

Statistical inference using hypothesis-testing methods

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This article is envisioned to form a base upon which a full-blown exhaustive discussion of hypothesis-testing could take place.

Statistical inference is the process of generating conclusions or generalizations about certain characteristics of the population on the basis of information obtained from a sample. Statistical inference can be conducted in two ways: by interval estimation of parameters and by testing the statistical hypothesis on parameters. These are actually reverse processes much like addition and subtraction or multiplication and division in arithmetic or differentiation and integration in calculus.

The following discussion is limited to the hypothesis-testing procedures applied to a selected batch of parameters such as (1) the population mean, μ , and (2) the population proportion, π , for one-sample tests of significance; (3) the difference of two population means, $\mu_1 - \mu_2$, for independent samples and (4) the difference of two population proportions, $\pi_1 - \pi_2$, for two-sample tests.

A statistical hypothesis is a statement about the numerical value of a population parameter.

There are six steps involved in the hypothesis-testing procedure, as shown in Figure 1.

Step 1. State the null hypothesis, H_0

The null hypothesis is usually the hypothesis that the researcher wants to gather evidence against or is the hypothesis to be tested. The H_0 carries the symbol = ; or at

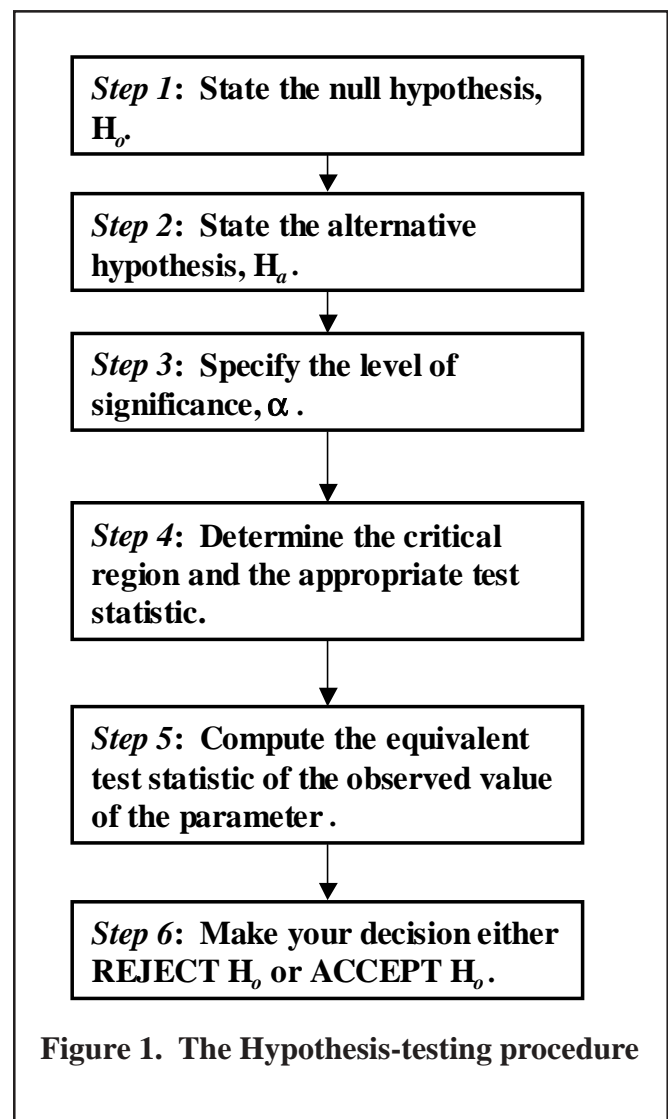


Figure 1. The Hypothesis-testing procedure

certain times bear two symbols such as \geq or \leq , thus, making H_o composite in nature.

Step 2: State the Alternative hypothesis, H_a .

The alternative hypothesis is usually the hypothesis for which the researcher wants to gather supporting evidence by way of observation that could be obtained from his sampling experiment.

The alternative hypothesis is the opposite of the null hypothesis. For a two tailed test, the symbol used in the H_a statement is \neq and for a one-tailed test, it is either $<$ or $>$. The alternative hypothesis carries *only one* symbol at all times.

Step 3: Specify the level of significance, α .

The significance level, denoted by α , is the probability of committing

a Type I Error: that of rejecting a null hypothesis which is, in reality, true.

Step 4: Determine the critical region and the appropriate test statistic.

The critical region is the rejection area for the null hypothesis. The rejection region covers a total area equal to α .

The appropriate test statistic is the standardized score of the sample statistic whose single critical value for a one-tailed test of significance or two critical values for a two-tailed test determines the boundary/boundaries between the acceptance and rejection regions for H_o . The test statistic could be Z , t , χ^2 , F , etc. depending on the appropriate sampling distribution of the sample statistic to be used. The critical value of the test statistic can be read from the statistical tables.

Step 5: Compute the equivalent test statistic of the observed value of the parameter.

The computed value of the test statistic can be generated from an appropriate formula as determined from the given conditions of the statistical experiment.

Step 6. Make your decision either REJECT H_o or ACCEPT H_o .

The null hypothesis, H_o , will be rejected if the computed value of the test statistic in STEP 5 falls within the rejection region or accepted, if it falls within the acceptance region for H_o .

The following table will summarize the symbols and terminologies to be used:

Measure of characteristic	Parameter (population measure)	Statistic (point estimator of parameter)	Standard error of the sample statistic
1. Mean *	μ	\bar{x}	$\sigma_{\bar{x}}$ or $s_{\bar{x}}$
2. Proporation*	π	p	σ_p
3. Difference of 2 mean**	$\mu_1 - \mu_2$	$\bar{x}_1 - \bar{x}_2$	$\sigma_{\bar{x}_1 - \bar{x}_2}$ OR $s_{\bar{x}_1 - \bar{x}_2}$
4. Difference of 2 proportions**	$\pi_1 - \pi_2$	$p_1 - p_2$	$\sigma_{p_1 - p_2}$

* applies to one-sample tests

** applies to two-sample tests

A. HYPOTHESIS –TESTING for μ :

Case 1. If the population standard deviation, σ , is known, use the standardized test statistic Z .

Illustrative Problem

The Supreme Restaurant chain claims that the waiting time of customers for service is normally distributed with a mean of 3.5 minutes and a standard deviation of 1.3 minutes. The quality – assurance department found in a sample of 40 customers at the hamburger branch in Malate that the mean waiting time was 4.8 minutes. Test the claim on the mean waiting time at the 0.05 significance level.

Given: $\mu_0 = 3.5$ minutes, $\sigma = 1.3$ minutes,
 $n = 40, \bar{x} = 4.8$ minutes, $\alpha = 0.05$

The symbol μ_0 represents the hypothesized or claimed population mean or true mean. There is no direction implied in the problem with regard to the parameter of interest which is the population mean waiting time, thus a two-tailed test will have to be conducted.

Compute the standard error of the mean $\sigma_{\bar{x}}$.

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.3 \text{ min.}}{\sqrt{40}} = 0.2055 \text{ min.}$$

Two-tailed test on μ :

Step1. $H_o : \mu = 3.5$ min.

The true or population mean waiting time of customers at the Supreme restaurant is 3.5 minutes.

Step 2. $H_a : \mu \neq 3.5$ min.

The true mean waiting time of customers at the Supreme restaurant is not 3.5 minutes.

Step 3. $\alpha = 0.05$

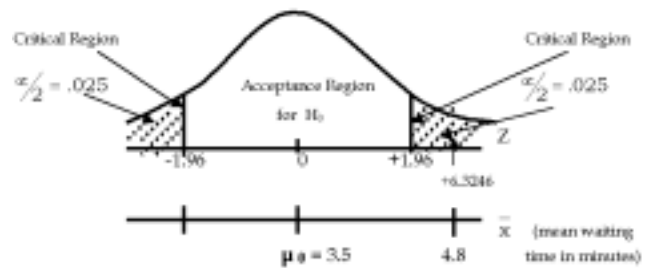
Step 4. $Crz = \pm z_{\alpha/2} = \pm z_{.025} = \pm 1.96$

Step 5 Computed

$$z = \frac{\bar{x} - \mu_o}{\sigma_{\bar{x}}} = \frac{4.8 \text{ min.} - 3.5 \text{ min.}}{0.2055 \text{ min.}} = +6.3246$$

Step 6. Decision: REJECT H_o

Sample evidence shows that the true mean waiting time of customers at the Supreme Restaurant is not 3.5 minutes, at $\alpha = 0.05$.



Case 2. If the sample standard deviation, s , is known, or could be known, use the test statistic t .

Illustrative Problem

Farm and power equipment dealers are typically dependent on a primary supplier organization for many of their business needs. These suppliers often demand control over many of the dealers’ decisions. To determine the degree to which dealers are dependent on suppliers, a survey of 12 farm and power equipment dealers was conducted. The study revealed the following data on the total number of suppliers engaged by the dealers:

4, 3, 2, 2, 3, 5, 2, 3, 3, 4, 4, 6.

Test the hypothesis that the true mean number of suppliers engaged by farm and power equipment dealers exceeds 3, using a 0.01 significance level.

Given: $\mu_0 = 3$ suppliers, $n = 12$,

$\bar{x} = 3.4167$ suppliers, $s = 1.2401$ suppliers

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = 0.3580 \text{ supplier}$$

The \bar{x} and s values are calculator-generated. The existence of the word “exceeds” referring to $\mu_0 = 3$ suggests a one-tailed or directional test.

One-tailed test on μ ;

Step 1. $H_o: \mu = 3$ suppliers

The true mean number of suppliers is 3.

Step 2. $H_a: \mu > 3$

The true mean number of suppliers exceeds 3.

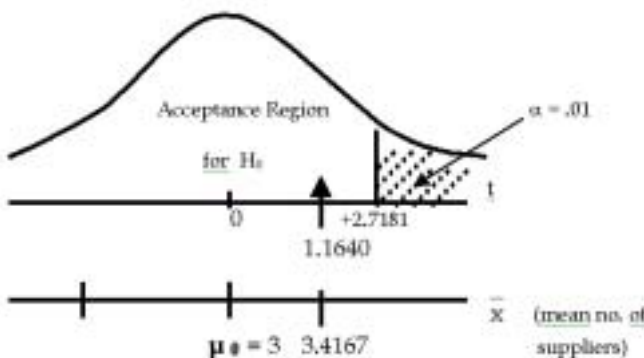
The symbol $>$ is determined by comparing the observed value of the mean \bar{x} to the hypothesized mean value, μ_0 .

Step 3. $\alpha = 0.01$

Step 4. $Cr\ t = +t_{df,\alpha} = +t_{11,01} = +2.7181$

There is only one critical value since this is a one-tailed test. The sign of $Cr\ t$ is $+$ because the critical region is singly located at the upper tail area of the t - distribution. The df or degree of freedom is given by $(n - 1)$.

Step 5. Computed $t = \frac{\bar{x} - \mu_0}{s_{\bar{x}}} = \frac{3.4167 - 3}{0.3580} = 1.1640$



Step 6. Decision: ACCEPT H_o .

∴ Sample evidence shows that the true mean number of suppliers engaged by the dealers is 3, at $\alpha = 0.01$. The population average number of suppliers does not exceed 3.

B. HYPOTHESIS –TESTING on the difference of two population means, $\mu_1 - \mu_2$, for independent samples:

Case 1. If the population standard deviation, σ_1 and σ_2 are known, use the **z**.

Illustrative Problem

The personnel department of Company A wishes to compare the efficiency of workers who are native-born with those who are foreign-born. Company records show that the standard deviations for outputs of native-born and foreign-born workers are 8 and 10, respectively. The company takes a sample of 32 native-born workers with a mean output of 50 units per worker, and a sample of 38 foreign-born workers with a mean output of 40 units per worker. Test to determine which group is more productive, at $\alpha = 0.01$.

Given:

Native Born Workers	Foreign-born Workers
$n_1 = 32$	$n_2 = 38$
$\bar{x}_1 = 50$ units	$\bar{x}_2 = 40$ units
$\sigma_1 = 8$ units	$\sigma_2 = 10$ units

The one-tailed will be done due to the presence of the words “more productive”.

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} = \sqrt{\frac{8^2}{32} + \frac{(10)^2}{38}} = 2.1521$$

One-tailed test on $\mu_1 - \mu_2$:

Step 1. $H_0 : \mu_1 - \mu_2 = 0$ (or $H_0 : \mu_1 = \mu_2$)

There is no significant difference between the true mean outputs of native-born and foreign-born workers.

Step 2. $H_a : \mu_1 - \mu_2 > 0$ (or $H_a : \mu_1 > \mu_2$)

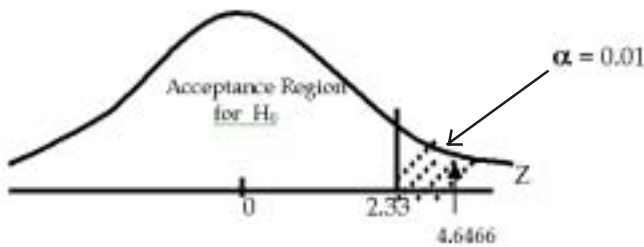
The true mean output of native-born workers is higher than that of foreign-born workers.

The symbol $>$ in H_a is determined by comparing H_a to \bar{x}_2 .

Step 3. $\alpha = 0.01$

Step 4. $Crz = +z_\alpha = z_{.01} = +2.33$

Step 5. Computed: $z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}} = \frac{50 - 40}{2.1521} = 4.6466$



Step 6. Decision: REJECT H_0 .

\therefore Sample evidence shows that the true mean output of native-born workers is higher than that of foreign-born workers, at $\alpha = 0.01$. Hence, the native-born workers are more efficient than the foreign-born workers.

Case 2 . If the sample standard deviations, S_1 and S_2 are known and if the samples come from populations having equal standard deviations or variances, use t .

Illustrative Problem

As a result of recent advances in educational telecommunications, many colleges and universities are utilizing instruction by interactive television for “distance” education. For example, each semester, Ball State University televises six graduate business courses to students at remote off-campus sites (Journal of Education for Business, Jan/Feb. 1991). To compare the performance of the off-campus MBA students at Ball State (who take the televised classes) to the on-campus MBA students (who have a “live” professor), a test devised by the Assembly of Collegiate Schools of Business (AACSB) was administered to a sample of both groups of students. (The test included seven exams covering accounting, business strategy, finance, human resources, marketing, management information systems, and production and operations management.) The AACSB test scores (50 points maximum) are summarized in the table. Based on these results, the researchers report that “there was no significant difference between the two groups of students.”

	Mean	Standard Deviation
On-Campus Students	41.93	2.86
Off-Campus Students	44.56	1.42

Source: Arndt, T.L. and LaFollette, W. R. “Interactive television and the non-traditional student.” Journal of Education for Business, Jan./Feb. 1991, p.184

Note the sample sizes were not given in the journal article. Assuming 40 students are sampled from each group, perform the desired analysis. Do you agree with the researchers’ findings? Use $\alpha = 0.05$.

Given:

On-Campus Students	Off-Campus Students
$n_1 = 40$	$n_2 = 40$
$\bar{x}_1 = 41.93$	$\bar{x}_2 = 44.56$
$s_1 = 2.86$	$s_2 = 1.42$

$$\begin{aligned}
 S_{\bar{x}_1 - \bar{x}_2} &= \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} \\
 &= \sqrt{\frac{(40 - 1)(2.86)^2 + (40 - 1)(1.42)^2}{40 + 40 - 2} \left(\frac{1}{40} + \frac{1}{40} \right)} \\
 &= 0.5049
 \end{aligned}$$

Two-tailed test on $\mu_1 - \mu_2$

Step 1. $H_0: \mu_1 = \mu_2$

The true mean scores of the On-Campus students and Off-Campus TV students are equal.

Step 2. $H_a: \mu_1 \neq \mu_2$

The true mean scores of the On-Campus students and Off-Campus TV students are not equal.

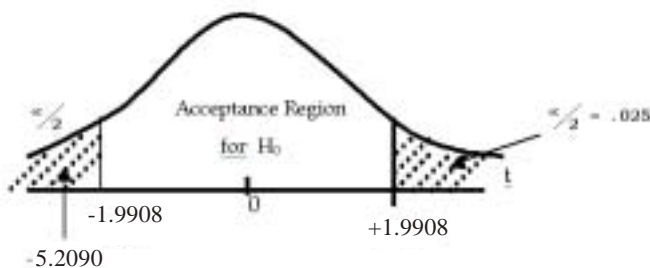
Step 3. $\alpha = 0.05$

Step 4. $\text{Cr } t = \pm t_{\frac{\alpha}{2}, \text{df}} = \pm t_{0.025, 78} = \pm 1.9908$

The degree of freedom for 2 samples is $(n_1 + n_2 - 2)$. Hence, $\text{df} = 40 + 40 - 2 = 78$

Step 5. Computed

$$t = \frac{\bar{x}_1 - \bar{x}_2}{S_{\bar{x}_1 - \bar{x}_2}} = \frac{41.93 - 44.56}{0.5049} = -5.2090$$



Step 6. Decision: REJECT H_0 .

\therefore Sample evidence shows that the true mean scores of the On-Campus students and Off-Campus TV students are not equal, at $\alpha = 0.05$. Or, there is a significant difference between the true mean scores of the two groups.

C. HYPOTHESIS-TESTING for one population proportion, π :

The following testing procedure applies to binomial or dichotomous distributions and to samples which are large, i.e., $n \geq 30$.

Illustrative Problem

The Red Grape Wine Company buys thousands of boxes of grapes each year. The Red Grape buyer visits vineyards and samples grapes on the vine. If the sample convinces the buyer that at least 80% of the total crop of a vineyard is of high wine-making quality, the buyer purchases the entire crop. Would the buyer recommend the purchase of a crop if a random sample of 600 grapes contains 468 high quality grapes? Use $\alpha = 0.05$.

Given:

$$\pi_0 = 0.80, \quad x = 468, \quad n = 600$$

$$p = \frac{x}{n} = \frac{468}{600} = 0.78$$

$$\sigma_p = \sqrt{\frac{\pi_0(1 - \pi_0)}{n}} = \sqrt{\frac{(0.80)(0.20)}{600}} = 0.0163$$

x : number of successes in the sample

n : sample size

π_0 : hypothesized population proportion or percentage

p : proportion of successes in the sample

σ_p : standard error of the proportion

A one-tailed test on π will be conducted for this problem due to the presence of the words "at least 80%".

One-tailed test on π :

Step 1. $H_0: \pi \geq 0.80$

At least 85% of the total crop of a vineyard is of high wine-making quality.

Step 2. $H_a: \pi < 0.80$

Less than 80% of the total crop of a vineyard is of high wine-making quality.

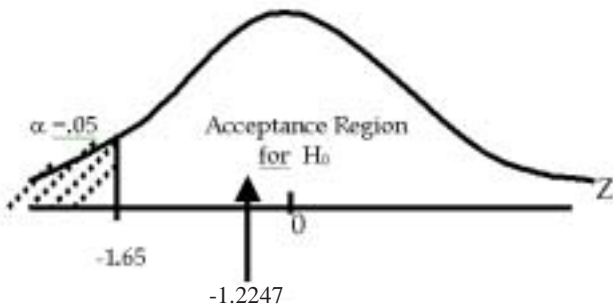
The symbol $<$ in the H_a statement is determined by comparing p to π_0 . H_0 is the opposite of H_a , hence we may complete the symbols in H_0 with \geq .

Step 3. $\alpha = 0.05$

Step 4. $Cr\ z = -z_\alpha = z_{.05} = -1.65$

Step 5. Computed

$$z = \frac{p - \pi_0}{\sigma_p} = \frac{0.78 - 0.80}{0.0163} = -1.2247$$



Step 6. Decision: ACCEPT H_0 .

\therefore Sample evidence shows that at least 80% of the total crop of a vineyard is of high wine-making quality, at $\alpha = 0.05$. Hence, the buyer will purchase the entire crop.

D. HYPOTHESIS –TESTING for the difference of two population proportions, π_1 and π_2

This applies to two-sample cases which have binomial distributions and have large ($n \geq 30$) sample sizes.

Illustrative Problem

A random sample of 200 men was selected from Metro Manila and 40 were found to be in favor of the divorce bill. A random sample of 250 women selected from the same place at the same time revealed that 85 were in favor of such a new bill. Is the proportion of women favoring the divorce bill higher than that of men in Metro Manila, given that $\alpha = 0.10$?

Given:

Men	Women
$x_1 = 40$	$x_2 = 85$
$n_1 = 200$	$n_2 = 250$
$p_1 = 0.2$	$p_2 = 0.34$

Compute π_0 , the combined estimate of the two proportions.

$$\pi_0 = \frac{x_1 + x_2}{n_1 + n_2} = \frac{40 + 85}{200 + 250} = 0.2778$$

Compute $\sigma_{p_1 - p_2}$, the standard error of the difference of two sample proportions.

$$\begin{aligned} \sigma_{p_1 - p_2} &= \sqrt{\pi_0 (1 - \pi_0) \left(\frac{1}{n_1} + \frac{1}{n_2} \right)} \\ &= \sqrt{0.2778 (1 - 0.2778) \left(\frac{1}{200} + \frac{1}{250} \right)} \\ &= 0.0425 \end{aligned}$$

This is a one-tailed test problem, as determined from the directional words: “higher than”.

One-tailed test on $\pi_1 - \pi_2$:

Step 1. $H_0: \pi_1 = \pi_2$

The proportions of men and women in Metro Manila who are in favor of the divorce bill are the same.

Step 2. $H_a: \pi_1 < \pi_2$

The proportion of men in Metro Manila who are in favor of the divorce law is lower than that of women.

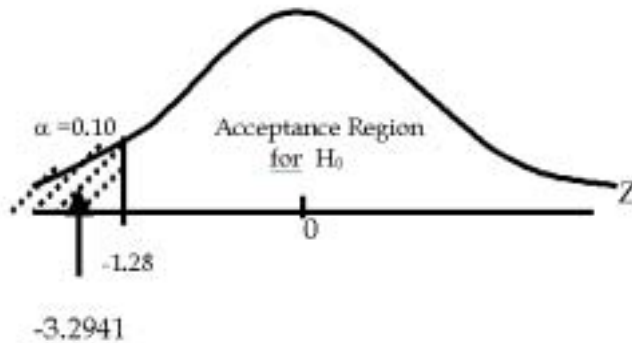
The symbol $<$ in H_a is determined by comparing p_1 to p_2 .

Step 3. $\alpha = 0.10$.

Step 4. $Cr\ z = -z_\alpha = -z_{0.10} = -1.28$

Step 5. Computed

$$z = \frac{p_1 - p_2}{\sigma_{p_1 - p_2}} = \frac{0.2 - 0.34}{0.0425} = -3.2941$$



Step 6. Decision: REJECT H_0

∴ Sample evidence shows that the proportion of men in Metro Manila who are in favor of the divorce bill is lower than that of women, at $\alpha = 0.10$.

The foregoing discussion is hoped to provide a useful tool and guide for the teachers and students of basic statistics. The above presentation was made in a compact manner. However, the author looks forward to a more detailed one for future uses.

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