



## Dynamical Modelling of Competing Firms

Brett N. Limsenkeng<sup>1,\*</sup> and Robert C. Roleda<sup>1</sup>

<sup>1</sup>Physics Department, De La Salle University, 2401 Taft Avenue, Manila

\*Corresponding Author: [brett.limsenkeng@gmail.com](mailto:brett.limsenkeng@gmail.com)

**Abstract:** Competition is the heart and soul of enterprise. Majority of the everyday goods we consume are products of a small number of competing firms (oligopolies). Competition of firms each producing a differentiated product is usually modelled using either the Cournot model—the quantity approach—or the Bertrand model—the price approach, both of which provide alternative viewpoints on the supply side of the market. Both models are however static in the sense that they are used to find equilibrium or the steady state prices. In this study, a logistic adjustment process for price setting was applied as extension to the Bertrand model; this, as a result, yields dynamical equations similar to the Lotka-Volterra equations (LVE). This set of equations describe the “movement” of prices over time given specific values as proxies for brand strength, customer loyalty, level of costs, and other factors. Although LVE was initially used to model predator-prey systems, and then modified to describe ecological competition, this study uses the same concept to describe the case of economical competition. The resulting Lotka-Volterra dynamics and flow maps are then analyzed and plotted using Mathematica; after which they are analyzed for stability and for different situations, e.g. starting price, brand strength, customer loyalty. The results coincide with the static steady state case and exhibit a Nash equilibrium as postulated by mathematician and game theorist John Forbes Nash. Several conclusions arise from this competition model. First, the resulting Lotka-Volterra dynamics tend towards a steady state similar to the static case; second, price levels depend on brand strength, customer loyalty, costs, and not on company adaptability; and finally, the conditions by which companies will run out of business and exit the market are provided by the model.

**Key Words:** competition; duopoly; Bertrand; Lotka-Volterra; Nash