data, the best model for each beta was of the form ARMA(p,q)+eGARCH(1,1). It is important to note that a different set of historical data may produce a different time series model for the beta parameters.

Forecast of interest rates was based on the assumption that yield curve would follow the Nelson-Siegel equation with the same shape parameter as produced from historical data and with beta parameters generated from the time series model. The dependence structure of the beta parameters was considered in generating their future values. This was carried out by finding the joint distribution of the error variables via appropriate copula.

Results showed that forecast of interest rates for different tenors, short, medium or long, was relatively good up to the next three months. From then on, the accuracy weakened. This showed that the model, wherein parameters were based on historical data, could be reliable only for the near future. For an active market, this is good enough since the models for the parameters can be adjusted every trading day.

As mentioned earlier, a model for forecasting the term structure of interest rates is essential in designing an optimal debt strategy for the government. This paper presents one such model and how it can be applied to the Philippine market data. Forecast of interest rates will help the policy makers determine the appropriate mix of debt instruments that will minimize cost subject to a prudent risk level.

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Table 5. *RMSE Per Tenor*

	1 Mo	**3 Mo**	**6 Mo**	**1 Yr**	**2 Yr**	**3 Yr**	**4 Yr**	**5 Yr**
RMSE	0.1106%	0.1556%	0.1489%	0.1071%	0.1821%	0.1460%	0.1310%	0.0635%
	6 Yr	**7 Yr**	**8 Yr**	**9 Yr**	**10 Yr**	**15 Yr**	**20 Yr**	**30 Yr**
RMSE	0.0830%	0.1399%	0.1921%	0.2378%	0.2743%	0.2636%	0.1507%	0.1655%

Table 6. *RMSE Per Week*

Week	1	2	3	4	5	6
RMSE	0.1236%	0.1280%	0.1163%	0.1100%	0.1265%	0.1389%
Week	7	8	9	10	11	12
RMSE	0.1407%	0.1491%	0.1321%	0.1640%	0.1719%	0.2246%

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