



The Game Chromatic Number of Some Classes of Graphs

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Abstract: The *coloring game* is played by two players called Alice and Bob on the vertices of a graph G as follows: Using colors from a set $X = \{1, 2, \dots, k\}$ of k distinct colors, the players take turns in

assigning colors to the vertices of G such that no two adjacent vertices will receive the same color. The two players play alternately with Alice always moving first. The game ends when either all the vertices have been colored, or it is no longer possible to color an uncolored vertex. Alice wins in the first case, and Bob wins otherwise.

The *game chromatic number* of a graph G is a graph invariant representing the smallest number of colors for which Alice has a guaranteed winning strategy. The game chromatic number of a graph G is denoted by $\chi_g(G)$.

The game chromatic number of various classes of graphs, including trees, cactuses and cartesian products of various types of graphs, have been determined. In this paper, the game chromatic number of some common classes of graphs, such as paths, cycles, complete graphs, complete bipartite graphs, star graphs, fans, wheels, Cartesian product graphs, and the Petersen graph, are determined. Some of these results are established by using the relationship between the game chromatic number with another graph invariant called the *game coloring number*. A previously published result is also modified.

Key words: coloring game; game chromatic number; cartesian product graph; graph invariant; marking game; game coloring number